Arizona’s Common Core Standards

Mathematics Curriculum Map

Algebra I

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS
State Board Approved June 2010

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Overview of the Common Core Standards Structure - HS

Conceptual Category	Number & Quantity (N)	Algebra (A)	Functions (F)	Geometry (G)	Statistics & Probability (S)	Modeling
Domains	The Real Number System (N-RN)	Seeing Structure in Expressions (A-SSE)	Interpreting Functions (F-IF)	Congruence (G-CO)	Interpreting Categorical & Quantitative Data (S-ID)	Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (☆)
	Quantities (N-Q)	Arithmetic with Polynomials & Rational Expressions (A-APR)	Building Functions (F-BF)	Similarity, Right Triangles, & Trigonometry (G-SRT)	Making Inferences & Justifying Conclusions (S-IC)	
	The Complex Number System (N-CN)	Creating Equations (A-CED)	Linear, Quadratic, & Exponential Models (F-LE)	Expressing Geometric Properties with Equations (G-GPE)	Conditional Probability & the Rules of Probability (S-CP)	
	Vector & Matrix Quantities (N-VM)	Reasoning with Equations & Inequalities (A-REI)	Trigonometric Functions (F-TF)	Geometric Measurement & Dimension (G-GMD)	Using Probability to Make Decisions (S-MD)	

Algebra 1

The Algebra I course outlined in this scope and sequence document begins with connections back to prior work with algebra, efficiently reviewing algebraic concepts that students have already studied while at the same time moving students forward into the new ideas described in the high school standards.
Students contrast exponential and linear functions as they explore exponential models using the familiar tools of tables, graphs, and symbols. Finally, they apply these same tools to a study of quadratic functions. Throughout, the connection between functions and equations is made explicit to give students more ways to model and make sense of problems.
Some standards may be revisited several times throughout the algebra course; others may be only partially addressed in different units, depending on the focus of the unit. Strikethroughs in the text indicate that only part of the standard is addressed in the unit.
The Mathematical Practices should become a natural way in which students come to understand and do mathematics. While, depending on the content to be understood or on the problem to be solved, any practice might be brought to bear, some practices may prove more useful than others. Opportunities for highlighting certain practices are indicated in different units in this document, but this highlighting should not be interpreted to mean that other practices should be neglected in those units.

	Units In Order of Instruction	Topic	Domains Addressed*	Mathematical Practices (Address all. Highlight bold.)	Suggested Unit Length (in days)
Q1	Unit 1	Representing relationships mathematically	★N-Q A-SSE ★A-CED A-REI F-IF F-BF	1, **2**, 3, **4**, 5, **6**, 7, 8	10
	Unit 2	Understanding functions	F-IF	1, **2**, 3, **4**, 5, **6**, 7, **8**	15
	Unit 3	Linear functions	F-IF F-BF ★F-LE ★S-ID	1, 2, 3, **4**, 5, 6, 7, **8**	15
Q2	Unit 4	Statistical models	★N-Q ★S-ID	**1,** 2, **3, 4, 5**, 6, 7, 8	20
	Unit 5	Linear equations and inequalities	★A-CED A-REI	**1,** 2, 3, **4,** 5, **6**, 7, 8	10
	Unit 6	Systems of linear equations and inequalities	★A-CED A-REI	**1**, **2**, 3, **4,** 5, **6**, 7, 8	15
Q3	Unit 7	Relationships that are not linear	N-RN F-IF	1, **2**, 3, **4,** 5, **6**, 7, 8	15
	Unit 8	Exponential functions and equations	A-SSE F-IF ★F-LE ★S-ID	1, **2, 3, 4, 5, 6,** 7, 8	15
Q4	Unit 9	Polynomial expressions and functions	A-SSE A-APR	**1**, 2, 3, 4, 5, 6, **7**, **8**	15
	Unit 10	Quadratic functions	A-SSE F-IF ★S-ID	1, 2, 3, **4, 5, 6, 7**, 8	15
	Unit 11	Quadratic equations	N-RN A-REI	**1, 2**, 3, **4, 5**, 6, **7**, 8	15

* Some standards in the domain are addressed in this unit. Other standards are addressed in other units.

★ Modeling standards are in this domain.

The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Overarching habits of mind of a productive math thinker	**1. Make sense of problems and persevere in solving them**	• Have or value sense-making • Use patience and persistence to listen to others • Be able to use strategies • Use self-evaluation and redirections • Be able to show or use multiple representations • Communicate both verbally and in written format • Be able to deduce what is a reasonable solution	• Provide open-ended and rich problems • Ask probing questions • Model multiple problem-solving strategies through Think- Alouds • Promotes and values discourse and collaboration • Cross-curricular integrations • Probe student responses (correct or incorrect) for understanding and multiple approaches • Provide solutions	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about...? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize...represent... show...?
	6. Attend to precision	• Communicate with precision-orally & written • Use mathematics concepts and vocabulary appropriately. • State meaning of symbols and use appropriately • Attend to units/labeling/tools accurately • Carefully formulate explanations • Calculate accurately and efficiently • Express answers in terms of context • Formulate and make use of definitions with others and their own reasoning.	• Think aloud/Talk aloud • Explicit instruction given through use of think aloud/talk aloud • Guided Inquiry including teacher gives problem, students work together to solve problems, and debriefing time for sharing and comparing strategies • Probing questions targeting content of study	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language...,definitions..., properties can you use to explain...? • How could you test your solution to see if it answers the problem?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book

The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	• Create multiple representations • Interpret problems in contexts • Estimate first/answer reasonable • Make connections • Represent symbolically • Visualize problems • Talk about problems, real life situations • Attending to units • Using context to think about a problem	• Develop opportunities for problem solving • Provide opportunities for students to listen to the reasoning of other students • Give time for processing and discussing • Tie content areas together to help make connections • Give real world situations • Think aloud for student benefit • Value invented strategies and representations • Less emphasis on the answer	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _______ related to ________? • What is the relationship between ______and ______? • What does_______mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use...? • Could we have used another operation or property to solve this task? Why or why not?
Reasoning and Explaining	**3. Construct viable arguments and critique the reasoning of others**	• Ask questions • Use examples and non-examples • Analyze data • Use objects, drawings, diagrams, and actions • Students develop ideas about mathematics and support their reasoning • Listen and respond to others • Encourage the use of mathematics vocabulary	• Create a safe environment for risk-taking and critiquing with respect • Model each key student disposition • Provide complex, rigorous tasks that foster deep thinking • Provide time for student discourse • Plan effective questions and student grouping	• What mathematical evidence would support your solution? • How can we be sure that...? / How could you prove that...? • Will it still work if...? • What were you considering when...? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? • Did you try a method that did not work? Why didn't it work? Could it work? • What is the same and what is different about...? • How could you demonstrate a counter-example?

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The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Modeling and Using Tools	**4. Model with mathematics**	• Realize they use mathematics (numbers and symbols) to solve/work out real-life situations • When approached with several factors in everyday situations, be able to pull out important information needed to solve a problem. • Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable. If not, go back and look for more information • Make sense of the mathematics	• Allow time for the process to take place (model, make graphs, etc.) • Model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) • Make appropriate tools available • Create an emotionally safe environment where risk taking is valued • Provide meaningful, real world, authentic, performance-based tasks (non-traditional work problems)	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the diagram, number line, chart, table, and your actions with the manipulatives? • Where did you see one of the quantities in the task in your equation or expression? What does each number in the equation mean? • How would it help to create a diagram, graph, table...? • What are some ways to visually represent...? • What formula might apply in this situation?
	5. Use appropriate tools strategically	• Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base 10 blocks, compass, protractor) • Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools)	• Maintain appropriate knowledge of appropriate tools • Effective modeling of the tools available, their benefits and limitations • Model a situation where the decision needs to be made as to which tool should be used	• What mathematical tools can we use to visualize and represent the situation? • Which tool is more efficient? Why do you think so? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? • Why was it helpful to use...? • What can using a ______ show us that _____may not? • In what situations might it be more informative or helpful to use...?

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The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Seeing structure and generalizing	**7. Look for and make use of structure**	• Look for, interpret, and identify patterns and structures • Make connections to skills and strategies previously learned to solve new problems/tasks • Reflect and recognize various structures in mathematics • Breakdown complex problems into simpler, more manageable chunks	• Be quiet and allow students to think aloud • Facilitate learning by using open-ended questioning to assist students in exploration • Careful selection of tasks that allow for students to make connections • Allow time for student discussion and processing • Foster persistence/stamina in problem solving • Provide graphic organizers or record student responses strategically to allow students to discover patters	• What observations do you make about...? • What do you notice when...? • What parts of the problem might you eliminate..., simplify...? • What patterns do you find in...? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to...? • In what ways does this problem connect to other mathematical concepts?
	8. Look for and express regularity in repeated reasoning	• Identify patterns and make generalizations • Continually evaluate reasonableness of intermediate results • Maintain oversight of the process	• Provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge • Provide adequate time for exploration • Provide time for dialogue and reflection • Ask deliberate questions that enable students to reflect on their own thinking • Create strategic and intentional check in points during student work time.	• Explain how this strategy works in other situations? • Is this always true, sometimes true or never true? • How would we prove that...? • What do you notice about...? • What is happening in this situation? • What would happen if...? • Is there a mathematical rule for...? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

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Algebra I

Unit 1: Representing relationships mathematically. Suggested number of days: 10

In this unit, students solidify their previous work with functional relationships as they begin to formalize the concept of a mathematical function. This unit provides an opportunity for students to reinforce their understanding of the various representations of a functional relationship—words, concrete elements, numbers, graphs, and algebraic expressions. Students review the distinction between independent and dependent variables in a functional relationship and connect those to the domain and range of a function. The standards listed here will be revisited multiple times throughout the course, as students encounter new function families.

Common Core State Standards for Mathematical Content

Quantities★— N---Q

A. Reason quantitatively and use units to solve problems

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.

Seeing Structure in Expressions — A---SSE

A. Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.

Creating equations★— A---CED

A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear ~~and quadratic functions, and simple rational and exponential~~ functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations ~~or inequalities, and by systems of equations and/or inequalities~~, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Comments

To make the strongest connection between students' previous work and the work of this course, the focus for **A---CED.A.1, A---CED.A.3** and **F---BF.A.1a** should be on linear functions and equations. Students will have solved linear equations using algebraic properties in their previous courses, but that should not be the focus of this unit. Instead, use students' work with **A---REI.D.10 F---IF.B.5,** and **F---IF.C.9** to reinforce students' understanding of the different kinds of information about a function that is revealed by its graph. This will build a solid foundation for students' ability to estimate solutions and their reasonableness using graphs.

Common Core State Standards for Mathematical Practice

2. Reason abstractly and quantitatively
4. Model with mathematics
6. Attend to precision

In this unit, students can begin to build proficiency with **MP.4** as they create mathematical models of contextual situations, while attending to limitations on those models. In order to create the models and interpret the results, students must attend to **MP.2**. As students create graphs of functional relationships, they

Explanations and Examples

N-Q 1 & 2

Include word problems where quantities are given in different units, which must be converted to make sense of the problem. For example, a problem might have an object moving 12 feet per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour:

$$24000\ \text{sec} \bullet \frac{1\text{min}}{60\,\text{sec}} \bullet \frac{1\text{hr}}{60\,\text{min}} \bullet \frac{1\text{day}}{24\text{hr}} \ldots$$

which is more than 8 miles per hour.

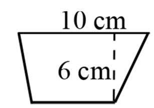
Graphical representations and data displays include, but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots, and multi-bar graphs.

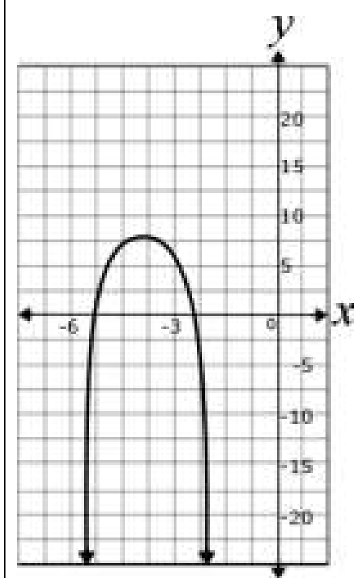
Examples:

- What type of measurements would one use to determine their income and expenses for one month?
- How could one express the number of accidents in Arizona?

A-SSE.1.a

Students should understand the vocabulary for the parts that make up the whole expression and be

Reasoning with Equations and Inequalities—A---REI D. Represent and solve equations ~~and inequalities graphically~~. 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **Interpreting Functions — F---IF** B. Interpret functions that arise in applications in terms of the context 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* C. Analyze functions using different representations 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **Building Functions — F---BF** A. Build a function that models a relationship between two quantities. 1. Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	must pay careful attention to quantities and scale, and so should be demonstrating **MP.6**.	able to identify those parts and interpret their meaning in terms of a context. **A-CED.A.1-3** Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. **Examples:** • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  **Example:** • A club is selling hats and jackets as a fundraiser. Their budget is \$1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs \$5 and each jacket costs \$8. o Write a system of inequalities to represent the situation. o Graph the inequalities. o If the club buys 150 hats and 100 jackets, will the conditions be satisfied? o What is the maximum number of jackets they can buy and still meet the conditions?
		A-REI.D.10 **Example:** • Which of the following points is on the circle with equation $(x-1)^2+(y+2)^2=5$?

		(a) (1, -2) (b) (2, 2) (c) (3, -1) (d) (3, 4) **F-IF.B.5 & F-IF.C.9** Students may explain orally, or in written format, the existing relationships. **Example:** • Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$  **F.BF.A.1.a** Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible,

		relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. **Examples:** • You buy a $10,000 car and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after *t* hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \le t \le 10$. Find the area of the oil slick as a function of time.

Unit 2: Understanding functions. Suggested number of days: 15

In this unit students build on their work in the previous unit to formalize the concept of a function. They will continue to explore continuous functions, but they will also investigate sequences as functions.

Common Core State Standards for Mathematical Content	**Comments**	**Explanations and Examples**

Interpreting Functions — F---IF
A. Understand the concept of a function and use function notation
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*
B. Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its
	In addition to sequences such as the one given in the example for **F---IF.A.3**, include arithmetic sequences and make the connection to linear functions. Geometric sequences could also be included as contrast to foreshadow work with exponential functions later in the course.
	
	Common Core State Standards for Mathematical Practice
	2. Reason abstractly and quantitatively
	4. Model with mathematics
	6. Attend to precision
	8. Look for and express regularity in repeated reasoning.
	
	In this unit, students investigate functions as mathematical models (MP.4). In order to analyze and communicate about these models, students must attend to MP.2 and MP.6. In developing symbolic representations of mathematical relationships, students might examine several specific instances of the relationship to find a generalizable regularity (MP.8).
		F-IF.A.1-3
		The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain.
		
		Examples:
		• If $f(x) = x^2 + 4x - 12$, find $f(2)$.
		• Let $f(x) = 2(x+3)^2$. Find $f(3)$, $f(-\frac{1}{2})$, $f(a)$, and $f(a-h)$
		• If P(*t*) is the population of Tucson *t* years after 2000, interpret the statements P(0) = 487,000 and P(10)-P(9) = 5,900.
		
		F-IF.B.4-5
		Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.
		
		Examples:
		• A rocket is launched from 180 feet above the ground at time *t* = 0. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where *t* is measured in seconds and *h* is the height above the ground measured in feet.
		  ○ What is a reasonable domain restriction for *t* in this context?
		  ○ Determine the height of the rocket two seconds after it was launched.
		  ○ How would you refine your answer to the first question based on your response to the second question?
		• Compare the graphs of $y = 3x^2$ and $y = 3x^3$.
		• Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of *R*(*x*). Also find the range, zeros, and asymptotes of *R*(*x*).
		• Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease.

graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*		• It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn’t rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday. Students may explain orally, or in written format, the existing relationships.

Unit 3: Linear functions. Suggested number of days: 15

This unit solidifies students' understanding of linear functions. It reviews the connection between the constant rate of change of a linear function, the slope of the line that is the linear function's graph, and the slope---intercept form for the equation of a line, **y = mx + b** before introducing the **x**---intercept, the standard form for the equation of a line, and the point---slope form for the equation of a line. This unit also introduces students to the idea that graphs of linear functions can be thought of as transformations on the graphs of other linear functions, setting the stage for the broader study of transformations of functions that continues in this and subsequent mathematics courses. This unit continues to reinforce the work with creating and representing equations described in **A---CED.A.2** and **A---REI.D.10** and with connecting the structure of expressions to contexts (**A---SSE.A.1.a**). This unit also deepens students' understanding of functions and their notation as described in F---IF.A.1 and F---IF.A.2. Students will investigate key features, domains, and ranges of linear functions as described in F---IF.B.4 **and F---IF.B.5**; write linear functions to model relationships between two quantities as in **F---BF.A.1a**; and compare properties of linear functions as in **F---IF.C.9**.

Common Core State Standards for Mathematical Content

Interpreting Functions — F---IF

B. Interpret functions that arise in applications in terms of the context.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Building Functions — F---BF

B. Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Linear, Quadratic, and Exponential Models★— F---LE

A. Construct and compare linear, quadratic, and ~~exponential~~ models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with ~~exponential functions~~.

Comments

The focus of **F---IF.B.6** should be on the constant rate of change of a linear function, although non---linear functions could be investigated for contrast. In this unit, focus **F---IF.C.7a** on linear functions and their intercepts; quadratic functions will be studied in Unit 10: Quadratic functions. Likewise, the focus of **F---BF.B.3, F---LE.A.1a, F---LE.A.2,** and **F---LE.B.5** should be on linear functions here; exponential functions will be studied in **Unit 8: Exponential functions and equations.**

Common Core State Standards for Mathematical Practice

4. Model with mathematics
8. Look for and express regularity in repeated reasoning.

In this unit, students continue to demonstrate their proficiency with **MP.4** as they create linear models of contextual situations, while attending to limitations on those models. Work with linear functions creates a number of opportunities to reinforce students' ability to recognize and leverage regularity in reasoning (**MP.8**), whether they are developing a general formula for finding the slope of a line or

Explanations and Examples

F-IF.B.6

The average rate of change of a function $y = f(x)$ over an interval [a,b] is

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

In addition to finding average rates of change from functions given symbolically, graphically, or in a table, Students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.

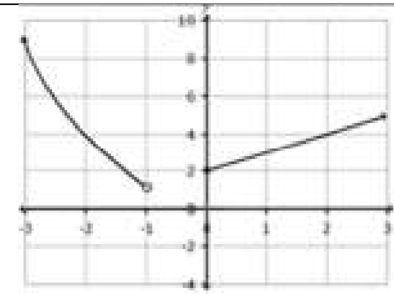
Examples:

- Use the following table to find the average rate of change of *g* over the intervals [-2, -1] and [0,2]:

x	*g(x)*
-2	2
-1	-1
0	-4
2	-10

- The table below shows the elapsed time when two different cars pass a 10, 20, 30, 40 and 50 meter mark on a test track.
    - For car 1, what is the average velocity (change in distance divided by change in time) between the 0 and 10 meter

a. Prove that linear functions grow by equal differences over equal intervals, ~~and that exponential functions grow by equal factors over equal intervals.~~ b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. 2. Construct linear ~~and exponential~~ functions, including arithmetic ~~and geometric~~ sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). B. Interpret expressions for functions in terms of the situation they made. 5. Interpret the parameters in a linear ~~or exponential~~ function in terms of a context. **Interpreting Categorical and Quantitative Data — S★---ID** C. Interpret linear models 7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	generalizing a pattern of repeated calculations to write a symbolic representation for a linear function.	mark? Between the 0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze the data to describe the motion of car 1. ○ How does the velocity of car 1 compare to that of car 2? <table><tr><td></td><td>***Car 1***</td><td>***Car 2***</td></tr><tr><td>***d***</td><td>***t***</td><td>***t***</td></tr><tr><td>10</td><td>4.472</td><td>1.742</td></tr><tr><td>20</td><td>6.325</td><td>2.899</td></tr><tr><td>30</td><td>7.746</td><td>3.831</td></tr><tr><td>40</td><td>8.944</td><td>4.633</td></tr><tr><td>50</td><td>10</td><td>5.348</td></tr><tr><td></td><td></td><td></td></tr></table> **F-IF.C.7** Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. **Examples:** • Describe key characteristics of the graph of $f(x) = \lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$

		 • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x – 1$. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?
		F-BF.B.3 Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. **Examples:** • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x)=x^2$ and $g(x)=2x^2$, and explain the differences in terms of the algebraic expressions for the functions.

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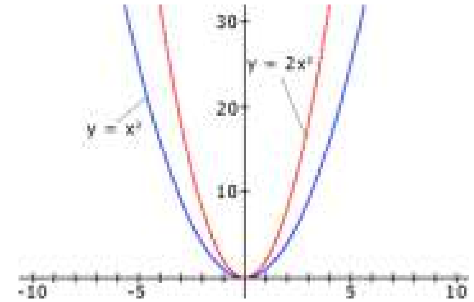
<td>

Describe effect of varying the parameters a, h, and k have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$.</td>
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<td>F-LE.A.1-2 and F-LE.B.5
Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions.

Examples:
A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in Plan 1? Plan 2? Plan 3?
$59.95/month for 700 minutes and $0.25 for each additional minute,
$39.95/month for 400 minutes and $0.15 for each additional minute, and
$89.95/month for 1,400 minutes and $0.05 for each additional minute.
A computer store sells about 200 computers at the price of $1,000 per computer. For each $50 increase in price, about ten fewer computers are sold. How much should the computer store charge per computer in order to maximize their profit?</td>
</tr>
</table>

		Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions. **Examples:** • Sara's starting salary is $32,500. Each year she receives a $700 raise. Write a sequence in explicit form to describe the situation. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.
		S-ID.C.7 Students may use spreadsheets or graphing calculators to create representations of data sets and create linear models. **Example:** • Lisa lights a candle and records its height in inches every hour. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.6), (3, 14.9), (4, 13.2), (5, 11.5), (7, 8.1), (9, 4.7), and (10, 3). Express the candle's height (*h*) as a function of time (*t*) and state the meaning of the slope and the intercept in terms of the burning candle. Solution: $h = -1.7t + 20$ Slope: The candle's height decreases by 1.7 inches for each hour it is burning. Intercept: Before the candle begins to burn, its height is 20 inches.

Unit 4: Statistical models. Suggested number of days: 20

This unit reviews the univariate data representations students studied previously and then introduces statistical models for bivariate categorical and quantitative data. Students have already addressed in previous units many of the standards in this unit, and they should now be able to apply their understandings from that previous work in the new work with the statistics standards in this unit. This unit provides opportunities to reinforce students' work from the previous unit with representing linear functions symbolically and graphically, as described in **A---SSE.A.1a, A---CED.A.2, F---IF.A.2, F---IF.B.4, F---IF.B.5, F---FI.C.9, F---BF.B.3, and F---LE.A.2,** and **F---LE.B.5**.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Quantities★— N---Q A. Reason quantitatively and use units to solve problems. 1. Use units as a way to understand problems and to guide the solution of multi---step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **Interpreting Categorical and Quantitative Data — S★---ID** A. Summarize, represent, and interpret data on a single count or measurement variable 1. Represent data with plots on the real number line (dot plots, histograms, and box plots). 2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. 3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). B. Summarize, represent, and interpret data on two categorical and quantitative variables. 5. Summarize categorical data for two categories in two---way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are	The work of **S---ID.B.6a** should focus on linear functions. Students will have the opportunity to create exponential and quadratic models for data when they study those functions in **Unit 8: Exponential functions and equations and Unit 10: Quadratic functions.** **Common Core State Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically In this unit, students make sense of problems through data (**MP.1**). They create statistical models (**MP.4**), sometimes using different tools such as spreadsheets and graphing technology (**MP.5**). Students must defend the appropriateness of their models and any conclusions they draw based on those models (**MP.3**).	**N-Q.A.1-3** Include word problems where quantities are given in different units, which must be converted to make sense of the problem. For example, a problem might have an object moving 12 feet per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour: $24000 \text{ sec} \bullet \frac{1\text{min}}{60\text{sec}} \bullet \frac{1\text{hr}}{60\text{min}} \bullet \frac{1\text{day}}{24\text{hr}}$ which is more than 8 miles per hour. Graphical representations and data displays include, but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots, and multi-bar graphs. **Examples:** • What type of measurements would one use to determine their income and expenses for one month? • How could one express the number of accidents in Arizona? The margin of error and tolerance limit varies according to the measure, tool used, and context. **Example:** Determining price of gas by estimating to the nearest cent is appropriate because you will not pay in fractions of a cent but the cost of gas is $\frac{\$3.479}{gallon}$.

related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, ~~quadratic, and exponential~~ models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. C. Interpret linear models 7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. 8. Compute (using technology) and interpret the correlation coefficient of a linear fit. 9. Distinguish between correlation and causation.		**S-ID.A.1-3** Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets. **Examples:** • The two data sets below depict the housing prices sold in the King River area and Toby Ranch areas of Pinal County, Arizona. Based on the prices below which price range can be expected for a home purchased in Toby Ranch? In the King River area? In Pinal County? o King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000} o Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000} • Given a set of test scores: 99, 96, 94, 93, 90, 88, 86, 77, 70, 68, find the mean, median and standard deviation. Explain how the values vary about the mean and median. What information does this give the teacher? Students may use spreadsheets, graphing calculators and statistical software to statistically identify outliers and analyze data sets with and without outliers as appropriate.
		S-ID.B.5-6 Students may use spreadsheets, graphing calculators, and statistical software to create frequency tables and determine associations or trends in the data.

Examples:

Two-way Frequency Table

A two-way frequency table is shown below displaying the relationship between age and baldness. We took a sample of 100 male subjects, and determined who is or is not bald. We also recorded the age of the male subjects by categories.

Two-way Frequency Table			
Bald	Age		Total
	Younger than 45	45 or older	
No	35	11	46
Yes	24	30	54
Total	59	41	100

The *total* row and *total* column entries in the table above report the marginal frequencies, while entries in the body of the table are the joint frequencies.

Two-way Relative Frequency Table

The relative frequencies in the body of the table are called conditional relative frequencies.

Two-way Relative Frequency Table			
Bald	Age		Total
	Younger than 45	45 or older	
No	0.35	0.11	0.46
Yes	0.24	0.30	0.54
Total	0.59	0.41	1.00

The residual in a regression model is the difference between the observed and the predicted y for some x (y the dependent variable and x the independent variable).

So if we have a model $y = ax + b$, and a data point (x_i, y_i) the residual is for this point is: $r_i = y_i - (ax_i + b)$. Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the

		variables are related, fit functions to data, perform regressions, and calculate residuals. **Example:** Measure the wrist and neck size of each person in your class and make a scatterplot. Find the least squares regression line. Calculate and interpret the correlation coefficient for this linear regression model. Graph the residuals and evaluate the fit of the linear equations.
		S-ID.C.7-9 Students may use spreadsheets or graphing calculators to create representations of data sets and create linear models. **Example:** • Lisa lights a candle and records its height in inches every hour. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.6), (3, 14.9), (4, 13.2), (5, 11.5), (7, 8.1), (9, 4.7), and (10, 3). Express the candle's height (h) as a function of time (t) and state the meaning of the slope and the intercept in terms of the burning candle. Solution: $h = -1.7t + 20$ Slope: The candle's height decreases by 1.7 inches for each hour it is burning. Intercept: Before the candle begins to burn, its height is 20 inches. Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients. **Example:** Collect height, shoe-size, and wrist circumference

		data for each student. Determine the best way to display the data. Answer the following questions: Is there a correlation between any two of the three indicators? Is there a correlation between all three indicators? What patterns and trends are apparent in the data? What inferences can be made from the data? Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment. **Example:** Diane did a study for a health class about the effects of a student's end-of-year math test scores on height. Based on a graph of her data, she found that there was a direct relationship between students' math scores and height. She concluded that "doing well on your end-of-course math tests makes you tall." Is this conclusion justified? Explain any flaws in Diane's reasoning.

Unit 5: Linear equations and inequalities. Suggested number of days: 10

Students have written and solved linear equations and inequalities in their previous mathematics courses. The work of this unit should be on bringing students to mastery of this area of their mathematical study. This unit leverages the connection between equations and functions and explores how different representations of a function lead to techniques to solve linear equations, including tables, graphs, concrete models, algebraic operations, and “undoing” (reasoning backwards). This unit provides opportunities for students to continue to practice their ability to create and graph equations in two variables, as described in **A-CED.A.2** and **A-REI.D.10.**

Common Core State Standards for Mathematical Content

Creating Equations★— A-CED

A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear ~~and quadratic~~ functions, ~~and simple rational and exponential functions.~~

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities-- A-REI

A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that

Comments

The work of **A-CED.A.1** should focus on linear equations and inequalities. Exponential equations will be addressed in **Unit 8: Exponential functions and equations**, and quadratic equations will be addressed **Unit 11: Quadratic equations**. Rational equations should be addressed in Algebra II.

The work of **A-CED.A.3, A-REI.D.11,** and **A-REI.D.12** should focus on single linear equations and inequalities, as systems of linear equations and inequalities are addressed in **Unit 6: Systems of linear equations and inequalities.**

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.

In this unit, students must be able to understand the questions they are being asked to answer, create appropriate equations and inequalities that

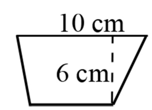
Explanations and Examples

A-CED.A.1,3,4

Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.

Examples:

- Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.

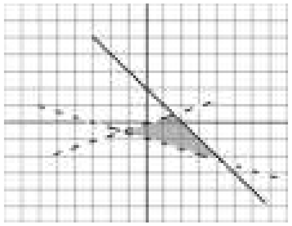


Example:

- A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8.
  - Write a system of inequalities to represent the situation.
  - Graph the inequalities.
  - If the club buys 150 hats and 100 jackets, will the conditions be satisfied?
  - What is the maximum number of jackets they can buy and still meet the conditions?

the original equation has a solution. Construct a viable argument to justify a solution method. B. Solve equations and inequalities in one variable 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. D. Represent and solve equations and inequalities graphically 11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, ~~polynomial, rational, absolute value, exponential, and logarithmic functions~~. 12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), ~~and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes~~.	will allow them to answer these questions, and be creative and flexible in the approaches they take to solve these equations and inequalities **(MP.1, MP.4)**. In order to create accurate equations and inequalities, students must be able to describe relationships precisely **(MP.6).**	
		A-REI.A.1 Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions. **Examples:** • Explain why the equation $x/2 + 7/3 = 5$ has the same solutions as the equation $3x + 14 = 30$. Does this mean that $x/2 + 7/3$ is equal to $3x$ +

		14? • Show that *x* = 2 and *x* = -3 are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning. **A-REI.B.3** **Examples:** • $-\frac{7}{3}y - 8 = 111$ • *3x > 9* • *ax + 7 = 12* • $\frac{3+x}{7} = \frac{x-9}{4}$ • Solve for *x*: 2/3*x* + 9 < 18
		A.REI.D.11-12 Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions. **Example:** • Given the following equations determine the *x* value that results in an equal output for both functions. $f(x) = 3x - 2$ $g(x) = (x+3)^2 - 1$ Students may use graphing calculators, programs, or applets to model and find solutions for inequalities or systems of inequalities. **Examples:** • Graph the solution: $y \le 2x + 3$. • A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women's magazines,

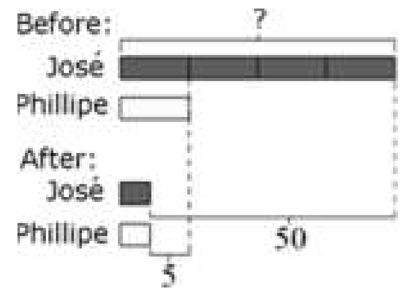
		but the company always publishes at least as many women's magazines as men's magazines. Find a system of inequalities that describes the possible number of men's and women's magazines that the company can produce each year consistent with these policies. Graph the solution set. • Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system. $\begin{cases} x-3y>0 \\ x+y\le 2 \\ x+3y>-3 \end{cases}$ Solution:  (3, 2) is not an element of the solution set (graphically or by substitution).

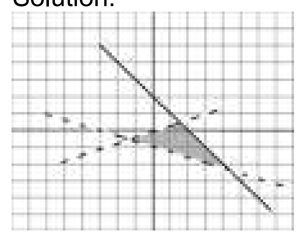
Unit 6: Systems of linear equations and inequalities. Suggested number of days: 15

In this unit students continue the study of systems of linear equations that they began in Grade 8. This unit should solidify their understanding of that prior work, and extend that understanding to creating and solving systems of linear inequalities. This unit provides opportunities for students to continue creating and graphing equations in two variables, as described in **A-CED.A.2**. They also extend their understanding of estimating solutions to equations graphically (**A-REI.D.11**) to estimating solutions of systems of equations.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples

Creating Equations★— A-CED
A. Create equations that describe numbers or relationships
3. Represent constraints ~~by equations or inequalities, and~~ by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
 	
Reasoning with Equations and Inequalities-- A-REI
C. Solve systems of equations
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
D. Represent and solve equations and inequalities graphically
12. Graph ~~the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph~~ the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
	For **A-CED.A.3** and **A-REI.D.12**, students should focus on systems of linear equations and inequalities in two variables.
	
	Common Core State Standards for Mathematical Practice
	1. Make sense of problems and persevere in solving them.
	2. Reason abstractly and quantitatively.
	4. Model with mathematics.
	6. Attend to precision.
	
	In this unit, students must be able to understand the problem they are being asked to solve and the constraints on the quantities in the problem (**MP.1, MP.4**). In order to model the constraints of the problem, students must be able to create precise algebraic representations (**MP.6**) in the form of equations or inequalities. Students must then be able to manipulate these representations and then interpret the results of that manipulation in the context of the problem being solved (**MP.2**)
		A-CED.A.3
		Example:
		• A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8.
		  o Write a system of inequalities to represent the situation.
		  o Graph the inequalities.
		  o If the club buys 150 hats and 100 jackets, will the conditions be satisfied?
		  o What is the maximum number of jackets they can buy and still meet the conditions?
		
		A-REI.C.5-6
		Example:
		Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that they two numbers, *x* and *y*, satisfy the equations $x + y = 10$ and $x – y = 4$.
		
		The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations.

		Examples: • José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.  • Solve the system of equations: $x + y = 11$ and $3x – y = 5$. • Use a second method to check your answer. • Solve the system of equations: • $x – 2y + 3z = 5$, $x + 3z = 11$, $5y – 6z = 9$. • The opera theater contains 1,200 seats, with three different prices. The seats cost \$45 dollars per seat, \$50 per seat, and \$60 per seat. The opera needs to gross \$63,750 on seat sales. There are twice as many \$60 seats as \$45 seats. How many seats in each level need to be sold? **A-REI.D.12** Students may use graphing calculators, programs, or applets to model and find solutions for inequalities or systems of inequalities. **Examples:**

		• Graph the solution: $y \leq 2x + 3$. • A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women's magazines, but the company always publishes at least as many women's magazines as men's magazines. Find a system of inequalities that describes the possible number of men's and women's magazines that the company can produce each year consistent with these policies. Graph the solution set. • Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system. $\begin{cases} x - 3y > 0 \\ x + y \leq 2 \\ x + 3y > -3 \end{cases}$ Solution:  (3, 2) is not an element of the solution set (graphically or by substitution).

Unit 7: Relationships that are not linear. Suggested number of days: 15		
In this unit students explore examples of nonlinear functions that exhibit some linear characteristics as they work with absolute value and step functions. Students also connect rational exponents to roots, and investigate square root and cube root functions as other special instances of nonlinear functions. This unit again provides opportunities for students to create and graph equations in two or more variables (**A-CED.A.2**) and use and interpret function notation (**F-IF.A.2**).		
Common Core State Standards for Mathematical Content	**Comments**	**Explanations and Examples**
The Real Number System—N-RN A. Extend the properties of exponents to rational exponents. 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.* 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. **Interpreting Functions—F-IF** C. Analyze functions using different representations 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	**Common Core State Standards for Mathematical Practice** 2. Reason abstractly and quantitatively. 4. Model with mathematics. 6. Attend to precision. When possible in this unit, students should work with real-world applications of absolute value, step, square root, and cube root functions to allow them to demonstrate their ability to reason abstractly and quantitatively (**MP.2**) and model with mathematics (**MP.4**). Their work in extending the properties of exponents to rational exponents will require careful use of definitions and precision in communicating their reasoning (**MP.6**).	**N-RN.A.1-2** Students may explain orally or in written format. **Examples:** • $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$; $5^{\frac{2}{3}} = \sqrt[3]{5^2}$ • Rewrite using fractional exponents: $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{\frac{4}{5}}$ • Rewrite $\frac{\sqrt{x}}{x^2}$ in at least three alternate forms. Solution: $x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{\sqrt{x^3}} = \frac{1}{x\sqrt{x}}$ • Rewrite $\sqrt[4]{2^{-4}}$ .using only rational exponents. • Rewrite $\sqrt[3]{x^3+3x^2+3x+1}$ in simplest form. **Examples:** • The Pythagorean Theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation $a^2 + b^2 = c^2$. ○ Why might the theorem need to be solved for c? ○ Solve the equation for c and write a problem situation where this form of the equation might be useful.

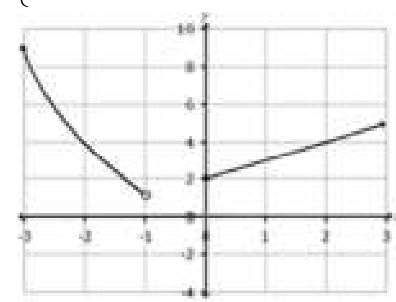
- o Solve $V = \frac{4}{3}\pi r^3$ for radius *r.*
- Motion can be described by the formula below, where *t* = time elapsed, *u*=initial velocity, *a* = acceleration, and *s* = distance traveled
  $s = ut + \frac{1}{2}at^2$
  - o Why might the equation need to be rewritten in terms of *a*?
  - o Rewrite the equation in terms of *a*.

F-IF.C.7
Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.

Examples:

- Describe key characteristics of the graph of *f(x)* = │*x* – 3│ + 5.
- Sketch the graph and identify the key characteristics of the function described below.

$$F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$$



- Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key

		characteristics of the graph. • Graph $f(x) = 2 \tan x – 1$. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?

Unit 8: Exponential functions and equations.   Suggested number of days: 15

This unit explores different situations that can be modeled with exponential functions and equations. Students use tables and graphs to contrast the repeated multiplication of exponential patterns with the repeated addition of linear patterns. This unit continues to reinforce the work with creating and representing equations described in **A-CED.A.2** and **A-REI.D.10** and with connecting the structure of expressions to contexts (**A-SSE.A.1.a**). This unit also deepens students' understanding of functions and their notation as described in **F-IF.A.2**. Students will investigate key features, domains, and ranges of exponential functions as described in **F-IF.B.4** and **F-IF.B.5**; write exponential functions to model relationships between two quantities as in **F-BF.A.1a**; use technology to explore simple transformations of exponential functions as described in **F-BF.B.3**; and compare properties of exponential functions as in **F-IF.C.9**.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Seeing Structure in Expressions—A-SSE B. Write expressions in equivalent forms to solve problems 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **Interpreting Functions—F-IF** C. Analyze functions using different representations 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude.~~ **Linear, Quadratic, and Exponential Models—F-LE** A. Construct and compare linear, quadratic, and exponential models and solve problems 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove ~~that linear functions grow by equal differences over equal intervals, and~~ that exponential functions grow by equal factors over	The work with **F-IF.C.7e**, **F-LE.A.1a**, **F-LE.A.1c**, and **F-LE.B.5** should focus on exponential functions with domains in the integers. **Common Core State Standards for Mathematical Practice** 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. In this unit, students continue to create mathematical models of contextual situations, while attending to limitations on those models and interpreting the results (**MP.2, MP.4, MP.6**). As they compare exponential to linear functions, students should make and justify conjectures (**MP.3**) They may use graphing technology as they explore transformations and fit exponential functions to data (**MP.5**).	**A-SSE.B.3** Students will use the properties of operations to create equivalent expressions. **Examples:** • Express $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$ in factored form and use your answer to say for what values of x the expression is zero. • Write the expression below as constant times a power of x and use your answer to decide whether the expression gets larger or smaller as x gets larger. ○ $\frac{(2x^3)^2(3x^4)}{(x^2)^3}$ **F-IF.C.7** Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. **Examples:** • Describe key characteristics of the graph of $f(x) = \lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key

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equal intervals.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
B. Interpret expressions for functions in terms of the situation they model
5. Interpret the parameters in a ~~linear or~~ exponential function in terms of a context.

Interpreting Categorical and Quantitative Data—S-ID
B. Summarize, represent, and interpret data on two categorical and quantitative variables
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
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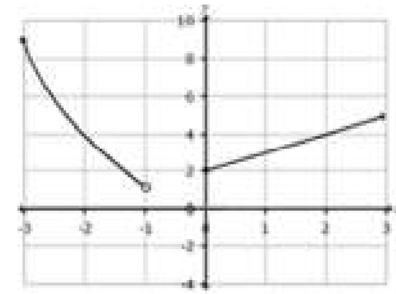
characteristics of the function described below.
$$F(x)=\begin{cases} x+2 \text{ for } x \ge 0 \\ -x^2 \text{ for } x < -1 \end{cases}$$


• Graph the function f(x) = 2^x by creating a table of values. Identify the key characteristics of the graph.
• Graph f(x) = 2 tan x – 1. Describe its domain, range, intercepts, and asymptotes.
• Draw the graph of f(x) = sin x and f(x) = cos x. What are the similarities and differences between the two graphs?
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F-LE.A.1,3
Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions.

Examples:
• A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in Plan 1? Plan 2? Plan 3?
4. $59.95/month for 700 minutes and $0.25 for each additional minute,
5. $39.95/month for 400 minutes and $0.15 for each additional minute, and
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		6. $89.95/month for 1,400 minutes and $0.05 for each additional minute. • A computer store sells about 200 computers at the price of $1,000 per computer. For each $50 increase in price, about ten fewer computers are sold. How much should the computer store charge per computer in order to maximize their profit? Students can investigate functions and graphs modeling different situations involving simple and compound interest. Students can compare interest rates with different periods of compounding (monthly, daily) and compare them with the corresponding annual percentage rate. Spreadsheets and applets can be used to explore and model different interest rates and loan terms. Students can use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions. • A couple wants to buy a house in five years. They need to save a down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. How much will they need to save each month in order to meet their goal? • Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth each type of interest has? o Lee borrows $9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest. o Lee borrows $9,000 from a bank to buy a car. The bank charges 5% interest compounded annually. • Calculate the future value of a given amount of money, with and without technology. • Calculate the present value of a certain

		amount of money for a given length of time in the future, with and without technology. **Example:** Contrast the growth of the $f(x)=x^3$ and $f(x)=3^x$.
		F-LE.B.5 Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions. Example: A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where n is the number of years since the initial deposit. What is the value of r? What is the meaning of the constant P in terms of the savings account? Explain either orally or in written format.
		S-ID.B.6 The residual in a regression model is the difference between the observed and the predicted y for some x (y the dependent variable and x the independent variable). So if we have a model $y = ax + b$, and a data point (x_i, y_i) the residual for this point is: $r_i = y_i - (ax_i + b)$. Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals. **Example:** Measure the wrist and neck size of each person in your class and make a scatterplot. Find the least squares regression line. Calculate and interpret the correlation coefficient for this linear regression model. Graph the residuals and evaluate the fit of the linear equations.

Unit 9: Polynomial expressions and functions. Suggested number of days: 15

In this unit students learn how to multiply, add, and subtract quadratic and cubic polynomials using concrete models and analytic techniques. They also learn how to factor quadratic trinomials and cubic polynomials using concrete models and analytic techniques. This work with polynomial expressions serves as a bridge to introductory work with polynomial functions, laying the foundation for deeper study of quadratic functions in **Unit 10** of this course and general polynomial functions in Algebra II. In this unit students will have many opportunities to interpret parts of an expression, as described in **A-SSE.A.1a**.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Seeing Structure in Expressions—A-SSE A. Interpret the structure of expressions 1. Interpret expressions that represent a quantity in terms of its context. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* 2. Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* B. Write expressions in equivalent forms to solve problems 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. **Arithmetic with Polynomial and Rational Expressions—A-APR** A. Perform arithmetic operations on polynomials 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. B. Understand the relationship between zeros and factors of polynomials 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	Limit the work under **A-APR.B.3** to quadratic and cubic polynomials having linear and quadratic factors. **Common Core State Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning. In order to be successful with factoring, students must become adept at seeing the structure in various expressions and making use of that structure (**MP.7**) and they must also be willing to try different strategies when an initial strategy does not work (**MP.1**). Their ability to look for regularity in calculations (**MP.8**) will be helpful as they explore the relationship between multiplication and factoring.	**A-SSE.A.1-2** Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context. Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. **Example:** Factor $x^3 - 2x^2 - 35x$ **A-SSE.B.3** Students will use the properties of operations to create equivalent expressions. **Examples:** • Express $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$ in factored form and use your answer to say for what values of x the expression is zero. • Write the expression below as constant times a power of x and use your answer to decide whether the expression gets larger or smaller as x gets larger. • $\frac{(2x^3)^2(3x^4)}{(x^2)^3}$

		A-APR.B.3 Graphing calculators or programs can be used to generate graphs of polynomial functions.

Unit 10: Quadratic functions. Suggested number of days: 15

This unit builds on students' previous exposure to quadratic functions, focusing on how to build quadratic functions that model real-world situations. Students learn how to use the method of completing the square to transform quadratic function rules to understand the behavior of the function. This unit provides opportunities for students to continue to engage with a number of standards they have encountered previously. They will work with **A-SSE.A.1** and **A-SSE.A.2** as they interpret and use the structure in quadratic expressions to understand quadratic functions. They will create and represent quadratic equations in two variables (**A-CED.1** and **A-REI.10**), apply function notation in the context of quadratic functions (**F-IF.A.2**), relate the domain of quadratic function to its graph (**F-IF.B.5**) and compare properties of quadratic functions represented in different ways (**F-IF.C.9**). They will build quadratic functions to model relationships between two quantities (**F-BF.A.1a**) and continue their study of transformations as they explore how changes in different parameters affect the graph of **y=x²** (**F-BF.B.3**).

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Seeing Structure in Expressions—A-SSE B. Write expressions in equivalent forms to solve problems 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. b. Complete the square in a quadratic expression to reveal the maximum value of the function it defines. **Interpreting Functions—F-IF** B. Interpret functions that arise in applications in terms of the context 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.* C. Analyze functions using different representations 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and	Students return to **F-IF.B.4, F-IF.C.7a,** and **S-ID.B.6a** as they analyze graphs of quadratic functions and fit quadratic models to data. **Common Core State Standards for Mathematical Practice** 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. In this unit, students will investigate data that can be modeled with quadratic functions and will create algebraic representations of those models that precisely communicate different characteristics of the situation being modeled (**MP.4, MP.6**). They may choose to use graphing technology to explore transformations or to fit quadratic functions to data (**MP.5**). They will make use of the structure of different quadratic expressions to make sense of the situations being modeled (**MP.7**).	**A-SSE.B.3** Students will use the properties of operations to create equivalent expressions. **Examples:** • Express $2(x^3 – 3x^2 + x – 6) – (x – 3)(x + 4)$ in factored form and use your answer to say for what values of *x* the expression is zero. **F-IF.B.4** Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. **Examples:** • A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = – 16t^2 + 96t + 180$, where *t* is measured in seconds and *h* is height above the ground measured in feet. ○ What is a reasonable domain restriction for *t* in this context? ○ Determine the height of the rocket two seconds after it was launched. ○ Determine the maximum height obtained by the rocket. ○ Determine the time when the rocket is 100 feet above the ground. ○ Determine the time at which the rocket hits the ground.

interpret these in terms of a context.

Interpreting Categorical and Quantitative Data—S-ID

B. Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

- How would you refine your answer to the first question based on your response to the second and fifth questions?

- Compare the graphs of $y = 3x^2$ and $y = 3x^3$.

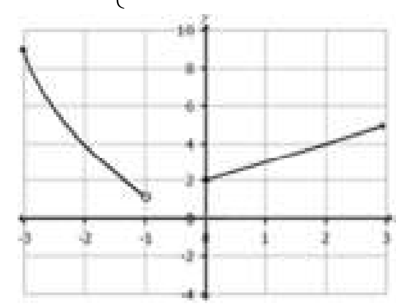
F-IF.C.7

Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.

Examples:

- Describe key characteristics of the graph of $f(x) = |x - 3| + 5$.
- Sketch the graph and identify the key characteristics of the function described below.

$$F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$$



- Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph.
- Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes.
- Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?

		S-ID.B.6 Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals.

Unit 11: Quadratic equations. Suggested number of days: 15

This unit focuses on quadratic equations in one variable (**A-CED.A.1**) that arise from quadratic functions. Students learn to solve these equations by graphing, factoring, and completing the square, and they see how the solution methods are connected as they connect the roots of an equation, the x-intercepts of a graph, and the zeros of a function. It also introduces students to the quadratic formula as a method for solving quadratic equations.

Common Core State Standards for Mathematical Content

The Real Number System—N-RN

B. Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers are rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Reasoning with Equations and Inequalities—A-REI

B. Solve equations and inequalities in one variable

4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Comments

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
7. Look for and make use of structure.

In this unit, students must be able to understand the problem they are being asked to solve, create equations that model the problem, and interpret the solutions in the context of the problem (**MP.1, MP.2, MP.4**). They must become adept at seeing the structure in various expressions and making use of that structure to choose efficient solution tools and techniques (**MP.5, MP.7**).

Explanations and Examples

N-RN.B.3

Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction (or between multiplication and addition).

A-REI.B.4

Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.

Value of Discriminant	Nature of Roots	Nature of Graph
$b^2 - 4ac = 0$	1 real roots	intersects *x*-axis once
$b^2 - 4ac > 0$	2 real roots	intersects *x*-axis twice
$b^2 - 4ac < 0$	2 complex roots	does not intersect *x*-axis

		Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation. • What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?