Kuta Software - Infinite Algebra 2

Name______________________________

Function Inverses

Date________________ Period____

State if the given functions are inverses.

1) $g(x) = 4 - \frac{3}{2}x$

$f(x) = \frac{1}{2}x + \frac{3}{2}$

2) $g(n) = \frac{-12 - 2n}{3}$

$f(n) = \frac{-5 + 6n}{5}$

3) $f(n) = \frac{-16 + n}{4}$

$g(n) = 4n + 16$

4) $f(x) = -\frac{4}{7}x - \frac{16}{7}$

$g(x) = \frac{3}{2}x - \frac{3}{2}$

5) $f(n) = -(n + 1)^3$

$g(n) = 3 + n^3$

6) $f(n) = 2(n - 2)^3$

$g(n) = \frac{4 + \sqrt[3]{4n}}{2}$

7) $f(x) = \frac{4}{-x - 2} + 2$

$h(x) = -\frac{1}{x + 3}$

8) $g(x) = -\frac{2}{x} - 1$

$f(x) = -\frac{2}{x + 1}$

Find the inverse of each function.

9) $h(x) = \sqrt[3]{x} - 3$

10) $g(x) = \frac{1}{x} - 2$

11) $h(x) = 2x^3 + 3$

12) $g(x) = -4x + 1$

13) $g(x) = \dfrac{7x + 18}{2}$

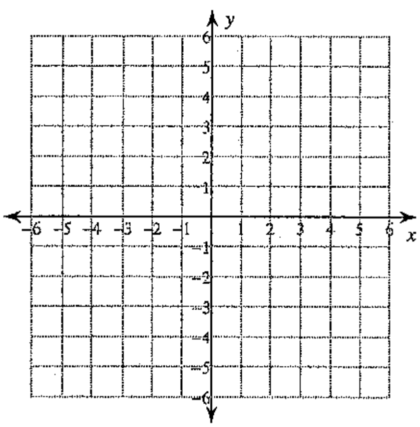
14) $f(x) = x + 3$

15) $f(x) = -x + 3$

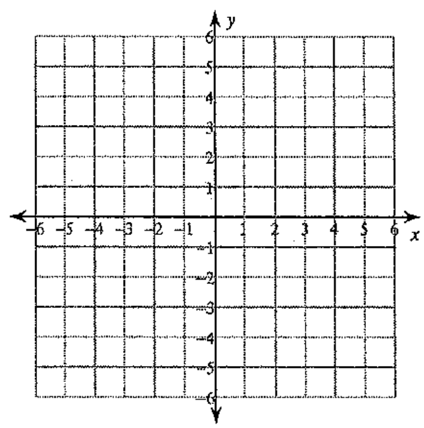
16) $f(x) = 4x$

Find the inverse of each function. Then graph the function and its inverse.

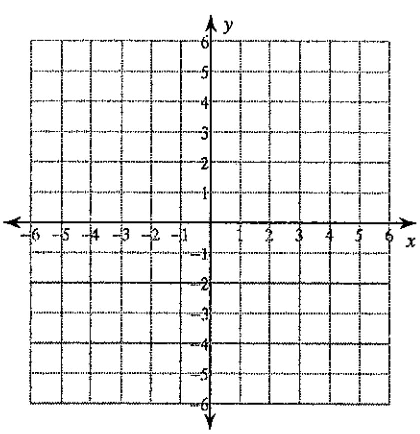
17) $f(x) = -1 - \dfrac{1}{5}x$



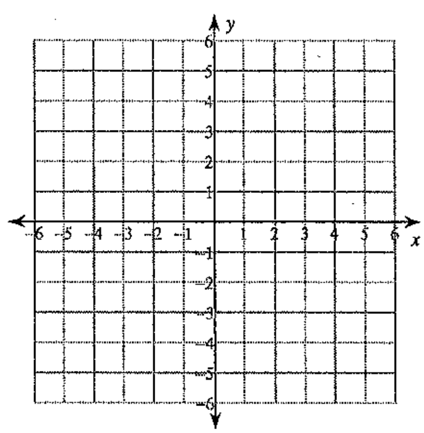
18) $g(x) = \dfrac{1}{x - 1}$



19) $f(x) = -2x^3 + 1$



20) $g(x) = \dfrac{-x - 5}{3}$



Day #

Date:

CLASSWORK

OBJECTIVE: Section 5.5—INVERSE FUNCTIONS

DEFINITION OF INVERSE FUNCTIONS...

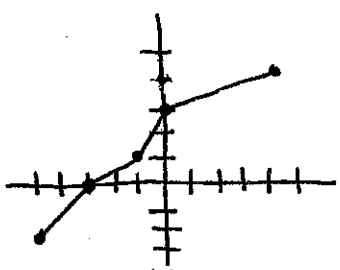
$f(x)=x+3$
$g(x)=x-3$

$f(x)=x^2$
$g(x)=\sqrt{x}$

INVERSE NOTATION:

Graphs of inverse functions...

aph the inverse:



TEST to see if functions are inverses of each other:

Are these functions inverses?

$f(x) = 2x - 5$
$g(x) = \frac{1}{2}x + \frac{5}{2}$

$f(x) = x^2 - 3$
$g(x) = \sqrt{x+3}$

$f(x) = 2x - 5$
$g(x) = \frac{1}{2}x + 5$

How do you FIND THE INVERSE Of a function?

Step 1:

Step 2:

Check-- (two options):

Find the inverse...

$f(x) = -\frac{1}{3}x + 1$

$g(x) = 2x - 1$

$h(x) = 2x^5 - 3$

Name________________________________

Function Inverses

Date________________ Period____

State if the given functions are inverses.

1) $g(x) = 4 - \frac{3}{2}x$

$f(x) = \frac{1}{2}x + \frac{3}{2}$

$f(g(x)) = \frac{1}{2}\left(4 - \frac{3}{2}x\right) + \frac{3}{2}$

$= 2 - \frac{3}{4}x + \frac{3}{2}$ $\boxed{\text{no!}}$

2) $g(n) = \frac{-12 - 2n}{3}$

$f(n) = \frac{-5 + 6n}{5}$

$f(g(n)) = \frac{-5 + 6\left(\frac{-12-2n}{3}\right)}{5}$

$= \frac{-5 + 2(-12 - 2n)}{5} = \frac{-5 - 24 - 4n}{5}$

$= \frac{-29 - 4n}{5}$ $\boxed{\text{no!}}$

3) $f(n) = \frac{-16 + n}{4}$

$g(n) = 4n + 16$

$f(g(n)) = \frac{-16 + 4n + 16}{4} = \frac{4n}{4} = n$ $\boxed{\text{yes!}}$

$g(f(n)) = 4\left(\frac{-16+n}{4}\right) + 16 = -16 + n + 16 = n$

4) $f(x) = -\frac{4}{7}x - \frac{16}{7}$

$g(x) = \frac{3}{2}x - \frac{3}{2}$ $\boxed{\text{no!}}$

$f(g(x)) = -\frac{4}{7}\left(\frac{3}{2}x - \frac{3}{2}\right) - \frac{16}{7}$

$= -\frac{12}{14}x + \frac{12}{14} - \frac{32}{14}$

5) $f(n) = -(n + 1)^3$

$g(n) = 3 + n^3$

$f(g(n)) = -(3 + n^3 + 1)^3$

$= -(n^3 + 4)^3$ $\boxed{\text{no!}}$

6) $f(n) = 2(n - 2)^3$ $\boxed{\text{yes!}}$

$g(n) = \frac{4 + \sqrt[3]{4n}}{2}$

$f(g(n)) = 2\left(\frac{4 + \sqrt[3]{4n}}{2} - \frac{4}{2}\right)^3$

$= 2\left(\frac{\cancel{4} + \sqrt[3]{4n} - \cancel{4}}{2}\right)^3 = 2\left(\frac{4n}{8}\right)$

$= \boxed{n}$

$g(f(n)) = \frac{4 + \sqrt[3]{4(2(n-2)^3)}}{2}$

$= \frac{4 + \sqrt[3]{8(n-2)^3}}{2}$

$= \frac{4 + 2(n-2)}{2}$

$= \frac{\cancel{4} + 2n - \cancel{4}}{2} = \boxed{n}$

7) $f(x) = \frac{4}{-x - 2} + 2$

$h(x) = -\frac{1}{x + 3}$

$h(f(x)) = \frac{1}{\left(\frac{4}{-x-2} + 2\right) + 3}$

$h(f(x)) = \frac{-1}{\frac{4}{-x-2} + 5}$

$f(h(x)) = \frac{4}{-\left(-\frac{1}{x+3}\right) - 2} + 2$ $\boxed{\text{no!}}$

$= \frac{4}{\frac{1}{x+3} - 2} + 2$

8) $g(x) = -\frac{2}{x} - 1$

$f(x) = -\frac{2}{x + 1}$

$f(g(x)) = -\frac{2}{\left(-\frac{2}{x} - 1\right) + 1}$

$= \frac{-2}{-\frac{2}{x} - 1 + 1} = \frac{-2}{-\frac{2}{x}} = \frac{-2}{1} \cdot \frac{-x}{2} = \boxed{x}$

$g(f(x)) = \frac{-2}{\frac{-2}{x+1}} - 1$

$= \frac{-2}{1} \cdot \frac{x+1}{-2} - 1 = x + 1 - 1 = x$

$\boxed{\text{yes!}}$

Find the inverse of each function.

9) $h(x) = \sqrt[3]{x} - 3$ $\boxed{h^{-1}(x) = (x + 3)^3}$

$y = \sqrt[3]{x} - 3$

$x = \sqrt[3]{y} - 3$

$x + 3 = \sqrt[3]{y}$

10) $g(x) = \frac{1}{x} - 2$

$y = \frac{1}{x} - 2$

$y(x + 2) = 1$

$x = \frac{1}{y} - 2$ $\boxed{g^{-1}(x) = \frac{1}{x + 2}}$

$\frac{x + 2}{1} = \frac{1}{y}$

11) $h(x) = 2x^3 + 3$

$y = 2x^3 + 3$

$x = 2y^3 + 3$ $\boxed{h^{-1}(x) = \sqrt[3]{\frac{x - 3}{2}}}$

$x - 3 = 2y^3$

$y^3 = \frac{x - 3}{2} = \left(\frac{x - 3}{2}\right)^{\frac{1}{3}}$

12) $g(x) = -4x + 1$

$y = -4x + 1$

$x = -4y + 1$

$x - 1 = -4y$

$\boxed{g^{-1}(x) = \frac{x - 1}{-4}}$ or $\boxed{-\frac{x}{4} + \frac{1}{4}}$

13) $g(x) = \frac{7x+18}{2}$

$y = \frac{7x+18}{2}$

$x = \frac{7y+18}{2}$

$2x = 7y + 18$

$2x - 18 = 7y$

$\boxed{\frac{2x-18}{7} = y = f^{-1}(x)}$

14) $f(x) = x + 3$

$x = y + 3$

$\boxed{f^{-1}(x) = x - 3}$

15) $f(x) = -x + 3$

$x = -y + 3$

$x - 3 = -y$

$\boxed{f^{-1}(x) = -x + 3}$

16) $f(x) = 4x$

$x = 4y$

$\boxed{y = \frac{x}{4}}$

Find the inverse of each function. Then graph the function and its inverse.

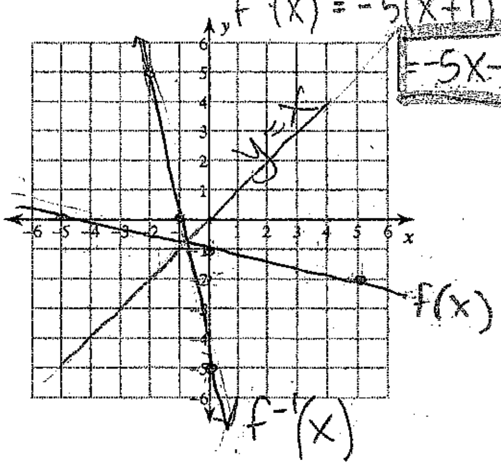
17) $f(x) = -1 - \frac{1}{5}x$

$x = -1 - \frac{1}{5}y$

$x + 1 = -\frac{1}{5}y$

$f^{-1}(x) = -5(x+1)$

$\boxed{-5x - 5}$



18) $g(x) = \frac{1}{x-1}$

$\frac{x}{1} = \frac{1}{y-1}$

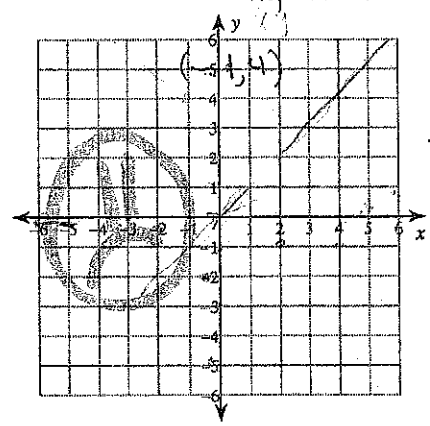
$x(y-1) = 1$

$xy - x = 1$

$xy = 1 + x$

$y = \frac{1+x}{x}$

$\boxed{y = \frac{1}{x} + 1}$



x	g(x)	$g^{-1}(x)$
-2	$-\frac{1}{3}$	$(-\frac{1}{3}, -2)$
0	-1	(-1, 0)
2	1	(1, 2)
-5	$-\frac{1}{6}$	$(-\frac{1}{6}, -5)$
5	$\frac{1}{4}$	$(\frac{1}{4}, 5)$

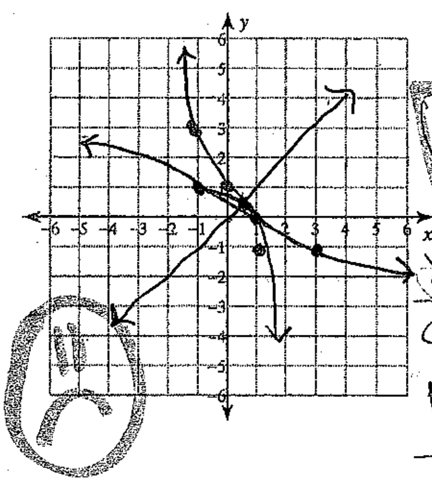
19) $f(x) = -2x^3 + 1$

$x = -2y^3 + 1$

$x - 1 = -2y^3$

$\frac{x-1}{-2} = y^3$

$\boxed{\sqrt[3]{\frac{x-1}{-2}} = y}$



x	f(x)	$f^{-1}(x)$
0	1	(1, 0)
1	-1	(-1, 1)
-1	3	(3, -1)

20) $g(x) = \frac{-x-5}{3}$

$-\frac{1}{3}x - \frac{5}{3}$

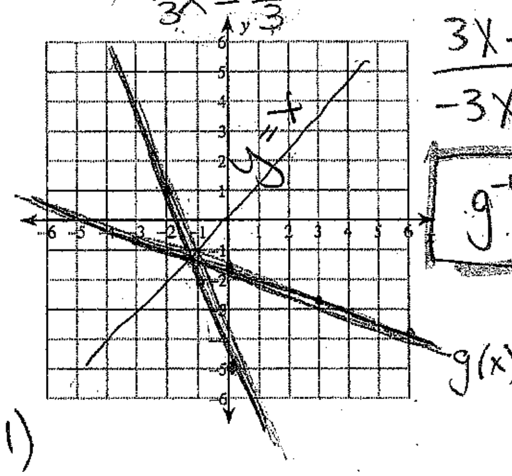
$x = \frac{-y-5}{3}$

$3x = -y - 5$

$3x + 5 = -y$

$-3x - 5 = y$

$\boxed{g^{-1}(x) = -3x - 5}$



Day #

Date:

CLASSWORK

OBJECTIVE: Section 5.5—INVERSE FUNCTIONS

DEFINITION OF INVERSE FUNCTIONS...

Inverse functions "undo" each other...

$f(x)=x+3$ (adds on 3)
$g(x)=x-3$ (takes off 3)

$f(x)=x^2$ squares x
$g(x)=\sqrt{x}$ unsquares x

(only works on positive x's!)

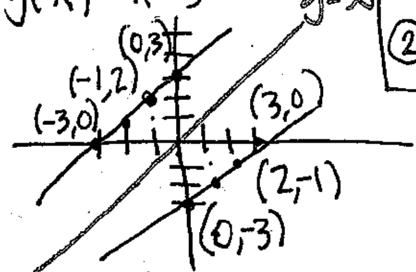
INVERSE NOTATION:

the inverse of $f(x)$ is $f^{-1}(x)$

-1 is not an exponent. Read: "f inverse of x"

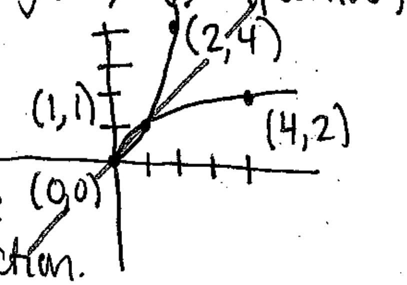
Graphs of inverse functions...

$f(x)=x+3$
$g(x)=x-3$



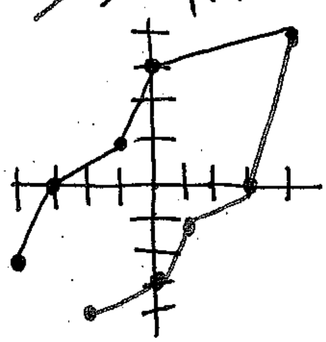
① reflection over $y=x$
② each $(x,y)\to(y,x)$

$f(x)=x^2$
$g(x)=\sqrt{x}$ (positive)



If we looked at all x values, the inverse wouldn't be a function.

Graph the inverse:



TEST to see if functions are inverses of each other:

If $g(x)$ and $f(x)$ are inverses, then $g(f(x))=f(g(x))=\boxed{x}$

If $f(x)=x+3$
$g(x)=x-3$

① $f(g(x))=(x-3)+3$
$=\boxed{x}$

② $g(f(x))=(x+3)-3$
$=\boxed{x}$

If $f(x)=x^2$
$g(x)=\sqrt{x}$

② $f(g(x))=(\sqrt{x})^2$
$=x$

$g(f(x))=\sqrt{x^2}$
$=x$

Are these functions inverses?

$f(x) = 2x - 5$
$g(x) = \frac{1}{2}x + \frac{5}{2}$

$f(g(x)) = 2\left(\frac{1}{2}x + \frac{5}{2}\right) - 5$
$= x + 5 - 5$
$= x$

$g(f(x)) = \frac{1}{2}(2x-5) + \frac{5}{2}$
$= x - \frac{5}{2} + \frac{5}{2}$
$= x$

$\boxed{\text{yes!}}$

$f(x) = x^2 - 3$
$g(x) = \sqrt{x+3}$

$f(g(x)) = (\sqrt{x+3})^2 - 3$
$= x + 3 - 3$
$= x$

$g(f(x)) = \sqrt{x^2 - 3 + 3}$
$= \sqrt{x^2}$
$= x$

$\boxed{\text{yes!}}$

$f(x) = 2x - 5$
$g(x) = \frac{1}{2}x + 5$

$f(g(x)) = 2\left(\frac{1}{2}x + 5\right) - 5$
$= x + 10 - 5$
$= x + 5$ no!

How do you FIND THE INVERSE Of a function?

Step 1: Swith the x and y around

Step 2: Solve for y

Check-- (two options): ① graph each (are they reflections over $y = x$?
② find $f(g(x))$ and $g(f(x))$: do they both equal x?

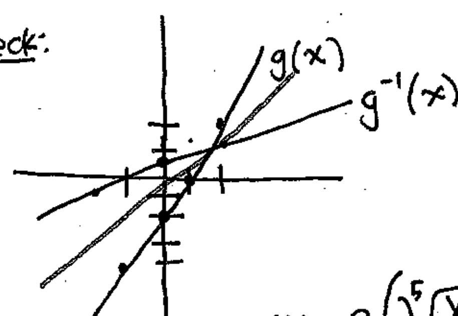
Find the inverse...

$f(x) = -\frac{1}{3}x + 1$
$y = -\frac{1}{3}x + 1$
$x = -\frac{1}{3}y + 1$
$-3\,(x-1) = -\frac{1}{3}y \;\cdot -3$
$\boxed{y = -3x + 3} = f^{-1}(x)$

Check: $f(f^{-1}(x)) = -\frac{1}{3}(-3x+3) + 1$
$= x - 1 + 1 = x$
$f^{-1}(f(x)) = -3\left(-\frac{1}{3}x + 1\right) + 3$
$= x - 3 + 3 = x$

$g(x) = 2x - 1$
$y = 2x - 1$
$x = 2y - 1$
$x + 1 = 2y$
$y = \frac{x+1}{2}$
$f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

Check:



$h(x) = 2x^5 - 3$
$y = 2x^5 - 3$
$x = 2y^5 - 3$
$\frac{x+3}{2} = \frac{2y^5}{2}$
$y^5 = \frac{x+3}{2}$ $\quad y = \sqrt[5]{\frac{x+3}{2}}$

Check: $f(f^{-1}(x)) = 2\left(\sqrt[5]{\frac{x+3}{2}}\right)^5 - 3$
$= 2\left(\frac{x+3}{2}\right) - 3 = x + 3 - 3 = \boxed{x}$

$f^{-1}(f(x)) = \sqrt[5]{\frac{2x^5 - 3 + 3}{2}} = \sqrt[5]{\frac{2x^5}{2}} = \sqrt[5]{x^5} = \boxed{x}$