Theoretical and Experimental Probability

Theoretical probability of an event.

In the game of Scrabble®, players use tiles bearing the letters of the alphabet to form words. Of the 100 tiles used in a Scrabble game, 12 have the letter *E* on them. What is the probability of drawing an *E* from a bag of 100 Scrabble tiles?

In this case, pulling an *E* from the bag is called a *favorable outcome.* A **favorable outcome** is an outcome that you are looking for when you conduct an experiment.

To find the probability of drawing an *E,* you can draw tiles from a bag and record your results, or you can find the *theoretical probability*. **Theoretical probability** is used to estimate the probability of an event when all the outcomes are equally likely.

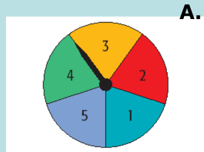
THEORETICAL PROBABILITY
probability = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$

If each possible outcome of an experiment is equally likely, then the experiment is said to be **fair**. Experiments involving number cubes and coins are usually assumed to be fair.

You can write probability as a fraction, a decimal, or a percent.

Additional Example 1A: Calculating Theoretical Probability

An experiment consists of spinning this spinner once.



A. What is the probability of spinning a 4?

The spinner is fair, so all 5 outcomes are equally likely. The probability of spinning a 4 is $P(4) = \frac{1}{5}$.

Additional Example 1B: Calculating Theoretical Probability

An experiment consists of spinning this spinner once.

B. What is the probability of spinning an even number?

There are 2 outcomes in the event of spinning an even number: 2 and 4.

$P(\text{spinning an even number}) = \dfrac{\text{number of possible even numbers}}{5}$

$= \dfrac{2}{5}$

Additional Example 1C: Calculating Theoretical Probability

An experiment consists of spinning this spinner once.

C. What is the probability of spinning a number less than 4?

There are 3 outcomes in the event of spinning a number less than 4: 1, 2, and 3.

$P(\text{spinning a number less than 4}) = \dfrac{3}{5}$

Try This: Example 1A

An experiment consists of spinning this spinner once.

A. What is the probability of spinning a 1?

The spinner is fair, so all 5 outcomes are equally likely. The probability of spinning a 1 is $P(1) = \dfrac{1}{5}$.

Try This: Example 1B

An experiment consists of spinning this spinner once.

B. What is the probability of spinning an odd number?

There are 3 outcomes in the event of spinning an odd number: 1, 3, and 5.

$P(\text{spinning an odd number}) = \dfrac{\text{number of possible odd numbers}}{5}$

$= \dfrac{3}{5}$

Try This: Example 1C

An experiment consists of spinning this spinner once.

C. What is the probability of spinning a number less than 3?

There are 2 outcomes in the event of spinning a number less than 3: 1 and 2.

$P(\text{spinning a number less than 3}) = \dfrac{2}{5}$

Additional Example 1A: Finding Theoretical Probability

Find the probability. Write your answer as a fraction, as a decimal, and as a percent

A. Andy has 20 marbles in a bag. Of these, 9 are clear. What is the probability of drawing a clear marble from the bag?

$P = \dfrac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$

$P(\text{clear}) = \dfrac{\text{number of clear marbles}}{\text{total number of marbles}}$ *Write the ratio.*

$= \dfrac{9}{20}$ *Substitute.*

$= 0.45 = 45\%$ *Write as a decimal and write as a percent.*

The theoretical probability of drawing a clear marble is $\dfrac{9}{20}$, 0.45, or 45%.

Additional Example 1B: Finding Theoretical Probability

B. What is the probability of rolling a number less than 4 on a fair number cube.

For a fair number cube, each of the six possible outcomes is equally likely. There are 3 ways to roll a number less than 4: 1, 2, or 3.

$$P = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

$$P(\text{number less than 4}) = \frac{\text{3 numbers less than 4}}{\text{6 possible outcomes}}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

$$= 0.5 = 50\%$$

The theoretical probability of rolling a number less than 4 is $\frac{1}{2}$, 0.50, or 50%.

Try This: Example 1A

Find the probability. Write your answer as a fraction, as a decimal, and as a percent.

A. Jane has 20 marbles in a bag. Of these 8 are green. What is the probability of drawing a green marble from the bag?

$$P = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

$$P(\text{green}) = \frac{\text{number of green marbles}}{\text{total number of marbles}}$$ *Write the ratio.*

$$= \frac{8}{20}$$ *Substitute.*

$$= 0.4 = 40\%$$ *Write as a decimal and write as a percent.*

The theoretical probability of drawing a green marble is $\frac{8}{20}$, 0.4, or 40%.

Try This: Example 1B

B. What is the probability of rolling a number more than 4 on a fair number cube.

For a fair number cube, each of the six possible outcomes is equally likely. There are 2 ways to roll a number greater than 4: 5 or 6.

$$P = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

$$P(\text{number more than 4}) = \frac{\text{2 numbers more than 4}}{\text{6 possible outcomes}}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

$$\approx 0.33 \approx 33\%$$

The theoretical probability of rolling a number more than 4 is $\frac{1}{3}$, 0.33, or 33%.

Additional Example 2A: *School Application*

The coach has written the names of each of the track team members on an index card. She draws randomly from these cards to choose a student to run a sprint and then replaces the card in the stack.

A. If there are 13 boys and 10 girls on the team, what is the theoretical probability that a girl's name will be drawn?

$$P(\text{girl}) = \frac{\text{number of girls on the team}}{\text{number of members on the team}}$$ *Find the theoretical probability.*

$$= \frac{10}{23}$$ *Substitute.*

Additional Example 2B: *School Application*

The coach has written the names of each of the track team members on an index card. She draws randomly from these cards to choose a student to run a sprint and then replaces the card in the stack.

B. If there are 13 boys and 10 girls on the team, what is the theoretical probability that a boy's name will be drawn?

$$P(\text{boy}) = \frac{\text{number of boys on the team}}{\text{number of members on the team}}$$ *Find the theoretical probability.*

$$= \frac{13}{23}$$ *Substitute.*

Try This: Example 2A

A teacher has written the name of each student on a piece of paper and placed the names in a box. She randomly draws a paper from the box to determine which student will present the answer to the problem of the day.

A. If there are 15 boys and 12 girls in the class, what is the theoretical probability that a girl's name will be drawn?

$$P(\text{girl}) = \frac{\text{number of girls in the class}}{\text{number of students in the class}}$$ *Find the theoretical probability.*

$$= \frac{12}{27}$$ *Substitute.*

Try This: Example 2B

A teacher has written the name of each student on a piece of paper and placed the names in a box. She randomly draws a paper from the box to determine which student will present the answer to the problem of the day.

B. If there are 15 boys and 12 girls in the class, what is the theoretical probability that a boy's name will be drawn?

$P(\text{boy}) = \dfrac{\text{number of boys in the class}}{\text{number of students in the class}}$ *Find the theoretical probability.*

$= \dfrac{15}{27}$ *Substitute.*

Additional Example 2A: Calculating Theoretical Probability for a Fair Die and a Fair Coin

An experiment consists of rolling one fair die and flipping a coin.

A. Show a sample space that has all outcomes equally likely.

The outcome of rolling a 5 and flipping heads can be written as the ordered pair (5, H). There are 12 possible outcomes in the sample space.

1H 2H 3H 4H 5H 6H

1T 2T 3T 4T 5T 6T

Additional Example 2B: Calculating Theoretical Probability for a Fair Die and a Fair Coin

An experiment consists of rolling one fair die and flipping a coin.

B. What is the probability of getting tails?

There are 6 outcomes in the event "flipping tails": (1, T), (2, T), (3, T), (4, T), (5, T), and (6, T).

$P(\text{tails}) = \dfrac{6}{12} = \dfrac{1}{2}$

Additional Example 2C: Calculating Theoretical Probability for a Fair Die and a Fair Coin

An experiment consists of rolling one fair die and flipping a coin.

C. What is the probability of getting an even number and heads?

There are 3 outcomes in the event "getting an even number and heads": (2, H), (4, H), (6, H).

$P(\text{even number and heads}) = \dfrac{3}{12} = \dfrac{1}{4}$

Try This: Example 2A

An experiment consists of flipping two coins.

A. Show a sample space that has all outcomes equally likely.

The outcome of flipping two heads can be written as HH. There are 4 possible outcomes in the sample space.

HH TH HT TT

Try This: Example 2 Continued

An experiment consists of flipping two coins.

B. What is the probability of getting one head and one tail?

There are 2 outcomes in the event **"getting one head and getting one tail":** (H, T) and (T, H).

$P(\text{head and tail}) = \dfrac{2}{4} = \dfrac{1}{2}$

Try This: Example 2C

An experiment consists of flipping two coins.

C. What is the probability of getting heads on both coins?

There is 1 outcome in the event "both heads": (H, H).

$P(\text{both heads}) = \frac{1}{4}$

Try This: Example 2D

An experiment consists of flipping two coins.

D. What is the probability of getting both tails?

There is 1 outcome in the event "both tails": (T, T).

$P(\text{both tails}) = \frac{1}{4}$

Experimental probability of an event.

Experimental probability is one way of estimating the probability of an event. **It is based on actual experience or observations.**

Experimental probability is found by comparing the number of times an event occurs to the total number of **trials**, the times an experiment is carried out or an observation is made. The more trials you have, the more accurate the estimate is likely to be.

EXPERIMENTAL PROBABILITY
$\text{probability} \approx \frac{\text{number of times an event occurs}}{\text{total number of trials}}$

Additional Example 1: *Sports Application*

During skating practice, Sasha landed 7 out of 12 jumps. What is the experimental probability that she will land her next jump?

$$P \approx \frac{\text{number of times an event occurs}}{\text{total number of trials}}$$

$$P \approx \frac{\text{number of jumps landed}}{\text{number of jumps attempted}}$$

$$\approx \frac{7}{12}$$ *Substitute.*

The experimental probability that Sasha will land her next jump is $\frac{7}{12}$.

Try This: Example 1

During basketball practice, Martha made 9 out of 10 free throws. What is the experimental probability that she will make her next attempt?

$$P \approx \frac{\text{number of times an event occurs}}{\text{total number of trials}}$$

$$P \approx \frac{\text{number of free throws made}}{\text{number of free throws attempted}}$$

$$\approx \frac{9}{10}$$ *Substitute.*

The experimental probability that Martha will make the next free throw is $\frac{9}{10}$.

Writing Math

"*P*(event)" represents the probability that an event will occur. For example, the probability of a flipped coin landing heads up could be written as "*P*(heads)."

Recall that an outcome of an experiment can be impossible, certain, or in between. If an event is impossible, it will never happen in any trial.

$$P(\text{impossible event}) = \frac{0}{\text{total number of trials}}$$

$$= 0$$

0 divided by any number except 0 equals 0.

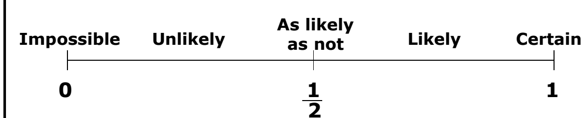
If an event is certain, it will always happen in every trial. This means that the number of times the event happens is equal to the total number of trials.

$$P(\text{certain event}) = \frac{\text{total number of trials}}{\text{total number of trials}}$$

$$= 1$$

Any number except 0 divided by itself equals 1.

All probabilities can be expressed numerically on a scale from 0 to 1.



Additional Example 2A: *Application*

Students have checked out 55 books from the library. Of these, 32 books are fiction.

A. What is the experimental probability that the next book checked out will be fiction?

$$P(\text{fiction}) \approx \frac{\text{number of fiction books checked out}}{\text{total number of books checked out}}$$

$$\approx \frac{32}{55}$$ *Substitute.*

The experimental probability that the next book checked out will be fiction is approximately $\frac{32}{55}$.

Additional Example 2B : *Application*

B. What is the experimental probability that the next book checked out will be nonfiction?

$$P(\text{nonfiction}) \approx \frac{\text{number of nonfiction books checked out}}{\text{total number of books checked out}}$$

$$\approx \frac{23}{55}$$ *Substitute.*

The experimental probability that the next book checked out will be nonfiction is approximately $\frac{23}{55}$.

Try This: Example 2A

Students have a fruit choice for lunch of an apple or a pear. So far 18 of 47 students have selected pears.

A. What is the experimental probability that the next fruit selected will be a pear?

$P(\text{pear}) \approx \dfrac{\text{number of pears selected}}{\text{total number of fruit selected}}$

$\approx \dfrac{18}{47}$ *Substitute.*

The experimental probability that the next fruit selected will be a pear is approximately $\mathbf{\dfrac{18}{47}}$.

Try This: Example 2B

B. What is the experimental probability that next fruit selected will be an apple?

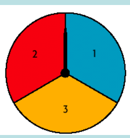
$P(\text{apple}) \approx \dfrac{\text{number of apples selected}}{\text{total number of fruit selected}}$

$\approx \dfrac{29}{47}$ *Substitute.*

The experimental probability that the next fruit selected will be an apple is approximately $\mathbf{\dfrac{29}{47}}$.

Additional Example 1A: Estimating the Probability of an Event

A. The table shows the results of 500 spins of a spinner. Estimate the probability of the spinner landing on 2.



Outcome	1	2	3
Spins	151	186	163

probability $\approx \dfrac{\text{number of spins that landed on 2}}{\text{total number of spins}} = \dfrac{186}{500}$

The probability of landing on 2 is about 0.372, or 37.2%.

Additional Example 1B: Estimating the Probability of an Event

B. A customs officer at the New York–Canada border noticed that of the 60 cars that he saw, 28 had New York license plates, 21 had Canadian license plates, and 11 had other license plates. Estimate the probability that a car will have Canadian license plates.

Outcome	New York	Canadian	Other
Observations	28	21	11

probability $\approx \dfrac{\text{number of Canadian license plates}}{\text{total number of license plates}} = \dfrac{21}{60}$

$= 0.35$

The probability that a car will have Canadian license plates is about 0.35, or 35%.

Try This: Example 1A

A. Jeff tosses a quarter 1000 times and finds that it lands heads 523 times. What is the probability that the next toss will land heads? Tails?

$P(\text{heads}) = \dfrac{523}{1000} = 0.523$

$P(\text{heads}) + P(\text{tails}) = 1$ *The probabilities must equal 1.*

$0.523 + P(\text{tails}) = 1$

$P(\text{tails}) = 0.477$

Try This: Example 1B

B. Josie sells TVs. On Monday she sold 13 plasma displays and 37 tube TVs. What is the probability that the first TV sold on Tuesday will be a plasma display? A tube TV?

probability $\approx \dfrac{\text{number of plasma displays}}{\text{total number of TVs}} = \dfrac{13}{13 + 37} = \dfrac{13}{50}$

$P(\text{plasma}) = 0.26$

$P(\text{plasma}) + P(\text{tube}) = 1$

$0.26 + P(\text{tube}) = 1$

$P(\text{tube}) = 0.74$

Additional Example 2: *Application*

Use the table to compare the probability that the Huskies will win their next game with the probability that the Knights will win their next game.

Team	Wins	Games
Huskies	79	138
Cougars	85	150
Knights	90	146

Additional Example 2 Continued

$$\text{probability} \approx \frac{\text{number of wins}}{\text{total number of games}}$$

probability for a Huskies win $\approx \frac{79}{138} \approx 0.572$

probability for a Knights win $\approx \frac{90}{146} \approx 0.616$

The Knights are more likely to win their next game than the Huskies.

Try This: Example 2

Use the table to compare the probability that the Huskies will win their next game with the probability that the Cougars will win their next game.

Team	Wins	Games
Huskies	79	138
Cougars	85	150
Knights	90	146

Try This: Example 2 Continued

$$\text{probability} \approx \frac{\text{number of wins}}{\text{total number of games}}$$

probability for a Huskies win $\approx \frac{79}{138} \approx 0.572$

probability for a Cougars win $\approx \frac{85}{150} \approx 0.567$

The Huskies are more likely to win their next game than the Cougars.

Experimental vs. Theoretical

Think about a single experiment, such as tossing a coin. There are two possible outcomes, heads or tails. What is *P*(heads) + *P*(tails)?

Experimental Probability (coin tossed 10 times)			Theoretical Probability
H	T		
//// /	////	$P(\text{heads}) = \frac{6}{10}$ $P(\text{tails}) = \frac{4}{10}$ $\frac{6}{10} + \frac{4}{10} = \frac{10}{10} = 1$	$P(\text{heads}) = \frac{1}{2}$ $P(\text{tails}) = \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$