



Arizona's Common Core Standards Mathematics

Standards - Mathematical Practices - Explanations and Examples
Second Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS

State Board Approved June 2010

August 2013 Publication

Second Grade Overview

Operations and Algebraic Thinking (OA)

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten (NBT)

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data (MD)

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry (G)

- Reason with shapes and their attributes.

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Second Grade: Mathematics Standards – Mathematical Practices – Explanations and Examples

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

- (1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
- (2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
- (3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
- (4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Operations and Algebraic Thinking (OA)

Represent and solve problems involving addition and subtraction.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.OA.A.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 1.)</p> <p>Connections: <i>2.NBT.5; 2.RI.3; 2.RI.4; 2.SL.2; ET02-S2C1-01</i></p>	<p><i>2.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>2.MP.4.</i> Model with mathematics.</p> <p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Word problems that are connected to students' lives can be used to develop fluency with addition and subtraction. Table 1 describes the four different addition and subtraction situations and their relationship to the position of the unknown.</p> <p>Examples:</p> <ul style="list-style-type: none"> • Take From example: David had 63 stickers. He gave 37 to Susan. How many stickers does David have now? $63 - 37 = \square$ • Add To example: David had \$37. His grandpa gave him some money for his birthday. Now he has \$63. How much money did David's grandpa give him? $\\$37 + \square = \\63 • Compare example: David has 63 stickers. Susan has 37 stickers. How many more stickers does David have than Susan? $63 - 37 = \square$ <ul style="list-style-type: none"> ○ Even though the modeling of the two problems above is different, the equation, $63 - 37 = ?$, can represent both situations (How many more do I need to make 63?) • Take From (Start Unknown) David had some stickers. He gave 37 to Susan. Now he has 26 stickers. How many stickers did David have before? $\square - 37 = 26$ <p>It is important to attend to the difficulty level of the problem situations in relation to the position of the unknown.</p> <ul style="list-style-type: none"> • Result Unknown, Total Unknown, and Both Addends Unknown problems are the least complex for students. • The next level of difficulty includes Change Unknown, Addend Unknown, and Difference Unknown • The most difficult are Start Unknown and versions of Bigger and Smaller Unknown (compare problems). <p>Second graders should work on ALL problem types regardless of the level of difficulty. Mastery is expected in second grade. Students can use interactive whiteboard or document camera to demonstrate and justify their thinking.</p> <p>This standard focuses on developing an algebraic representation of a word problem through addition and subtraction --the intent is not to introduce traditional algorithms or rules.</p>

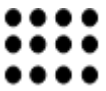

Operations and Algebraic Thinking (OA)

Add and subtract within 20.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.OA.B.2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. (See standard 1.OA.6 for a list of mental strategies.)</p> <p>Connections: 2.NBT.5; 2.NBT.9; ET02-S2C1-01</p>	<p>2.MP.2. Reason abstractly and quantitatively.</p> <p>2.MP.7. Look for and make use of structure.</p> <p>2.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>This standard is strongly connected to all the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 20. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.</p> <p>Mental strategies help students make sense of number relationships as they are adding and subtracting within 20. The ability to calculate mentally with efficiency is very important for all students.</p> <p>Mental strategies may include the following:</p> <ul style="list-style-type: none"> • Counting on • Making tens ($9 + 7 = 10 + 6$) • Decomposing a number leading to a ten ($14 - 6 = 14 - 4 - 2 = 10 - 2 = 8$) • Fact families ($8 + 5 = 13$ is the same as $13 - 8 = 5$) • Doubles • Doubles plus one ($7 + 8 = 7 + 7 + 1$) <p>However, the use of objects, diagrams, or interactive whiteboards, and various strategies will help students develop fluency.</p>

Operations and Algebraic Thinking (OA)

Work with equal groups of objects to gain foundations for multiplication.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.OA.C.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.</p> <p>Connections: <i>2.OA.4; 2.RI.3; 2.RI.4; ET02-S1C1-01; ET02-S2C1-01</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning</p>	<p>Students explore odd and even numbers in a variety of ways including the following: students may investigate if a number is odd or even by determining if the number of objects can be divided into two equal sets, arranged into pairs or counted by twos. After the above experiences, students may derive that they only need to look at the digit in the ones place to determine if a number is odd or even since any number of tens will always split into two even groups.</p> <p>Example:</p> <ul style="list-style-type: none"> Students need opportunities writing equations representing sums of two equal addends, such as: $2 + 2 = 4$, $3 + 3 = 6$, $5 + 5 = 10$, $6 + 6 = 12$, or $8 + 8 = 16$. This understanding will lay the foundation for multiplication and is closely connected to <i>2.OA.4</i>. <p>The use of objects and/or interactive whiteboards will help students develop and demonstrate various strategies to determine even and odd numbers.</p>
<p>2.OA.C.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.</p> <p>Connections: <i>2.OA.3, 2.RI.3; ET02-S1C2-01; ET02-S1C2-02; ET02-S2C1-01</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Students may arrange any set of objects into a rectangular array. Objects can be cubes, buttons, counters, etc. Objects do not have to be square to make an array.</p> <ul style="list-style-type: none"> Geoboards can also be used to demonstrate rectangular arrays. Students then write equations that represent the total as the sum of equal addends as shown below. <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>$4 + 4 + 4 = 12$</p> </div> <div style="text-align: center;">  <p>$5 + 5 + 5 + 5 = 20$</p> </div> </div> <p>Interactive whiteboards and document cameras may be used to help students visualize and create arrays.</p>

Number and Operations in Base Ten (NBT)

Understand place value.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.NBT.A.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:</p> <ol style="list-style-type: none"> 100 can be thought of as a bundle of ten tens—called a “hundred.” The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). <p>Connections: <i>2.NBT.5; 2.RI.3; 2.RI.4; 2.SL.3; ET02-S1C2-01; ET02-S1C2-01; ET02-S2C1-01</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Understanding that 10 ones make one ten and that 10 tens make one hundred is fundamental to students’ mathematical development. Students need multiple opportunities counting and “bundling” groups of tens in first grade. In second grade, students build on their understanding by making bundles of 100s with or without leftovers using base ten blocks, cubes in towers of 10, ten frames, etc. This emphasis on bundling hundreds will support students’ discovery of place value patterns.</p> <p>As students are representing the various amounts, it is important that emphasis is placed on the language associated with the quantity. For example, 243 can be expressed in multiple ways such as 2 groups of hundred, 4 groups of ten and 3 ones, as well as 24 tens and 3 ones. When students read numbers, they should read in standard form as well as using place value concepts. For example, 243 should be read as “two hundred forty-three” as well as two hundreds, 4 tens, 3 ones.</p> <p>A document camera or interactive whiteboard can also be used to demonstrate “bundling” of objects. This gives students the opportunity to communicate their thinking.</p>
<p>2.NBT.A.2. Count within 1000; skip-count by 5s, 10s, and 100s.</p> <p>Connections: <i>2.NBT.8; ET02-S1C3-01</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Students need many opportunities counting, up to 1000, from different starting points. They should also have many experiences skip counting by 5s, 10s, and 100s to develop the concept of place value.</p> <p>Examples:</p> <ul style="list-style-type: none"> The use of the 100s chart may be helpful for students to identify the counting patterns. The use of money (nickels, dimes, dollars) or base ten blocks may be helpful visual cues. The use of an interactive whiteboard may also be used to develop counting skills. <p>The ultimate goal for second graders is to be able to count in multiple ways with no visual support.</p>

Number and Operations in Base Ten (NBT)

Understand place value.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.NBT.A.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p> <p>Connections: <i>2.SL.2; 2.RI.3</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Students need many opportunities reading and writing numerals in multiple ways.</p> <p>Examples:</p> <ul style="list-style-type: none"> • Base-ten numerals 637 (standard form) • Number names six hundred thirty seven (written form) • Expanded form 600 + 30 + 7 (expanded notation) <p>When students say the expanded form, it may sound like this: “6 hundreds plus 3 tens plus 7 ones” OR “600 plus 30 plus 7.”</p>
<p>2.NBT.A.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p> <p>Connections: <i>2.NBT.03; 2.RI.3; ET02-S1C2-02</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.6.</i> Attend to precision.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Students may use models, number lines, base ten blocks, interactive whiteboards, document cameras, written words, and/or spoken words that represent two three-digit numbers. To compare, students apply their understanding of place value. They first attend to the numeral in the hundreds place, then the numeral in tens place, then, if necessary, to the numeral in the ones place.</p> <p>Comparative language includes but is not limited to: more than, less than, greater than, most, greatest, least, same as, equal to and not equal to. Students use the appropriate symbols to record the comparisons.</p>

Number and Operations in Base Ten (NBT)

Use place value understanding and properties of operations to add and subtract.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.NBT.B.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>Connections: <i>2.OA.2; 2.NBT.1; 2.NBT.3; 2.RI.3; 2.W.2; 2.SL.3</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students should have experiences solving problems written both horizontally and vertically. They need to communicate their thinking and be able to justify their strategies both verbally and with paper and pencil.</p> <p>Addition strategies based on place value for $48 + 37$ may include:</p> <ul style="list-style-type: none"> • Adding by place value: $40 + 30 = 70$ and $8 + 7 = 15$ and $70 + 15 = 85$. • Incremental adding (breaking one number into tens and ones); $48 + 10 = 58$, $58 + 10 = 68$, $68 + 10 = 78$, $78 + 7 = 85$ • Compensation (making a friendly number): $48 + 2 = 50$, $37 - 2 = 35$, $50 + 35 = 85$ <p>Subtraction strategies based on place value for $81 - 37$ may include:</p> <ul style="list-style-type: none"> • Adding up (from smaller number to larger number): $37 + 3 = 40$, $40 + 40 = 80$, $80 + 1 = 81$, and $3 + 40 + 1 = 44$. • Incremental subtracting: $81 - 10 = 71$, $71 - 10 = 61$, $61 - 10 = 51$, $51 - 7 = 44$ • Subtracting by place value: $81 - 30 = 51$, $51 - 7 = 44$ <p>Properties that students should know and use are:</p> <ul style="list-style-type: none"> • Commutative property of addition (Example: $3 + 5 = 5 + 3$) • Associative property of addition (Example: $(2 + 7) + 3 = 2 + (7 + 3)$) • Identity property of 0 (Example: $8 + 0 = 8$) <p>Students in second grade need to communicate their understanding of why some properties work for some operations and not for others.</p> <ul style="list-style-type: none"> • Commutative Property: In first grade, students investigated whether the commutative property works with subtraction. The intent was for students to recognize that taking 5 from 8 is not the same as taking 8 from 5. Students should also understand that they will be working with numbers in later grades that will allow them to subtract larger numbers from smaller numbers. This exploration of the commutative property continues in second grade. • Associative Property: Recognizing that the associative property does not work for subtraction is difficult for students to consider at this grade level as it is challenging to determine all the possibilities.

Number and Operations in Base Ten (NBT)

Use place value understanding and properties of operations to add and subtract.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.NBT.B.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.</p> <p>Connections: <i>2.NBT.5; 2.RI.3; 2.W.2; 2.SL.2; ET02-S2C1-01</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Students demonstrate addition strategies with up to four two-digit numbers either with or without regrouping. Problems may be written in a story problem format to help develop a stronger understanding of larger numbers and their values. Interactive whiteboards and document cameras may also be used to model and justify student thinking.</p>
<p>2.NBT.B.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p> <p>Connections: <i>2.NBT.5; 2.NBT.6; 2.RI.3; 2.SL.3; 2.W.2; ET02-S1C2-01; ET02-S2C1-01</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.4.</i> Model with mathematics.</p> <p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>There is a strong connection between this standard and place value understanding with addition and subtraction of smaller numbers. Students may use concrete models or drawings to support their addition or subtraction of larger numbers. Strategies are similar to those stated in 2.NBT.5, as students extend their learning to include greater place values moving from tens to hundreds to thousands. Interactive whiteboards and document cameras may also be used to model and justify student thinking.</p>

Number and Operations in Base Ten (NBT)

Use place value understanding and properties of operations to add and subtract.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.NBT.B.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</p> <p>Connections: <i>2.RI.3; 2.SL.1; 2.SL.2; 2.SL.3; ET02-S2C1-01</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Students need many opportunities to practice mental math by adding and subtracting multiples of 10 and 100 up to 900 using different starting points. They can practice this by counting and thinking aloud, finding missing numbers in a sequence, and finding missing numbers on a number line or hundreds chart. Explorations should include looking for relevant patterns.</p> <p>Mental math strategies may include:</p> <ul style="list-style-type: none"> • counting on; 300, 400, 500, etc. • counting back; 550, 450, 350, etc. <p>Examples:</p> <ul style="list-style-type: none"> • 100 more than 653 is ____ (753) • 10 less than 87 is ____ (77) • “Start at 248. Count up by 10s until I tell you to stop.” <p>An interactive whiteboard or document camera may be used to help students develop these mental math skills.</p>

Number and Operations in Base Ten (NBT)

Use place value understanding and properties of operations to add and subtract.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.NBT.B.9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)</p> <p>Connections: <i>2.NBT.1; 2.RI.3; 2.RI.4; 2.W.2; 2.SL.2; 2.SL.3; ET02-S2C1-01</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>2.MP.4.</i> Model with mathematics.</p> <p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Students need multiple opportunities explaining their addition and subtraction thinking. Operations embedded within a meaningful context promote development of reasoning and justification.</p> <p>Example:</p> <ul style="list-style-type: none"> • Mason read 473 pages in June. He read 227 pages in July. How many pages did Mason read altogether? <ul style="list-style-type: none"> ○ Karla's explanation: $473 + 227 = \underline{\quad}$. I added the ones together ($3 + 7$) and got 10. Then I added the tens together ($70 + 20$) and got 90. I knew that $400 + 200$ was 600. So I added $10 + 90$ for 100 and added $100 + 600$ and found out that Mason had read 700 pages altogether. ○ Debbie's explanation: $473 + 227 = \underline{\quad}$. I started by adding 200 to 473 and got 673. Then I added 20 to 673 and I got 693 and finally I added 7 to 693 and I knew that Mason had read 700 pages altogether. ○ Becky's explanation: I used base ten blocks on a base ten mat to help me solve this problem. I added 3 ones (units) plus 7 ones and got 10 ones which made one ten. I moved the 1 ten to the tens place. I then added 7 tens rods plus 2 tens rods plus 1 tens rod and got 10 tens or 100. I moved the 1 hundred to the hundreds place. Then I added 4 hundreds plus 2 hundreds plus 1 hundred and got 7 hundreds or 700. So Mason read 700 books. <p>Students should be able to connect different representations and explain the connections. Representations can include numbers, words (including mathematical language), pictures, number lines, and/or physical objects. Students should be able to use any/all of these representations as needed.</p> <p>An interactive whiteboard or document camera can be used to help students develop and explain their thinking.</p>


Measurement and Data (MD)

Measure and estimate lengths in standard units.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.MD.A.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.</p> <p>Connections: <i>2.SL.3; SC02-S1C2-03</i></p>	<p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.6.</i> Attend to precision.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p>	<p>Students in second grade will build upon what they learned in first grade from measuring length with non-standard units to the new skill of measuring length in metric and U.S. Customary with standard units of measure. They should have many experiences measuring the length of objects with rulers, yardsticks, meter sticks, and tape measures. They will need to be taught how to actually use a ruler appropriately to measure the length of an object especially as to where to begin the measuring. Do you start at the end of the ruler or at the zero?</p>
<p>2.MD.A.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.</p> <p>Connections: <i>2.MD.1; 2.MD.3; 2.MD.4; 2.RI.3; 2.RI.4; 2.W.2; 2.SL.3; SC02-S1C2-03; ET02-S2C1-02</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.6.</i> Attend to precision.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p>	<p>Students need multiple opportunities to measure using different units of measure. They should not be limited to measuring within the same standard unit. Students should have access to tools, both U.S. Customary and metric. The more students work with a specific unit of measure, the better they become at choosing the appropriate tool when measuring.</p> <p>Students measure the length of the same object using different tools (ruler with inches, ruler with centimeters, a yardstick, or meter stick). This will help students learn which tool is more appropriate for measuring a given object. They describe the relationship between the size of the measurement unit and the number of units needed to measure something. For instance, a student might say, "The longer the unit, the fewer I need." Multiple opportunities to explore provide the foundation for relating metric units to customary units, as well as relating within customary (inches to feet to yards) and within metric (centimeters to meters).</p>

Measurement and Data (MD)

Measure and estimate lengths in standard units.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.MD.A.3. Estimate lengths using units of inches, feet, centimeters, and meters.</p> <p>Connections: <i>2.MD.1; 2.W.2; 2.SL.3</i></p>	<p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.6.</i> Attend to precision.</p>	<p>Estimation helps develop familiarity with the specific unit of measure being used. To measure the length of a shoe, knowledge of an inch or a centimeter is important so that one can approximate the length in inches or centimeters. Students should begin practicing estimation with items which are familiar to them (length of desk, pencil, favorite book, etc.).</p> <p>Some useful benchmarks for measurement are:</p> <ul style="list-style-type: none"> • First joint to the tip of a thumb is about an inch • Length from your elbow to your wrist is about a foot • If your arm is held out perpendicular to your body, the length from your nose to the tip of your fingers is about a yard 
<p>2.MD.A.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.</p> <p>Connections: <i>2.MD.1; 2.RI.3; 2.RI.4; 2.W.2; 2.SL.3; ET02-S2C1-01; SC02-S1C1-03</i></p>	<p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.6.</i> Attend to precision.</p>	<p>Second graders should be familiar enough with inches, feet, yards, centimeters, and meters to be able to compare the differences in lengths of two objects. They can make direct comparisons by measuring the difference in length between two objects by laying them side by side and selecting an appropriate standard length unit of measure. Students should use comparative phrases such as “It is longer by 2 inches” or “It is shorter by 5 centimeters” to describe the difference between two objects. An interactive whiteboard or document camera may be used to help students develop and demonstrate their thinking.</p>

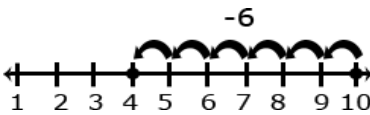
Measurement and Data (MD)

Relate addition and subtraction to length.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.MD.B.5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.</p> <p>Connections: <i>2.OA.1; 2.NBT.5; 2.RI.3; 2.W.2; 2.SL.2; 2.SL.3; ET02-S1C2-02</i></p>	<p><i>2.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.4.</i> Model with mathematics.</p> <p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Students need experience working with addition and subtraction to solve word problems which include measures of length. It is important that word problems stay within the same unit of measure. Counting on and/or counting back on a number line will help tie this concept to previous knowledge. Some representations students can use include drawings, rulers, pictures, and/or physical objects. An interactive whiteboard or document camera may be used to help students develop and demonstrate their thinking.</p> <p>Equations include:</p> <ul style="list-style-type: none"> • $20 + 35 = c$ • $c - 20 = 35$ • $c - 35 = 20$ • $20 + b = 55$ • $35 + a = 55$ • $55 = a + 35$ • $55 = 20 + b$ <p>Example:</p> <ul style="list-style-type: none"> • A word problem for $5 - n = 2$ could be: Mary is making a dress. She has 5 yards of fabric. She uses some of the fabric and has 2 yards left. How many yards did Mary use? <p>There is a strong connection between this standard and demonstrating fluency of addition and subtraction facts. Addition facts through $10 + 10$ and the related subtraction facts should be included.</p>

Measurement and Data (MD)

Relate addition and subtraction to length.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.MD.B.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.</p> <p>Connections: <i>2.NBT.2; 2.OA.1; 2.MD.5; 2.RI.3; 2.SL.3; ET02-S1C2-02</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.4.</i> Model with mathematics.</p> <p><i>2.MP.5.</i> Use appropriate tools strategically.</p>	<p>Students represent their thinking when adding and subtracting within 100 by using a number line. An interactive whiteboard or document camera can be used to help students demonstrate their thinking.</p> <p>Example:</p> <ul style="list-style-type: none"> • $10 - 6 = 4$ 

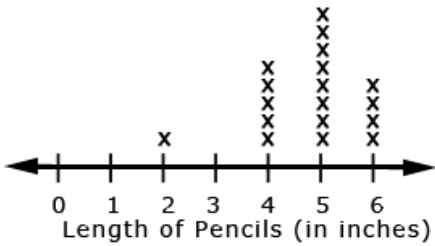
Measurement and Data (MD)

Work with time and money.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.MD.C.7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p> <p>Connections: 2.NBT.2; 2.RI.3; 2.W.2; 2.SL.2; ET02-S1C2-01; ET02-S1C2-02</p>	<p>2.MP.5. Use appropriate tools strategically.</p> <p>2.MP.6. Attend to precision.</p>	<p>In first grade, students learned to tell time to the nearest hour and half-hour. Students build on this understanding in second grade by skip-counting by 5 to recognize 5-minute intervals on the clock. They need exposure to both digital and analog clocks. It is important that they can recognize time in both formats and communicate their understanding of time using both numbers and language. Common time phrases include the following: quarter till ____, quarter after ____, ten till ____, ten after ____, and half past ____.</p> <p>Students should understand that there are 2 cycles of 12 hours in a day - a.m. and p.m. Recording their daily actions in a journal would be helpful for making real-world connections and understanding the difference between these two cycles. An interactive whiteboard or document camera may be used to help students demonstrate their thinking.</p>
<p>2.MD.C.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. <i>Example: If you have 2 dimes and 3 pennies, how many cents do you have?</i></p> <p>Connections: 2.NBT.1; 2.NBT.5; 2.RI.3; 2.RI.4; 2.W.2; 2.SL.2; ET02-S1C2-01; ET02-S1C2-02</p>	<p>2.MP.1. Make sense of problems and persevere in solving them.</p> <p>2.MP.2. Reason abstractly and quantitatively.</p> <p>2.MP.4. Model with mathematics.</p> <p>2.MP.5. Use appropriate tools strategically.</p> <p>2.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>Since money is not specifically addressed in kindergarten, first grade, or third grade, students should have multiple opportunities to identify, count, recognize, and use coins and bills in and out of context. They should also experience making equivalent amounts using both coins and bills. "Dollar bills" should include denominations up to one hundred (\$1.00, \$5.00, \$10.00, \$20.00, \$100.00).</p> <p>Students should solve story problems connecting the different representations. These representations may include objects, pictures, charts, tables, words, and/or numbers. Students should communicate their mathematical thinking and justify their answers. An interactive whiteboard or document camera may be used to help students demonstrate and justify their thinking.</p> <p>Example:</p> <ul style="list-style-type: none"> Sandra went to the store and received \$ 0.76 in change. What are three different sets of coins she could have received?

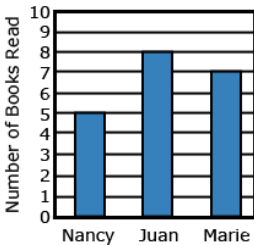
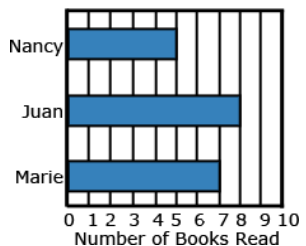
Measurement and Data (MD)

Represent and interpret data.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.MD.D.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</p> <p>Connections: <i>2.RI.3; 2.RI.4; 2.W.2; SC02-S1C2-04; SC02-S1C3-01; ET02-S2C1-01</i></p>	<p><i>2.MP.4.</i> Model with mathematics.</p> <p><i>2.MP.5.</i> Use appropriate tools strategically.</p> <p><i>2.MP.6.</i> Attend to precision.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>This standard emphasizes representing data using a line plot. Students will use the measurement skills learned in earlier standards to measure objects. Line plots are first introduced in this grade level. A line plot can be thought of as plotting data on a number line. An interactive whiteboard may be used to create and/or model line plots.</p> <div style="text-align: center;"> <p>Number of Pencils Measured</p>  </div>














Measurement and Data (MD)

Represent and interpret data.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>																								
<p>2.MD.D.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (See Table 1.)</p> <p>Connections: <i>2.RI.3; 2.RI.4; 2.W.2; 2.SL.2; 2.SL.3; SC02-S1C2-04; SC02-S1C3-01; SC02-S1C3-03; ET02-S2C1-01</i></p>	<p>2.MP.1. Make sense of problems and persevere in solving them.</p> <p>2.MP.2. Reason abstractly and quantitatively.</p> <p>2.MP.4. Model with mathematics.</p> <p>2.MP.5. Use appropriate tools strategically.</p> <p>2.MP.6. Attend to precision.</p> <p>2.MP.8. Look for and express regularity in repeated reasoning.</p>	<p>Students should draw both picture and bar graphs representing data that can be sorted up to four categories using single unit scales (e.g., scales should count by ones). The data should be used to solve put together, take-apart, and compare problems as listed in Table 1.</p> <p>In second grade, picture graphs (pictographs) include symbols that represent single units. Pictographs should include a title, categories, category label, key, and data.</p> <table border="1" data-bbox="1192 532 1619 722"> <thead> <tr> <th colspan="2">Number of Books Read</th> </tr> </thead> <tbody> <tr> <td>Nancy</td> <td>☆ ☆ ☆ ☆ ☆</td> </tr> <tr> <td>Juan</td> <td>☆ ☆ ☆ ☆ ☆ ☆ ☆ ☆</td> </tr> <tr> <td colspan="2">☆ = 1 Book</td> </tr> </tbody> </table> <p>Second graders should draw both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="1081 812 1333 1096"> <p>Books Read</p>  <table border="1"> <caption>Books Read (Vertical Bar Graph)</caption> <thead> <tr> <th>Name</th> <th>Number of Books Read</th> </tr> </thead> <tbody> <tr> <td>Nancy</td> <td>5</td> </tr> <tr> <td>Juan</td> <td>8</td> </tr> <tr> <td>Marie</td> <td>7</td> </tr> </tbody> </table> </div> <div data-bbox="1428 812 1732 1096"> <p>Books Read</p>  <table border="1"> <caption>Books Read (Horizontal Bar Graph)</caption> <thead> <tr> <th>Name</th> <th>Number of Books Read</th> </tr> </thead> <tbody> <tr> <td>Nancy</td> <td>5</td> </tr> <tr> <td>Juan</td> <td>8</td> </tr> <tr> <td>Marie</td> <td>7</td> </tr> </tbody> </table> </div> </div>	Number of Books Read		Nancy	☆ ☆ ☆ ☆ ☆	Juan	☆ ☆ ☆ ☆ ☆ ☆ ☆ ☆	☆ = 1 Book		Name	Number of Books Read	Nancy	5	Juan	8	Marie	7	Name	Number of Books Read	Nancy	5	Juan	8	Marie	7
Number of Books Read																										
Nancy	☆ ☆ ☆ ☆ ☆																									
Juan	☆ ☆ ☆ ☆ ☆ ☆ ☆ ☆																									
☆ = 1 Book																										
Name	Number of Books Read																									
Nancy	5																									
Juan	8																									
Marie	7																									
Name	Number of Books Read																									
Nancy	5																									
Juan	8																									
Marie	7																									

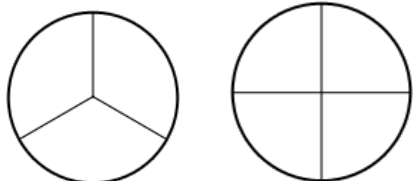
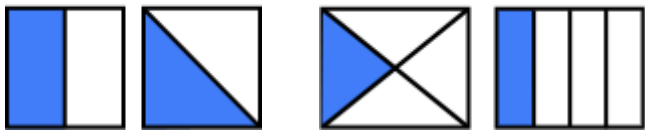
Geometry (G)

Reason with shapes and their attributes.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>								
<p>2.G.A.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)</p> <p>Connections: <i>2.RI.3; 2.RI.4; 2.W.2; 2.SL.2; 2.SL.3; SC02-S5C1-01; ET02-S2C1-01</i></p>	<p><i>2.MP.4.</i> Model with mathematics.</p> <p><i>2.MP.7.</i> Look for and make use of structure.</p>	<p>Students identify, describe, and draw triangles, quadrilaterals, pentagons, and hexagons. Pentagons, triangles, and hexagons should appear as both regular (equal sides and equal angles) and irregular. Students recognize all four sided shapes as quadrilaterals. Students use the vocabulary word “angle” in place of “corner” but they do not need to name angle types. Interactive whiteboards and document cameras may be used to help identify shapes and their attributes. Shapes should be presented in a variety of orientations and configurations.</p> <div data-bbox="1234 558 1566 812" style="border: 1px solid black; padding: 5px; text-align: center;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 2px;">triangle</td> <td style="border: 1px solid black; padding: 2px;">quadrilaterals</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;"></td> <td style="border: 1px solid black; padding: 2px;"></td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">pentagons</td> <td style="border: 1px solid black; padding: 2px;">hexagons</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;"></td> <td style="border: 1px solid black; padding: 2px;"></td> </tr> </table> </div>	triangle	quadrilaterals			pentagons	hexagons		
triangle	quadrilaterals									
										
pentagons	hexagons									
										
<p>2.G.A.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.</p> <p>Connections: <i>2.OA.4; 2.SL.2; 2.RI.3; ET02-S1C2-02</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.6.</i> Attend to precision.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>This standard is a precursor to learning about the area of a rectangle and using arrays for multiplication. An interactive whiteboard or manipulatives such as square tiles, cubes, or other square shaped objects can be used to help students partition rectangles.</p> <p>Rows are horizontal and columns are vertical.</p> <div data-bbox="1306 990 1503 1136" style="border: 1px solid black; padding: 5px; text-align: center;">  </div>								

Geometry (G)

Reason with shapes and their attributes.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>2.G.A.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words <i>halves</i>, <i>thirds</i>, <i>half of</i>, <i>a third of</i>, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</p> <p>Connections: <i>2.RI.3</i>; <i>2.RI.4</i>; <i>2.W.2</i>; <i>2.SL.2</i>; <i>2.SL.3</i>; <i>ET02-S1C2-02</i></p>	<p><i>2.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>2.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>2.MP.6.</i> Attend to precision.</p> <p><i>2.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>This standard introduces fractions in an area model. Students need experiences with different sizes, circles, and rectangles.</p> <ul style="list-style-type: none"> For example, students should recognize that when they cut a circle into three equal pieces, each piece will equal one third of its original whole. In this case, students should describe the whole as three thirds. If a circle is cut into four equal pieces, each piece will equal one fourth of its original whole and the whole is described as four fourths. <div style="text-align: center;">  <p style="margin-left: 100px;">$\frac{3}{3}$ $\frac{4}{4}$</p> </div> <ul style="list-style-type: none"> Students should see circles and rectangles partitioned in multiple ways so they learn to recognize that equal shares can be different shapes within the same whole. An interactive whiteboard may be used to show partitions of shapes. <div style="text-align: center;"> <p>halves fourths</p>  </div>

Standards for Mathematical Practice (MP)		
<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u> <i>are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.</i>	<u>Explanations and Examples</u>
2.MP.1. Make sense of problems and persevere in solving them.		In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They make conjectures about the solution and plan out a problem-solving approach.
2.MP.2. Reason abstractly and quantitatively.		Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and relate addition and subtraction to length.
2.MP.3. Construct viable arguments and critique the reasoning of others.		Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?”, “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but listen to others’ explanations. They decide if the explanations make sense and ask appropriate questions.
2.MP.4. Model with mathematics.		In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
2.MP.5. Use appropriate tools strategically.		In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.
2.MP.6. Attend to precision.		As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.



Arizona’s Common Core Standards – Mathematics – Second Grade

Standards for Mathematical Practice (MP)		
<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u> <i>are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.</i>	<u>Explanations and Examples</u>
2.MP.7. Look for and make use of structure.		Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).
2.MP.8. Look for and express regularity in repeated reasoning.		Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as rounding up and then adjusting the answer to compensate for the rounding. Students continually check their work by asking themselves, “Does this make sense?”

Table 1. Common addition and subtraction situations.⁶

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown ¹
Put Together / Take Apart²	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare³	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

⁶Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

¹These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

²Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

³For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.