High School Algebra IIAB (2-Year Sequence)

Teacher Blueprint Pages

Chandler Unified School District #80

Please Note—Changes related to the structure of the Teacher Blueprint Pages:

- ❖ A sequence within each quarter.
- ❖ Multiple standards are located in the same row; these standards are intended to be taught in tandem (concurrently) to maximize student learning and retention.
- ❖ To help teachers understand the groupings or clusters, a topic name was provided in Year 3, like "Quadratic Equations and Functions". This is followed by preskills that support the instruction of the topic.
- ❖ Embedded Standards that support teaching conceptually. These help teachers understand key standards that will be taught in tandem throughout an entire topic. These are *not Standards for Mathematical practice not Process Integration Objectives*, but are <u>Content standards</u>, like the standards they are placed next to.
- ❖ While changes in the provided sequence are not intended, it is understood that changes may be made to serve the needs of individual students.
- ❖ There is also a document called, "**High School Overview of the 2010 Standards**" to support teacher teams in looking ahead at the Common Core State Standards and understanding what will be required to transition to those standards.

ALL Semesters Standards for Mathematical Practice	
Standards	***Explanations and Examples***
HS.MP.1. Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2. Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3. Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
HS.MP.4. Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
HS.MP.5. Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

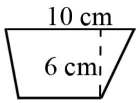
ALL Semesters
Standards for Mathematical Practice

Standards	***Explanations and Examples***
HS.MP.6. Attend to precision.	High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
HS.MP.7. Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
HS.MP.8. Look for and express regularity in repeated reasoning.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Year 3 Semester 1 (FIRST Year)

Semester 1 Topic 1: Linearity and Functions

Preskills: Linear expressions and equations (one-step through multi-step), graphing linearity (slope, y-intercept, x-intercept), an introduction to graphing technology.

Standards	***Embedded Standards for Semester 1***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.A-CED.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	**HS.F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*** Connections: ETHS-S6C2.03; 9-10.RST.7; 11-12.RST.7	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.	Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. Examples: • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  • Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of a piece of lava *t* seconds after it is ejected from the volcano is given by $h(t) = -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?

Semester 1 Topic 1: Linearity and Functions			
Preskills: Linear expressions and equations (one-step through multi-step), graphing linearity (slope, y-intercept, x-intercept), an introduction to graphing technology.			
Standards	***Embedded Standards for Semester 1***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **a.** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Connection: 11-12.WHST.1a-1e	**HS.N-VM.10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.** **HS.N-CN.1. Know there is a complex number i such that $i^2 = -1$, and every complex number**	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions. Examples: • A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in Plan 1? Plan 2? Plan 3? 1. \$59.95/month for 700 minutes and \$0.25 for each additional minute, 2. \$39.95/month for 400 minutes and \$0.15 for each additional minute, and 3. \$89.95/month for 1,400 minutes and \$0.05 for each additional minute. • A computer store sells about 200 computers at the price of \$1,000 per computer. For each \$50 increase in price, about ten fewer computers are sold. How much should the computer store charge per computer in order to maximize their profit?

Semester 1 Topic 1: Linearity and Functions

Preskills: Linear expressions and equations (one-step through multi-step), graphing linearity (slope, y-intercept, x-intercept), an introduction to graphing technology.

Standards	***Embedded Standards for Semester 1***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. Connection: 11-12.RST.4	**has the form *a* + *bi* with *a* and *b* real.** **HS.N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.** Connection: 11-12.RST.4 **HS.N-CN.3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.** Connection: 11-12.RST.3 **HS.N-CN.5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument 120°.***	HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.	Students can investigate functions and graphs modeling different situations involving simple and compound interest. Students can compare interest rates with different periods of compounding (monthly, daily) and compare them with the corresponding annual percentage rate. Spreadsheets and applets can be used to explore and model different interest rates and loan terms. Students can use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions. • A couple wants to buy a house in five years. They need to save a down payment of \$8,000. They deposit \$1,000 in a bank account earning 3.25% interest, compounded quarterly. How much will they need to save each month in order to meet their goal? • Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth each type of interest has? o Lee borrows \$9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest. o Lee borrows \$9,000 from a bank to buy a car. The bank charges 5% interest compounded annually. • Calculate the future value of a given amount of money, with and without technology. • Calculate the present value of a certain amount of money for a given length of time in the future, with and without technology.

Embedded Standards Examples and Explanations

Semester 1 Topic 1: Linearity and Functions

Preskills: Linear expressions and equations (one-step through multi-step), graphing linearity (slope, y-intercept, x-intercept), an introduction to graphing technology.

Standards	***Embedded Standards for Semester 1***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			

Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.
Examples:

- A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet.
  - What is a reasonable domain restriction for t in this context?
  - Determine the height of the rocket two seconds after it was launched.
  - Determine the maximum height obtained by the rocket.
  - Determine the time when the rocket is 100 feet above the ground.
  - Determine the time at which the rocket hits the ground.
  - How would you refine your answer to the first question based on your response to the second and fifth questions?
- Compare the graphs of $y = 3x^2$ and $y = 3x^3$.
- Let $R(x) = \dfrac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$.
- Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease.

It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.

Example:

- Given $w = 2 - 5i$ and $z = 3 + 4i$
  - *a.* Use the conjugate to find the modulus of w.
  - *b.* Find the quotient of z and w.

Solution:

a.
$$|w|^2 = w\overline{w}$$
$$|w|^2 = (2-5i)(2+5i)$$
$$|w|^2 = 4 + 10i - 10i - 25i^2$$
$$|w|^2 = 4 - 25i^2$$
$$|w|^2 = 4 - 25(-1)$$
$$|w|^2 = 4 + 25$$
$$|w|^2 = 29$$
$$|w| = \sqrt{29}$$

b.
$$\frac{z}{w} = \frac{3+4i}{2-5i}$$
$$\frac{z}{w} = \frac{3+4i}{2-5i}\left(\frac{2+5i}{2+5i}\right)$$
$$\frac{z}{w} = \frac{6+15i+8i-20}{4+25}$$
$$\frac{z}{w} = \frac{-14+23i}{29}$$

Example:

- Simplify the following expression. Justify each step using the commutative, associative and distributive properties.
  $(3-2i)(-7+4i)$

Solutions may vary; one solution follows:

$(3-2i)(-7+4i)$	
$3(-7+4i) - 2i(-7+4i)$	Distributive Property
$-21 + 12i + 14i - 8i^2$	Distributive Property
$-21 + (12i + 14i) - 8i^2$	Associative Property
$-21 + i(12 + 14) - 8i^2$	Distributive Property
$-21 + 26i - 8i^2$	Computation
$-21 + 26i - 8(-1)$	$i^2 = -1$
$-21 + 26i + 8$	Computation
$-21 + 8 + 26i$	Commutative Property
$-13 + 26i$	Computation

Semester 1 Topic 2: Systems of Equations (Functions)			
Preskills: Graphing linear systems, substitution and elimination, matrices, systems of linear inequalities, linear programming			
Standards	***Embedded Standards for Semester1***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* Connection: 11-12.RST.7	**HS.F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*** Connections: ETHS-S6C2.03; 9-10.RST.7; 11-12.RST.7	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to	The residual in a regression model is the difference between the observed and the predicted y for some x(y the dependent variable and $x$the independent variable). So if we have a model $y = ax + b$, and a data point (x_i, y_i) the residual is for this point is: $r_i = y_i - (ax_i + b)$. Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals. Example: • Measure the wrist and neck size of each person in your class and make a scatter plot. Find the least squares regression line. Calculate and interpret the correlation coefficient for this linear regression model. Graph the residuals and evaluate the fit of the linear equations.

Semester 1 Topic 2: Systems of Equations (Functions)

Preskills: Graphing linear systems, substitution and elimination, matrices, systems of linear inequalities, linear programming

Standards	***Embedded Standards for Semester1***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. b. Informally assess the fit of a function by plotting and analyzing residuals. Connections: 11-12.RST.7; 11-12.WHST.1b-1c	**HS.N-VM.10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.** **HS.N-CN.1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real.** **HS.N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.** Connection: 11-12.RST.4	precision. HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning. Connections: SCHS-S1C2-05; SCHS-S1C3-01; ETHS-S1C2-01; ETHS-S1C3-01; ETHS-S6C2-03	

Semester 1 Topic 2: Systems of Equations (Functions)

Preskills: Graphing linear systems, substitution and elimination, matrices, systems of linear inequalities, linear programming

Standards	***Embedded Standards for Semester1***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. c. Fit a linear function for a scatter plot that suggests a linear association. Connection: 11-12.RST.7	**HS.N-CN.3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.** Connection: 11-12.RST.3 **HS.N-CN.5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* *$(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.***		
HS.S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit. Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.RST.5; 11-12.WHST.2e		HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.8. Look for and express regularity in repeated reasoning.	Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients. Example: • Collect height, shoe-size, and wrist circumference data for each student. Determine the best way to display the data. Answer the following questions: Is there a correlation between any two of the three indicators? Is there a correlation between all three indicators? What patterns and trends are apparent in the data? What inferences can be made from the data?

Embedded Standards Examples and Explanations

Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. **Continued on next page** **Continued for previous page**	Example: • Given $w = 2 - 5i$ and $z = 3 + 4i$ *a.* Use the conjugate to find the modulus of *w*. *b.* Find the quotient of *z* and *w*. **Continued for previous page**	Example: • Simplify the following expression. Justify each step using the commutative, associative and distributive properties. $(3-2i)(-7+4i)$ **Continued on next page** **Continued for previous page**

Semester 1 Topic 2: Systems of Equations (Functions)			
Preskills: Graphing linear systems, substitution and elimination, matrices, systems of linear inequalities, linear programming			
Standards	***Embedded Standards for Semester1***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			

Examples: • A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet. o What is a reasonable domain restriction for t in this context? o Determine the height of the rocket two seconds after it was launched. o Determine the maximum height obtained by the rocket. o Determine the time when the rocket is 100 feet above the ground. o Determine the time at which the rocket hits the ground. o How would you refine your answer to the first question based on your response to the second and fifth questions? • Compare the graphs of $y = 3x^2$ and $y = 3x^3$. • Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$. • Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease. It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.	Solution: a. $\|w\|^2 = w\overline{w}$ $\|w\|^2 = (2-5i)(2+5i)$ $\|w\|^2 = 4 + 10i - 10i - 25i^2$ $\|w\|^2 = 4 - 25i^2$ $\|w\|^2 = 4 - 25(-1)$ $\|w\|^2 = 4 + 25$ $\|w\|^2 = 29$ $\|w\| = \sqrt{29}$ b. $\frac{z}{w} = \frac{3+4i}{2-5i}$ $\frac{z}{w} = \frac{3+4i}{2-5i}\left(\frac{2+5i}{2+5i}\right)$ $\frac{z}{w} = \frac{6+15i+8i-20}{4+25}$ $\frac{z}{w} = \frac{-14+23i}{29}$	Solutions may vary; one solution follows: $(3-2i)(-7+4i)$ $3(-7+4i)-2i(-7+4i)$ Distributive Property $-21+12i+14i-8i^2$ Distributive Property $-21+(12i+14i)-8i^2$ Associative Property $-21+i(12+14)-8i^2$ Distributive Property $-21+26i-8i^2$ Computation $-21+26i-8(-1)$ $i^2=-1$ $-21+26i+8$ Computation $-21+8+26i$ Commutative Property $-13+26i$ Computation

Year 3 Semester 2 (FIRST Year)

Semester 2 Topic 1: Quadratic Equations and Functions

Preskills: Solving quadratic equations, finding roots (factoring, completing the square, using the quadratic formula), graphing quadratics, complex numbers and imaginary roots, systems of quadratics and systems of quadratics and linear equations.

<table>
<tr><th><u>Standards</u></th><th><u>Graphing Standards for Semester2</u></th><th><u>Mathematical Practices</u></th><th><u>Explanations and Examples</u></th></tr>
<tr><td>Students are expected to:</td><td></td><td></td><td></td></tr>
<tr><td>HS.A-REI.4. Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.</td><td>HS.A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value,</td><td>HS.MP.2. Reason abstractly and quantitatively.

HS.MP.3. Construct viable arguments and critique the reasoning of others.

HS.MP.7. Look for and make use of structure.</td><td>Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.

<table><tr><th>Value of Discriminant</th><th>Nature of Roots</th><th>Nature of Graph</th></tr><tr><td>$b^2 – 4ac = 0$</td><td>1 real roots</td><td>intersects x-axis once</td></tr><tr><td>$b^2 – 4ac > 0$</td><td>2 real roots</td><td>intersects x-axis twice</td></tr><tr><td>$b^2 – 4ac < 0$</td><td>2 complex roots</td><td>does not intersect x-axis</td></tr></table>
Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation.
What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?</td></tr>
</table>

Semester 2 Topic 1: Quadratic Equations and Functions

Preskills: Solving quadratic equations, finding roots (factoring, completing the square, using the quadratic formula), graphing quadratics, complex numbers and imaginary roots, systems of quadratics and systems of quadratics and linear equations.

Standards	***Graphing Standards for Semester2***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.A-REI.4. Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	exponential, and logarithmic functions. Connection: ETHS-S6C2-03 HS.F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\, f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	HS.MP.8. Look for and express regularity in repeated reasoning.	

Semester 2 Topic 1: Quadratic Equations and Functions

Preskills: Solving quadratic equations, finding roots (factoring, completing the square, using the quadratic formula), graphing quadratics, complex numbers and imaginary roots, systems of quadratics and systems of quadratics and linear equations.

Standards	***Graphing Standards for Semester2***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Connection: 11-12.RST.7	Connections: ETHS-S6C2-03; 11-12.WHST.2e HS.F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.		
HS.N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.	HS.F-IF.7. Graph functions expressed symbolically and show key features of the graph. **a. Graph linear and quadratic functions and show intercepts, maxima, and minima.** Connections: ETHS-S6C1-03; ETHS-S6C2-03 HS.F-IF.7. Graph functions expressed symbolically and show		Examples: • Within which number system can $x^2 = -2$ be solved? Explain how you know. • Solve $x^2 + 2x + 2 = 0$ over the complex numbers. • Find all solutions of $2x^2 + 5 = 2x$ and express them in the form $a + bi$.

Semester 2 Topic 1: Quadratic Equations and Functions			
Preskills: Solving quadratic equations, finding roots (factoring, completing the square, using the quadratic formula), graphing quadratics, complex numbers and imaginary roots, systems of quadratics and systems of quadratics and linear equations.			
Standards	***Graphing Standards for Semester2***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.N-CN.8. Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.*	key features of the graph. **b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.** Connections: ETHS-S6C1-03; ETHS-S6C2-03		
HS.N-CN.9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Connection: 11-12.WHST.1c	HS.F-IF.7. Graph functions expressed symbolically and show key features of the graph. **c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.** Connections: ETHS-S6C1-03; ETHS-S6C2-03		Examples: • How many zeros does $-2x^2 + 3x - 8$ have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra. • How many complex zeros does the following polynomial have? How do you know? $p(x) = (x^2 - 3)(x^2 + 2)(x - 3)(2x - 1)$

Semester 2 Topic 1: Quadratic Equations and Functions

Preskills: Solving quadratic equations, finding roots (factoring, completing the square, using the quadratic formula), graphing quadratics, complex numbers and imaginary roots, systems of quadratics and systems of quadratics and linear equations.

Standards	***Graphing Standards for Semester2***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	HS.F-BF.1. Write a function that describes a relationship between two quantities. c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* Connections: ETHS-S6C1-03;ETHS-S6C2-03	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Graphing calculators or programs can be used to generate graphs of polynomial functions. Example: • Factor the expression $x^3+4x^2-59x-126$ and explain how your answer can be used to solve the equation $x^3+4x^2-59x-126=0$. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x)=x^3+4x^2-59x-126$.

Semester 2 Topic 1: Quadratic Equations and Functions

Preskills: Solving quadratic equations, finding roots (factoring, completing the square, using the quadratic formula), graphing quadratics, complex numbers and imaginary roots, systems of quadratics and systems of quadratics and linear equations.

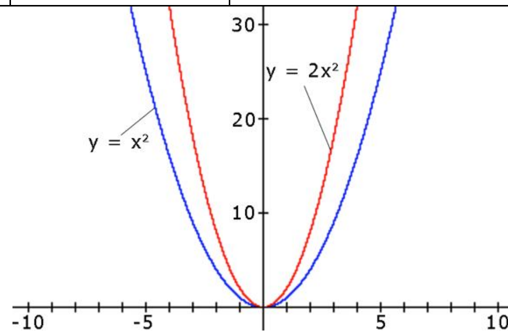
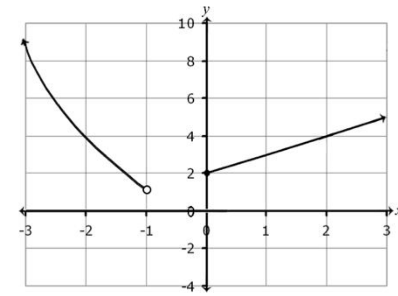
Standards	***Graphing Standards for Semester2***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.A-APR.2. Know and apply the Remainder Theorem: For a polynomial *p*(*x*) and a number *a*, the remainder on division by *x* – *a* is *p*(*a*), so *p*(*a*) = 0 if and only if (*x* – *a*) is a factor of *p*(*x*).			The Remainder theorem says that if a polynomial *p*(*x*) is divided by *x* – *a*, then the remainder is the constant *p*(*a*). That is, $p(x) = q(x)(x - a) + p(a)$. So if p(a) = 0 then p(x) = q(x)(x-a). • Let $p(x) = x^5 - 3x^4 + 8x^2 - 9x + 30$. Evaluate p(-2). What does your answer tell you about the factors of p(x)? [Answer: p(-2) = 0 so x+2 is a factor.]

Graphing Standards Examples and Explanations

Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions. Example: • Given the following equations determine the *x*	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is *f*(*x*) = $x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format.. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions	Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Describe key characteristics of the graph of *f(x)* = $\lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below.

Semester 2 Topic 1: Quadratic Equations and Functions

Preskills: Solving quadratic equations, finding roots (factoring, completing the square, using the quadratic formula), graphing quadratics, complex numbers and imaginary roots, systems of quadratics and systems of quadratics and linear equations.

Standards	***Graphing Standards for Semester2***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
value that results in an equal output for both functions. $f(x) = 3x - 2$ $g(x) = (x+3)^2 - 1$ Example: Contrast the growth of the $f(x)=x^3$ and $f(x)=3^x$.	 • Describe effect of varying the parameters *a, h,* and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$.		$F(x) = \begin{cases} x+2 \text{ for } x \geq 0 \\ -x^2 \text{for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?

Year 3 Semester 3 (SECOND Year)

Semester 3 Topic 1: Algebra and Functions **Preskills:** Factoring, functions and function notation, graphing and translations with graphing, technology.			
Standards	***Embedded Standards for Semester 3***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
Review for students: **Functions and Function Notation** **Factoring monomials, binomials, trinomials, and polynomials.** **Graphing Functions and Translating those graphs.** **Using Technology**			

Semester 3 Topic 1: Algebra and Functions

Preskills: Factoring, functions and function notation, graphing and translations with graphing, technology.

Standards	***Embedded Standards for Semester 3***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a. Use the properties of exponents to interpret expressions for exponential functions.** ***For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*** Connection: 11-12.RST.7	HS.F-BF.4. Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics.	

Semester 3 Topic 1: Algebra and Functions			
Preskills: Factoring, functions and function notation, graphing and translations with graphing, technology.			
Standards	***Embedded Standards for Semester 3***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* Connections: 11-12.RST.4; 11-12.RST.9; 11-12.WHST.2d	HS.F-BF.5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. Connection: ETHS-S6C2-03 HS.A-APR.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. HS.F-IF.7. Graph	5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.	Students may explain orally or in written format.

Semester 3 Topic 1: Algebra and Functions

Preskills: Factoring, functions and function notation, graphing and translations with graphing, technology.

<table>
<tr><th><u>Standards</u></th><th><u>Embedded Standards for Semester 3</u></th><th><u>Mathematical Practices</u></th><th><u>Explanations and Examples</u></th></tr>
<tr><td>Students are expected to:</td><td></td><td></td><td></td></tr>
<tr><td>HS.F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Connections: ETHS-S6C1-03; ETHS-S6C2-03; 11-12.RST.4</td><td>functions expressed symbolically and show key features of the graph.
d. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</td><td></td><td>A couple wants to buy a house in five years. They need to save a down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. How much will they need to save each month in order to meet their goal?
Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth each type of interest has?
Lee borrows $9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest.
Lee borrows $9,000 from a bank to buy a car. The bank charges 5% interest compounded annually.
Calculate the future value of a given amount of money, with and without technology.
Calculate the present value of a certain amount of money for a given length of time in the future, with and without technology.</td></tr>
<tr><td>HS.F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Connections: ETHS-S6C1-03; ETHS-S6C2-03; 11-12.RST.4; SSHS-S5C5-03</td><td>HS.F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases
e. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Connections:</td><td></td><td>Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.

Examples:
Determine an exponential function of the form $f(x) = ab^x$ using data points from the table. Graph the function and identify the key characteristics of the graph.
<table><tr><th>x</th><th>f(x)</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>3</td></tr><tr><td>3</td><td>27</td></tr></table>
Sara’s starting salary is $32,500. Each year she receives a $700 raise. Write a sequence in explicit form to describe the situation.</td></tr>
</table>

Semester 3 Topic 1: Algebra and Functions

Preskills: Factoring, functions and function notation, graphing and translations with graphing, technology.

Standards	***Embedded Standards for Semester 3***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context. Connections: ETHS-S6C1-03; ETHS-S6C2-03; SSHS-S5C5-03; 11-12.WHST.2e	ETHS-S6C1-03; ETHS-S6C2-03		Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions. Example: • A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where n is the number of years since the initial deposit. What is the value of r? What is the meaning of the constant P in terms of the savings account? Explain either orally or in written format.
HS.A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* Connection: 11-12.RST.4	HS.F-LE.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Connections: ETHS-S6C1-03; ETHS-S6C2-03; 11-12.RST.3		Example: In February, the Bezanson family starts saving for a trip to Australia in September. The Bezanson's expect their vacation to cost \$5375. They start with \$525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip?
HS.F-BF.1. Write a function that describes a relationship between two quantities. **a. Determine an explicit expression, a recursive process, or steps for calculation from a context.** Connections: ETHS-S6C1-03; ETHS-S6C2-03; 9-10.RST.7; 11-12.RST.7			Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: • You buy a \$10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of \$250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between

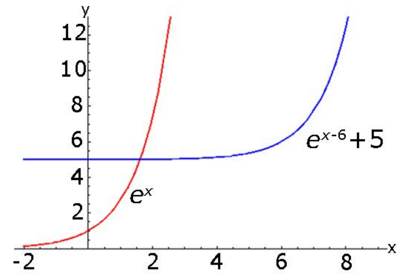
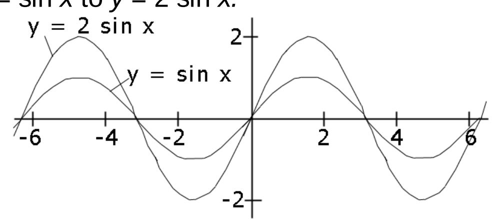
Semester 3 Topic 1: Algebra and Functions **Preskills:** Factoring, functions and function notation, graphing and translations with graphing, technology.			
Standards	***Embedded Standards for Semester 3***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.F-BF.1. Write a function that describes a relationship between two quantities. b. **Combine standard function types using arithmetic operations.** *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* Connections: ETHS-S6C1-03; ETHS-S6C2-03			its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after *t* hours is given in feet by $r = 10t^2 - 0.5t$, for 0 ≤ *t* ≤ 10. Find the area of the oil slick as a function of time.

Semester 3 Topic 1: Algebra and Functions **Preskills:** Factoring, functions and function notation, graphing and translations with graphing, technology.			
Standards	***Embedded Standards for Semester 3***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases **e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.** Connections: ETHS-S6C1-03; ETHS-S6C2-03			
HS.F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Connection: 11-12.RST.7			
Embedded Standards Examples and Explanations			

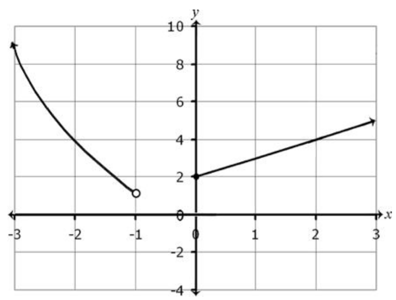
Semester 3 Topic 1: Algebra and Functions

Preskills: Factoring, functions and function notation, graphing and translations with graphing, technology.

Standards	***Embedded Standards for Semester 3***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms. **Example:** • Solve 200 $e^{0.04t}$ = 450 for *t*. Solution: We first isolate the exponential part by dividing both sides of the equation by 200. $e^{0.04t} = 2.25$ Now we take the natural logarithm of both sides. $ln\ e^{0.04t} = ln\ 2.25$ The left hand side simplifies to 0.04*t*, by logarithmic identity 1. $0.04t = ln\ 2.25$ Lastly, divide both sides by 0.04 $t = ln\ (2.25) / 0.04$ $t \approx 20.3$	The polynomial *q*(*x*) is called the quotient and the polynomial *r*(*x*) is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. Examples: • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. • Express $f(x)=\frac{2x+1}{x-1}$ in a form that reveals the horizontal asymptote of its graph. [Answer: $f(x)=\frac{2x+1}{x-1}=\frac{2(x-1)+2}{x-1}=2+\frac{2}{x-1}$, so the horizontal asymptote is *y* = 2.]	• Compare the shape and position of the graphs of $f(x)=e^x$ to $g(x)=e^{x-6}+5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions  • Describe the effect of varying the parameters *a, h,* and *k* on the shape and position of the graph f(*x*) = $ab^{(x+h)}$ + k., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of *y* = sin *x* to *y* = 2 sin *x.* 
Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to solve problems involving logarithms and exponents. Example:	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples:

Semester 3 Topic 1: Algebra and Functions			
Preskills: Factoring, functions and function notation, graphing and translations with graphing, technology.			
Standards	***Embedded Standards for Semester 3***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			

graph functions. **Examples:** • Describe key characteristics of the graph of $f(x) = \lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?	• Find the inverse of $f(x) = 3(10)^{2x}$.	• For the function $h(x) = (x - 2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. • Graph $h(x)$ and $h^{-1}(x)$ and explain how they relate to each other graphically. • Find a domain for $f(x) = 3x^2 + 12x - 8$ on which it has an inverse. Explain why it is necessary to restrict the domain of the function.

Year 3 Semester 4 (SECOND Year)

Semester4 Topic 1: Algebra and Functions			
Standards	***Embedded Standards for Semester4***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).* Connections: ETHS-S1C2-01; 9-10.WHST.1a-1e; 11-12.WHST.1a-1e	HS.N-CN.4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. Connection: 11-12.RST.3 HS.N-CN.6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. Connection: 11-12.RST.3	H S.MP.1. Make sense of problems and persevere in solving them. HS.MP.2. Reason abstractly and quantitatively. HS.MP.3 Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Students may use geometric simulation software to model figures and prove simple geometric theorems. Example: Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram.

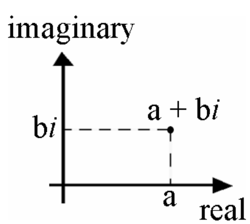
Semester4 Topic 1: Algebra and Functions			
Standards	***Embedded Standards for Semester4***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. Connections: ETHS-S1C2-01; 11-12.RST.4		HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.	Students may use geometric simulation software to explore the connection between circles and the Pythagorean Theorem. Examples: • Write an equation for a circle with a radius of 2 units and center at (1, 3). • Write an equation for a circle given that the endpoints of the diameter are (-2, 7) and (4, -8). Find the center and radius of the circle $4x^2 + 4y^2 - 4x + 2y - 1 = 0$.
HS.G-GPE.2. Derive the equation of a parabola given a focus and directrix. Connections: ETHS-S1C2-01; 11-12.RST.4			Students may use geometric simulation software to explore parabolas. Examples: Write and graph an equation for a parabola with focus (2, 3) and directrix $y = 1$.
HS.G-GPE.3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. Connections: ETHS-S1C2-01; 11-12.RST.4			Students may use geometric simulation software to explore conic sections. Example: Write an equation in standard form for an ellipse with foci at (0, 5) and (2, 0) and a center at the origin.
HS.G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. Connection: ETHS-S1C2-01			Students may use geometric simulation software to model figures and create cross sectional views. Example: Identify the shape of the vertical, horizontal, and other cross sections of a cylinder.

Semester4 Topic 1: Algebra and Functions

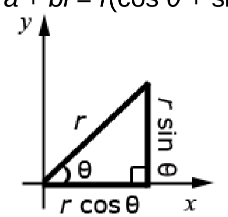
Standards	***Embedded Standards for Semester4***	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:			
HS.G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). Connection: ETHS-S1C2-01			Students may use simulation software and modeling software to explore which model best describes a set of data or situation.
HS.G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). Connection: ETHS-S1C2-01			Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

Embedded Standards Examples and Explanations

Students will represent complex numbers using rectangular and polar coordinates.



$a + bi = r(\cos\theta + \sin\theta)$

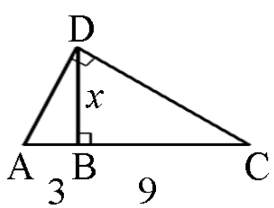


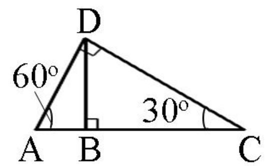
Examples:

- Plot the points corresponding to $3 - 2i$ and $1 + 4i$. Add these complex numbers and plot the result. How is this point related to the two others?
- Write the complex number with modulus (absolute value) 2 and argument π/3 in rectangular form.

Find the modulus and argument ($0 < \theta < 2\pi$) of the number $\sqrt{6} + \sqrt{-6}$.

Semester 4 Topic 2: Trigonometry and Trigonometric Functions

Standards	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:		
HS.F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.		
HS.F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. Connections: ETHS-S1C2-01; 11-12.WHST.2b; 11-12.WHST.2e	HS.MP.2. Reason abstractly and quantitatively.	Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or in written format) their understanding.
HS.F-TF.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for π /3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π-*x*, π+*x*, and 2π-*x* in terms of their values for *x*, where *x* is any real number. Connection: 11-12.WHST.2b	HS.MP.2. Reason abstractly and quantitatively. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	Examples: • Evaluate all six trigonometric functions of $\theta = \frac{\pi}{3}$. • Evaluate all six trigonometric functions of $\theta = 225^{o}$. • Find the value of x in the given triangle where $\overline{AD} \perp \overline{DC}$ and $\overline{AC} \perp \overline{DB}$ $m\angle A = 60°, m\angle C = 30°$. Explain your process for solving the problem including the use of trigonometric ratios as appropriate.  **Continued on the next page** **Continued from previous page**

Semester 4 Topic 2: Trigonometry and Trigonometric Functions		
Standards	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:		
		• Find the measure of the missing segment in the given triangle where $\overline{AD} \perp \overline{DC}$, $\overline{AC} \perp \overline{DB}$, $m\angle A = 60°, m\angle C = 30°, \overline{AC} = 12, \overline{AB} = 3$. Explain (orally or in written format) your process for solving the problem including use of trigonometric ratios as appropriate. 
HS.F-TF.4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. Connections: ETHS-S1C2-01; 11-12.WHST.2c	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or written format) their understanding of symmetry and periodicity of trigonometric functions.
HS.F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. Connection: ETHS-S1C2-01	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and periodic phenomena. Example: • The temperature of a chemical reaction oscillates between a low of 20° C and a high of 120° C. The temperature is at its lowest point when *t* = 0 and completes one cycle over a six hour period. a) Sketch the temperature, *T*, against the elapsed time, *t*, over a 12 hour period. b) Find the period, amplitude, and the midline of the graph you drew in part a). c) Write a function to represent the relationship between time and temperature. d) What will the temperature of the reaction be 14 hours after it began? e) At what point during a 24 hour day will the reaction have a temperature of 60° C?

Semester 4 Topic 2: Trigonometry and Trigonometric Functions		
Standards	***Mathematical Practices***	***Explanations and Examples***
Students are expected to:		
HS.F-TF.6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. Connections: ETHS-S1C2-01; 11-12.WHST.2e		Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions. Examples: • Identify a domain for the sine function that would permit an inverse function to be constructed. • Describe the behavior of the graph of the sine function over this interval. • Explain (orally or in written format) why the domain cannot be expanded any further.
HS.F-TF.7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. Connections: ETHS-S1C2-01; 11-12.WHST.1a	HS.MP.2. Reason abstractly and quantitatively. HS.MP.5. Use appropriate tools strategically.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and solve trigonometric equations. Example: • Two physics students set up an experiment with a spring. In their experiment, a weighted ball attached to the bottom of the spring was pulled downward 6 inches from the rest position. It rose to 6 inches above the rest position and returned to 6 inches below the rest position once every 6 seconds. The equation $h = -6\cos\left(\frac{\pi}{2}t\right)$ accurately models the height above and below the rest position every 6 seconds. Students may explain, orally or in written format, when the weighted ball first will be at a height of 3 inches, 4 inches, and 5 inches above rest position.
HS.F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. Connection: 11-12.WHST.1a-1e	HS.MP.3. Construct viable arguments and critique the reasoning of others.	
HS.F-TF.9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. Connection: 11-12.WHST.1a-1e	HS.MP.3. Construct viable arguments and critique the reasoning of others.	