Arizona’s Common Core Standards

Mathematics Curriculum Map

PreCalculus

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS
State Board Approved June 2010

Overview of the Common Core Standards Structure - HS

Conceptual Category	**Number & Quantity (N)**	**Algebra (A)**	**Functions (F)**	**Geometry (G)**	**Statistics & Probability (S)**	**Modeling**
Domains	The Real Number System (N-RN)	Seeing Structure in Expressions (A-SSE)	Interpreting Functions (F-IF)	Congruence (G-CO)	Interpreting Categorical & Quantitative Data (S-ID)	Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (☆)
	Quantities (N-Q)	Arithmetic with Polynomials & Rational Expressions (A-APR)	Building Functions (F-BF)	Similarity, Right Triangles, & Trigonometry (G-SRT)	Making Inferences & Justifying Conclusions (S-IC)	
	The Complex Number System (N-CN)	Creating Equations (A-CED)	Linear, Quadratic, & Exponential Models (F-LE)	Expressing Geometric Properties with Equations (G-GPE)	Conditional Probability & the Rules of Probability (S-CP)	
	Vector & Matrix Quantities (N-VM)	Reasoning with Equations & Inequalities (A-REI)	Trigonometric Functions (F-TF)	Geometric Measurement & Dimension (G-GMD)	Using Probability to Make Decisions (S-MD)	

Algebra 1

The PreCalculusI course outlined in this scope and sequence document begins with connections back to prior work Algebra 2 and Geometry, efficiently reviewing algebraic concepts that students have already studied while at the same time moving students forward into the new ideas described in the high school fourth year standards.

Students contrast linear, quadratic, exponential, absolute value, polynomial, rational, and radical functions as they compare/contrast these models using the familiar tools of tables, graphs, and symbols. They apply these same tools to understand inverse functions and composed functions. Through the concept of transformations of functions, students develop and understanding of vectors, and matrices, using these tools to efficiently solve real world problems.

Students will build on prior knowledge of basic trigonometry to develop an understanding of trigonometric functions and trig identities. Students will learn about a variety of conic sections, further extending their knowledge of trigonometry to help them make sense of this set of geometric figures.

The Mathematical Practices should become a natural way in which students come to understand and do mathematics. While, depending on the content to be understood or on the problem to be solved, any practice might be brought to bear, some practices may prove more useful than others. Opportunities for highlighting certain practices are indicated in different units in this document, but this highlighting should not be interpreted to mean that other practices should be neglected in those units.

	Units In Order of Instruction	Topic	Domains Addressed*	Mathematical Practices (Address all. Highlight bold.)	Suggested Unit Length (in days)
Q1	Unit 1	Sequences and Series	★F-BF F-LE ★A-SSE	1, **2**, 3, **4**, 5, **6**, 7, **8**	10
	Unit 2	Algebraic Expressions, Equations and Inequalities (divided into 6 sections)	(see below)		See below
	Section 2.1	Linear, Piecewise, Quadratic, and Absolute Value Functions	F-IF N-VM A-REI N-CN	**1**, 2, 3, **4**, 5, 6, **7**, **8**	10
	Section 2.2	Polynomial and Exponential Functions	F-LE A-SSE F-IF A-REI A-APR N-CN	**1**, 2, 3, **4**, 5, 6, **7**, **8**	10
	Section 2.3	Radical Equations and Functions		**1**, 2, 3, **4**, 5, 6, **7**, **8**	5
	Section 2.4	Rational Equations and Functions	F-IF A-APR	**1**, 2, 3, **4**, 5, 6, **7**, **8**	10
Q2	Section 2.5	Inverse Functions	F-BF	**1**, **2**, **3**, 4, 5, 6, **7**, **8**	10
	Section 2.6	Composed Functions		**1**, **2**, **3**, 4, 5, **6**, **7**, **8**	10
	Unit 3	Transformations of Functions	N-VM	**1**, 2, 3, **4**, 5, 6, 7, **8**	10
	Unit 4	Exponential and Logarithmic Functions	F-BF	**1**, 2, **3**, **4**, **5**, 6, 7, 8	10
Q3	Unit 5	Trigonometry	(see below)		See below
	Section 5.1	Trigonometry of General Triangles	G-SRT	**1**, 2, 3, **4**, 5, 6, **7**, **8**	10
	Section 5.2	Trigonometric Functions	F-BF	1, 2, 3, **4**, **5**, **6**, **7**, 8	15

			F-TF		
	Section 5.3	Trigonometric Identities	F-TF	**1, 2**, 3, **4, 5**, 6, **7**, 8	10
Q4	Unit 6	Conic Sections	G-GPE	**1, 2**, 3, **4, 5**, 6, **7**, **8**	20
	Unit 7	Statistics and Probability	S-CP S-MD	**1, 2**, 3, **4, 5**, 6, **7**, 8	15

* Some standards in the domain are addressed in this unit. Other standards are addressed in other units.

★ Modeling standards are in this domain.

The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Overarching habits of mind of a productive math thinker	1.Make sense of problems and persevere in solving them	• Have or value sense-making • Use patience and persistence to listen to others • Be able to use strategies • Use self-evaluation and redirections • Be able to show or use multiple representations • Communicate both verbally and in written format • Be able to deduce what is a reasonable solution	• Provide open-ended and rich problems • Ask probing questions • Model multiple problem-solving strategies through Think- Alouds • Promotes and values discourse and collaboration • Cross-curricular integrations • Probe student responses (correct or incorrect) for understanding and multiple approaches • Provide solutions	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about...? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize...represent... show...?
	6.Attend to precision	• Communicate with precision-orally & written • Use mathematics concepts and vocabulary appropriately. • State meaning of symbols and use appropriately • Attend to units/labeling/tools accurately • Carefully formulate explanations • Calculate accurately and efficiently • Express answers in terms of context • Formulate and make use of definitions with others and their own reasoning.	• Think aloud/Talk aloud • Explicit instruction given through use of think aloud/talk aloud • Guided Inquiry including teacher gives problem, students work together to solve problems, and debriefing time for sharing and comparing strategies • Probing questions targeting content of study	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language...,definitions..., properties can you use to explain...? • How could you test your solution to see if it answers the problem?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book

The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	• Create multiple representations • Interpret problems in contexts • Estimate first/answer reasonable • Make connections • Represent symbolically • Visualize problems • Talk about problems, real life situations • Attending to units • Using context to think about a problem	• Develop opportunities for problem solving • Provide opportunities for students to listen to the reasoning of other students • Give time for processing and discussing • Tie content areas together to help make connections • Give real world situations • Think aloud for student benefit • Value invented strategies and representations • Less emphasis on the answer	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _______ related to ________? • What is the relationship between ______and ______? • What does_______mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use…? • Could we have used another operation or property to solve this task? Why or why not?
Reasoning and Explaining	**3. Construct viable arguments and critique the reasoning of others**	• Ask questions • Use examples and non-examples • Analyze data • Use objects, drawings, diagrams, and actions • Students develop ideas about mathematics and support their reasoning • Listen and respond to others • Encourage the use of mathematics vocabulary	• Create a safe environment for risk-taking and critiquing with respect • Model each key student disposition • Provide complex, rigorous tasks that foster deep thinking • Provide time for student discourse • Plan effective questions and student grouping	• What mathematical evidence would support your solution? • How can we be sure that…? / How could you prove that…? • Will it still work if…? • What were you considering when…? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? • Did you try a method that did not work? Why didn't it work? Could it work? • What is the same and what is different about…? • How could you demonstrate a counter-example?

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The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

<table>
<tr><th colspan="2">Mathematics Practices</th><th>Student Dispositions</th><th>Teacher Actions</th><th>Related Questions</th></tr>
<tr><td rowspan="2">Modeling and Using Tools</td><td>4. Model with mathematics</td><td>• Realize they use mathematics (numbers and symbols) to solve/work out real-life situations
• When approached with several factors in everyday situations, be able to pull out important information needed to solve a problem.
• Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable. If not, go back and look for more information
• Make sense of the mathematics</td><td>• Allow time for the process to take place (model, make graphs, etc.)
• Model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written)
• Make appropriate tools available
• Create an emotionally safe environment where risk taking is valued
• Provide meaningful, real world, authentic, performance-based tasks (non-traditional work problems)</td><td>• What number model could you construct to represent the problem?
• What are some ways to represent the quantities?
• What is an equation or expression that matches the diagram, number line, chart, table, and your actions with the manipulatives?
• Where did you see one of the quantities in the task in your equation or expression? What does each number in the equation mean?
• How would it help to create a diagram, graph, table...?
• What are some ways to visually represent...?
• What formula might apply in this situation?</td></tr>
<tr><td>5. Use appropriate tools strategically</td><td>• Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base 10 blocks, compass, protractor)
• Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools)</td><td>• Maintain appropriate knowledge of appropriate tools
• Effective modeling of the tools available, their benefits and limitations
• Model a situation where the decision needs to be made as to which tool should be used</td><td>• What mathematical tools can we use to visualize and represent the situation?
• Which tool is more efficient? Why do you think so?
• What information do you have?
• What do you know that is not stated in the problem?
• What approach are you considering trying first?
• What estimate did you make for the solution?
• In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative?
• Why was it helpful to use...?
• What can using a ______ show us that _____may not?
• In what situations might it be more informative or helpful to use...?</td></tr>
</table>

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The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Seeing structure and generalizing	**7. Look for and make use of structure**	• Look for, interpret, and identify patterns and structures • Make connections to skills and strategies previously learned to solve new problems/tasks • Reflect and recognize various structures in mathematics • Breakdown complex problems into simpler, more manageable chunks	• Be quiet and allow students to think aloud • Facilitate learning by using open-ended questioning to assist students in exploration • Careful selection of tasks that allow for students to make connections • Allow time for student discussion and processing • Foster persistence/stamina in problem solving • Provide graphic organizers or record student responses strategically to allow students to discover patters	• What observations do you make about...? • What do you notice when...? • What parts of the problem might you eliminate..., simplify...? • What patterns do you find in...? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to...? • In what ways does this problem connect to other mathematical concepts?
	8. Look for and express regularity in repeated reasoning	• Identify patterns and make generalizations • Continually evaluate reasonableness of intermediate results • Maintain oversight of the process	• Provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge • Provide adequate time for exploration • Provide time for dialogue and reflection • Ask deliberate questions that enable students to reflect on their own thinking • Create strategic and intentional check in points during student work time.	• Explain how this strategy works in other situations? • Is this always true, sometimes true or never true? • How would we prove that...? • What do you notice about...? • What is happening in this situation? • What would happen if...? • Is there a mathematical rule for...? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

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Unit 1: Sequences and Series Suggested number of days: 10		
In this unit, through context and later without context, students will build sequences, both arithmetic and geometric, using recursive, explicit and graphical models. Students will compare and contrast arithmetic and geometric sequences.		
Arizona College and Career Ready Standards for Mathematical Content	**Comments**	**Explanations and Examples**
Build, compare, and contrast arithmetic and geometric sequences (understanding that arithmetic sequences are linear and geometric sequences are exponential). Investigate limits of sequences and series. Be able to find specific terms of a sequence or series. • **F-BF.2 - build sequences, both arithmetic and geometric, using recursive, explicit, and graphical models.** • **F-LE.2 – Construct linear and exponential functions, including arithmetic and geometric sequences given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).** Solve series problems • **A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For***	Students should be challenged to build on prior learning about sequences, both arithmetic and geometric, applying their understandings about sequences to applications, building on fluency in interpreting the meaning of common difference or common ratio in terms of a variety of contexts. This unit introduces students to the concept of limits, which will be further explored in later units. **Common Core State Standards for Mathematical Practice** 2. Reason abstractly and quantitatively 4. Model with mathematics 6. Attend to precision 8. Look for and express regularity in repeated reasoning. In this unit, students investigate sequences and series as mathematical models (MP.4). In order to analyze and communicate about these models, students must attend to	•

example, calculate mortgage payments. **(★)** • **(Used throughout) HS.F-BF.A.1. Write a function that describes a relationship between two quantities.** **c. Compose functions.** ***For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.***	MP.2 and MP.6. In developing symbolic representations of mathematical relationships, students might examine several specific instances of the relationship to find a generalizable regularity (MP.8).	

Unit 2: Algebraic Expressions, Equations and Inequalities – Throughout this unit, students will build on their prior experiences with a variety of equations to strengthen problem-solving fluency. Students will be encouraged to view these skills as tools to be used to solve real world problems. Context-based problems should be presented and students should apply their understanding of linear, quadratic, absolute value, polynomial, exponential, radical and rational equations to solve real world problems. Connections to prior learning should be emphasized, but it is important that students extend their learning beyond past experience to construct new understandings. Throughout the unit students use models and multiple representations, including graphs, tables, and symbols to solve or interpret solutions.

Section 2.1 – Linear, Piecewise, Quadratic and Absolute Value Suggested number of days: 10

In this section, first with context and later without context, students will solve and interpret solutions to equations and systems of equations of functions, including linear, piecewise, quadratic and absolute value functions. Both with and without context, students will solve and interpret solutions to inequalities and systems of inequalities of functions including linear, piecewise, quadratic and absolute value functions. Students will define reasonable limits for the domain and range of each function. Students will solve systems of equations using matrices.

Arizona College and Career Ready Standards for Mathematical Content	Comments	Explanations and Examples
First with context and later without context students will solve and interpret solutions to equations and systems of equations of functions, including linear, piecewise, quadratic and absolute value functions. Students will define reasonable limits for the domain and range of each function. Students will learn to use matrices to solve systems of equations as an added strategy, identifying situations where matrices will make the solving process more efficient. • **HS.F-IF.C.7. Graph functions expressed symbolically and show key features of the**	**Arizona College and Career Ready Standards for Mathematical Practice** 4. Model with mathematics 8. Look for and express regularity in repeated reasoning. In this unit, students continue to demonstrate their proficiency with **MP.4** as they create mathematical models of contextual situations, while attending to limitations on those models. Work with linear, piecewise, quadratic and absolute value functions	**HS.N-VM.C.6** Students may use graphing calculators and spreadsheets to create and perform operations on matrices. • The adjacency matrix of a simple graph is a matrix with rows and columns labeled by graph vertices, with a 1 or a 0 in position (v_i, v_j) according to whether v_i and v_j are adjacent or not. A "1" indicates that there is a connection between the two vertices, and a "0" indicates that there is no connection. **Example:** ○ Write an inventory matrix for the following situation. A teacher is buying supplies for two art classes. For class 1, the teacher buys 24 tubes of paint, 12 brushes, and 17 canvases. For class 2, the teacher buys 20 tubes of paint, 14 brushes and 15 canvases. Next year, she has 3 times as many students in each class. What affect does this have on the amount of supplies? ▪ **<u>Solution</u>** ▪ **Year 1**

graph, by hand in simple cases and using technology for more complicated cases.

First with context and later without context students will solve and interpret solutions to inequalities and systems of inequalities of functions including linear, piecewise, quadratic and absolute value functions. Students will define reasonable limits for the domain and range of each function.

Solve systems of equations using matrices

- **HS.N-VM.C.6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.**
- **HS.N-VM.C.7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.**
- **HS.N-VM.C.8. Add, subtract, and multiply matrices of appropriate dimensions.**
- **HS.N-VM.C.9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.**

creates a number of opportunities to reinforce students' ability to recognize and leverage regularity in reasoning (**MP.8**), whether they are developing a general formula for finding the slope of a line or generalizing a pattern of repeated calculations to write a symbolic representation for a function.

$$\begin{array}{c} \text{Class 1} \\ \text{Class 2} \end{array} \begin{bmatrix} 24 & 12 & 17 \\ 20 & 14 & 15 \end{bmatrix}$$

- **Year 2**
- Class 1 $\begin{bmatrix} 72 & 36 & 51 \\ 60 & 42 & 45 \end{bmatrix}$
- Class 2

HS.N-VM.C.7

Students may use graphing calculators and spreadsheets to create and perform operations on matrices.

Examples:

$$-3\begin{bmatrix} -7 & 19 & 15 \\ 41 & -63 & 20 \\ 2 & 0 & -8 \end{bmatrix}$$

- The following is an inventory matrix for Company A's jellybean, lollipop, and gum flavors. The price per unit is $0.03 for jelly beans, gum, and lollipops. Determine the gross profit for each flavor and for the entire lot.

$$\begin{array}{c} \\ \text{C1} \\ \text{C2} \\ \text{C3} \end{array} \begin{array}{c} \begin{array}{ccccccc} \text{F1} & \text{F2} & \text{F3} & \text{F4} & \text{F5} & \text{F6} & \text{F7} \end{array} \\ \begin{bmatrix} 327 & 818 & 465 & 211 & 127 & 134 & 705 \\ 513 & 222 & 312 & 446 & 645 & 671 & 101 \\ 878 & 901 & 51 & 156 & 711 & 423 & 344 \end{bmatrix} \end{array}$$

C1 = Jelly beans
C2 = Lollipops
C3 = Gum

F1 = Vanilla
F2 = Banana
F3 = Strawberry
F4 = Tangerine
F5 = Coconut
F6 = Mint
F7 = Licorice

◦ **HS.N-VM.C.10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.** ◦ **HS.A-REI.C.8. Represent a system of linear equations as a single matrix equation in a vector variable.** ◦ **HS.A-REI.C.9. Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 X 3 or greater).** ◦ **HS.N-CN.A.3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.** ◦ **HS.N-CN.B.4. Represent complex numbers on the complex plane in rectangular ~~and polar~~ form (including real and imaginary numbers), ~~and explain why the rectangular and polar forms of a given complex number represent the same number.~~** ◦ **HS.N-CN.B.6. Calculate the distance between numbers in the complex plane as the modulus**		**HS.N-VM.C.8** Students may use graphing calculators and spreadsheets to create and perform operations on matrices. **Example:** Find 2*A* – *B* + C and $A \cdot B$ given Matrices *A*, *B* and *C* below. Matrix A $\begin{bmatrix} -7 & 19 & 15 \\ 41 & -63 & 20 \\ 2 & 0 & -8 \end{bmatrix}$ Matrix B $\begin{bmatrix} 23 & 18 & 55 \\ -18 & -47 & 11 \\ 39 & -6 & -8 \end{bmatrix}$ Matrix C $\begin{bmatrix} -4 & 7 & 12 \\ 51 & 9 & 80 \\ 13 & 72 & 8 \end{bmatrix}$ **HS.A-REI.C.8** **Example:** Write the system as a matrix equation. $\begin{cases} -b + 2c = 4 \\ a + b - c = 0 \\ 2a + 3c = 11 \end{cases}$ Identify the coefficient matrix, the variable matrix, and the constant matrix. **HS.N-CN.A.3** **Example:** Given *w* = 2 – 5*i* and *z* = 3 + 4*i* *a. Use the conjugate to find the modulus of w.* *b.* Find the quotient of *z* and *w*. **HS.N-CN.C.9** **Examples:** ◦ How many zeros does $-2x^2 + 3x - 8$ have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra.

of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. ○ **HS.N-CN.C.9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.**		○ How many complex zeros does the following polynomial have? How do you know? $p(x) = (x^2 - 3)(x^2 + 2)(x - 3)(2x - 1)$
		•
		•

Section 2.2 - Polynomial and Exponential Functions and Equations. Suggested number of days: 10		
First with context and later without context students will solve and interpret solutions to equations and systems of equations of functions, including polynomial and exponential functions. First with context and later without context students will solve and interpret solutions to inequalities and systems of inequalities of functions including polynomial and exponential functions. Students will define reasonable limits for the domain and range of each function. Throughout this section, students will compare and contrast with linear, quadratic and absolute value functions.		
Arizona College and Career Ready Standards for Mathematical Content	**Comments**	**Explanations and Examples**
• First with context and later without context students will solve and interpret solutions to equations and systems of equations of functions, including polynomial and exponential functions. ○ **F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (★)** ○ **HS.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.** (★) ○ **HS.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.** ***For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*** ○ **F.IF.4 For a function that models a relationship between two**	MP 2: Reason abstractly and quantitatively. Students will use a variety of representations to compare polynomial and exponential equations, interpreting solutions and key features of the graphs in terms of the context they represent. MP 8: Look for and express regularity in repeated reasoning. Students should be encouraged to identify patterns in equations, graphs and tables, connecting these patterns between the different representations. Teachers should provide students with a variety of tasks that allow students to generalize relationships and methods, extending prior learning regarding rates of change in linear, quadratic and exponential models to connect with the variety of equations represented in this section.	Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and polynomial functions to show how the *y* (output) values of the exponential function eventually exceed those of polynomial functions. • Have students draw the graphs of exponential and other polynomial functions on a graphing calculator or computer utility and examine the fact that the exponential curve will eventually get higher than the polynomial function's graph. A simple example would be to compare the graphs (and tables) of the functions $y = x^2$ and $y = 2^x$ to find that the *y* values are greater for the exponential function when $x > 4$.

quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the functions is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity. (★) ○ **F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* (★)** • First with context and later without context students will solve and interpret solutions to inequalities and systems of inequalities of functions including polynomial and exponential functions. ○ **A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions**		

of the equation *f(x) = g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (★)
 	
• Students will define reasonable limits for the domain and range of each function.
• Compare and contrast with linear, quadratic and absolute value functions.
    ○ **A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.**
    ○ **A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.**
    ○ **A.APR.4 Prove polynomial identities and use them to describe numerical relationships.**		

For example, the polynomial identity $(x^2+y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples* ○ **HS.N-CN.C.8. Extend polynomial identities to the complex numbers.** ***For example, rewrite*** $x^2 + 4$ ***as*** $(x + 2i)(x - 2i)$***.*** • Students will use Binomial Theorem to distribute efficiently.		

Section 2.3 - Radical Equations and Functions. Suggested number of days: 5		
First with context and later without context students will solve and interpret solutions to equations and systems of equations of radical functions. First with context and later without context students will solve and interpret solutions to inequalities and systems of inequalities of radical functions. Students will define reasonable limits for the domain and range of each function. Compare and contrast with linear, quadratic, absolute value, and exponential functions.		
Arizona College and Career Ready Standards for Mathematical Content	**Comments**	**Explanations and Examples**
• First with context and later without context students will solve and interpret solutions to equations and systems of equations of radical functions. • First with context and later without context students will solve and interpret solutions to inequalities and systems of inequalities of radical functions. • Students will define reasonable limits for the domain and range of each function. • Compare and contrast with linear, quadratic, absolute value, and exponential functions.	**Arizona College and Career Ready Standards for Mathematical Practice** 4. Model with mathematics 8. Look for and express regularity in repeated reasoning. In this unit, students continue to demonstrate their proficiency with **MP.4** as they create mathematical models of contextual situations, while attending to limitations on those models. Work with radical functions creates a number of opportunities to reinforce students' ability to recognize and leverage regularity in reasoning (**MP.8**), whether they are identifying extraneous solutions or generalizing a pattern of repeated calculations to write a symbolic representation for a function	

Section 2.4 - Rational Equations and Functions. Suggested number of days: 10		
First with context and later without context students will solve and interpret solutions to equations and systems of equations of rational functions. First with context and later without context students will solve and interpret solutions to inequalities and systems of inequalities of rational functions. Students will build on prior learning by comparing and contrasting rational functions with linear, quadratic, absolute value, and exponential functions. Students should understand the properties of and be able to create the graphs of rational functions, including limits, domain, range, asymptotes, and end behavior. Throughout this section students will use models and representations of functions, including graphs, tables, and symbols to solve or interpret solutions.		
Arizona College and Career Ready Standards for Mathematical Content	**Comments**	**Explanations and Examples**
• First with context and later without context students will solve and interpret solutions to equations and systems of equations of rational functions. • First with context and later without context students will solve and interpret solutions to inequalities and systems of inequalities of rational functions. • Compare and contrast with linear, quadratic, absolute value, and exponential functions. • Students should understand the properties of and be able to create the graphs of rational functions, including limits, domain, range, asymptotes, and end behavior. ○ **(HS.F-IF.C.7. *continued*)** **d. Graph rational functions, identifying zeros and asymptotes when suitable**	**Arizona College and Career Ready Standards for Mathematical Practice** 4. Model with mathematics 8. Look for and express regularity in repeated reasoning. In this unit, students continue to demonstrate their proficiency with **MP.4** as they create mathematical models of contextual situations, while attending to limitations on those models. Work with rational equations and functions creates a number of opportunities to reinforce students' ability to recognize and leverage regularity in reasoning (**MP.8**), whether they are developing a general rule for finding exceptions to the domain or generalizing a pattern of repeated calculations to write a symbolic representation for a function.	

factorizations are available, and showing end behavior. • Throughout the unit students use models and representations of functions, including graphs, tables, and symbols to solve or interpret solutions. ○ **HS.A-APR.D.7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.**		

Section 2.5 – Inverse Functions Suggested number of days: 10

Using function notation, students manipulate functions to find the inverse of a function. Students interpret the meaning of the domain and range of a function and its inverse. Students will define reasonable limits for the domain and range of each function.

Arizona College and Career Ready Standards for Mathematical Content	Comments	Explanations and Examples
• Using function notation, students manipulate functions to find the inverse of a function. Students interpret the meaning of the domain and range of a function and its inverse. • Students will define reasonable limits for the domain and range of each function. ○ **HS.F-BF.B.4 Find inverse functions.** **a. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*** **b. Verify by composition that one function is the inverse of another.** **c. Read values of an inverse function from a graph or a table, given that the function has an inverse.** **d. Produce an invertible function from a non-invertible function by restricting the domain.**	**Arizona College and Career Ready Standards for Mathematical Practice** 2. Reason abstractly and quantitatively 4. Model with mathematics 5. Use appropriate tools strategically 7. Look for and make use of structure	•

Section 2.6 – Composition of Functions. Suggested number of days: 10		
Students will compose functions and calculate a value for a set of composed functions. Functions to be composed include linear, quadratic, absolute value, polynomial, exponential, radical and rational functions.		
Arizona College and Career Ready Standards for Mathematical Content	**Comments**	**Explanations and Examples**
• Students will compose functions and calculate a value for a set of composed functions. Functions to be composed include linear, quadratic, absolute value, polynomial, exponential, radical and rational. • Seeing/Using Structure in equations	**Arizona College and Career Ready Standards for Mathematical Practice** 2. Reason abstractly and quantitatively 4. Model with mathematics 7. Look for and make use of structure.	•

<table>
<tr><th colspan="3">Unit 3: Transformations of Functions. Suggested number of days: 10</th></tr>
<tr><td colspan="3">Through context, experiment with transformations of functions, and represent the transformations using a variety of models (symbols, graphs, tables, diagrams, physical models…). Students will recognize and perform transformations of functions (linear, piecewise, quadratic, absolute value, polynomial, rational and radical functions). Students will experiment with vectors as a form of transformation. Students will use vectors to solve real world problems. Students will use vector matrices as a tool to solve real world problems.</td></tr>
<tr><th>Arizona College and Career Ready Standards for Mathematical Content</th><th>Comments</th><th>Explanations and Examples</th></tr>
<tr><td>

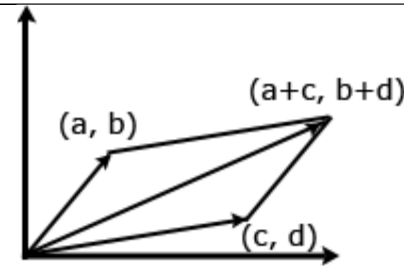
Through context, experiment with transformations of functions, and represent the transformations using a variety of models (symbols, graphs, tables, diagrams, physical models…).
Students will recognize and perform transformations of functions (linear, piecewise, quadratic, absolute value, polynomial, rational and radical functions).
Students will experiment with vectors as a form of transformation.
Students will use vectors to solve real world problems.
Students use vector matrices (etc…) to solve problems….

HS.N-VM.A.1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
HS.N-VM.A.2. Find the components of a vector by subtracting the coordinates of

</td><td>
Arizona College and Career Ready Standards for Mathematical Practice
2. Reason abstractly and quantitatively
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
</td><td>
HS.N-VM.A.3
Examples:

A motorboat traveling from one shore to the other at a rate of 5 m/s east encounters a current flowing at a rate of 3.5 m/s north.
What is the resultant velocity?
If the width of the river is 60 meters wide, then how much time does it take the boat to travel to the opposite shore?
What distance downstream does the boat reach the opposite shore?
A ship sails 12 hours at a speed of 15 knots (nautical miles per hour) at a heading of 68º north of east. It then turns to a heading of 75º north of east and travels for 5 hours at 8 knots. Find its position north and east of its starting point.
(For this problem, assume the earth is flat.)
The solution(s) may require an explanation, orally or in written form, that includes understanding of velocity and other relevant quantities.

HS.N-VM.B.4
Examples
Addition of vectors is used to determine the resultant of two given vectors. This can be done by lining up the vectors end to end, adding the components, or using the parallelogram rule. Students may use applets to help them visualize operations of vectors given in rectangular or polar form.
</td></tr>
</table>

an initial point from the coordinates of a terminal point. ○ **HS.N-VM.A.3. Solve problems involving velocity and other quantities that can be represented by vectors.** ○ **HS.N-VM.B.4. Add and subtract vectors.** **a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.** **b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.** **c. Understand vector subtraction *v* – *w* as *v* + (–*w*), where –*w* is the additive inverse of *w*, with the same magnitude as *w* and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.** ○ ○ **HS.N-VM.B.5. Multiply a vector by a scalar.** **a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.**		 **Examples:** ○ Given two vectors u and v, can the magnitude of the resultant be found by adding the magnitude of each vector? Use an example to illustrate your explanation. ○ If u = $\langle -2, -8\rangle$ and v = $\langle 2,8\rangle$, find *u* + *v*, *u* + (-*v*), and *u* – *v*. Explain the relationship between *u* + (-*v*) and *u* – *v* in terms of the vector components. A plane is flying due east at an average speed of 600 miles per hour. There is a crosswind from the south at 50 miles per hour. What is the magnitude and direction of the resultant? **HS.N-VM.B.5** The result of multiplying a vector *v* by a positive scalar *c* is a vector in the same direction as *v* with a magnitude of *cv*. If *c* is negative, then the direction of *v* is reversed by scalar multiplication. Students will represent scalar multiplication graphically and component-wise. Students may use applets to help them visualize operations of vectors given in rectangular or polar form. **Example:** Given u = $\langle 2,4\rangle$, write the components and draw the vectors for *u*, 2*u*, ½*u*, and – *u*. How are the vectors related?

b. Compute the magnitude of a scalar multiple *cv* using ||*cv*|| = |*c*|*v*. Compute the direction of *cv* knowing that when |*c*|*v* ≠ 0, the direction of *cv* is either along *v* (for *c* > 0) or against *v* (for *c* < 0). ○ **HS.N-VM.C.11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.** ○ **HS.N-VM.C.12. Work with 2 ⍰ 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.**		

Unit 4: Exponential and Logarithmic Functions. Suggested number of days: 10		
In this unit, students will use logarithms to solve exponential equations. Students will explore the meaning of the natural number and use it to solve real world problems. Through context, students will explore the inverse relationship of exponents and logarithms, including e and Ln. Students should compare and contrast exponential and polynomial functions.		
Arizona College and Career Ready Standards for Mathematical Content	**Comments**	**Explanations and Examples**
• Use logarithms to solve exponential equations • Explore the meaning of the natural number and use it to solve real world problems. • Explore the inverse relationship of exponents and logarithms, including e and Ln. • Compare/contrast exponential and polynomial functions. ○ **HS.F-BF.B.5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.**	**Arizona College and Career Ready Standards for Mathematical Practice** 2. Reason abstractly and quantitatively 4. Model with mathematics 5. Use appropriate tools strategically 7. Look for and make use of structure.	

Unit 5: Trigonometry

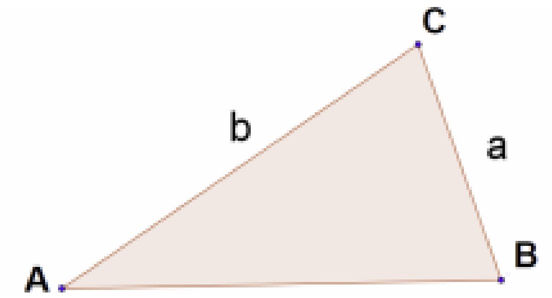
In this unit, students will explore the meaning of radians as a measure of arc length. Students will explore the relationship between sine and cosine of angles on the unit circle to develop an understanding of the relationship between these values. Students should be able to reconstruct their understanding the unit circle accurately. Students should be able to fluently demonstrate an understanding of the basic trig functions and their inverses, applying the basic trig ratios and identities both in right triangles and on the unit circle. Students will apply trig in various problem solving contexts, graph trig functions, identify properties of trig functions, explain the meaning of trig functions (amplitude, period…), and transform trig functions, symbolically and graphically. Students should be able to explain the meaning of the changes in terms of the impact on period, amplitude, phase shift, and displacement, relating these changes to real world contexts.

Students should be able to solve trig equations, write complex numbers in trig form, and graph using polar coordinates

Section 5.1 – Trigonometry of General Triangles. Suggested Number of Days: 10

In this section students will build on prior understandings of basic trigonometric relationships between sides and angles of a triangle to derive the formula for the area of a triangle, applying this understanding to real world contexts. Students will prove the Law of Sines and Law of Cosines, and use them to solve problems. Students will apply the Law of Sines and Law of Cosines and use them to find unknown measurements in right and non-right triangles.

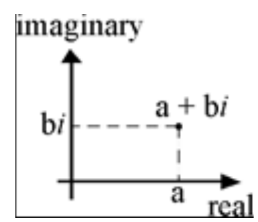
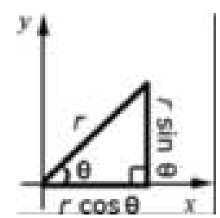
Arizona College and Career Ready Standards for Mathematical Content	Comments	Explanations and Examples
• Students will apply trigonometry to general triangles ○ **G.SRT.9 – Derive the formula $A=\frac{1}{2}ab\cdot sinC$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.** ○ **G.SRT.10 – Prove the Law of Sines and Law of Cosines and use them to solve problems** ○ **G.SRT.11 – Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces…).**	**Arizona College and Career Ready Standards for Mathematical Practice** 2. Reason abstractly and quantitatively 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning	**Applying Law of Cosines:** Two airplanes leave an airport, and the angle between their flight paths is 40º. An hour later, one plane has traveled 300 miles while the other has traveled 200 miles. How far apart are the planes at this time? A triangle has sides of 8 and 7 and the angle between these sides is 35°. Solve the triangle. (Find all missing angles and sides.)

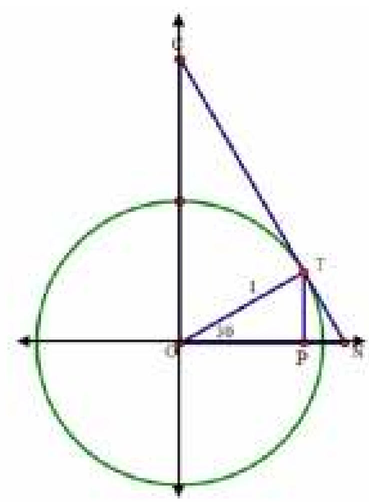
		Three soccer players are practicing on a field. The triangle they create has side lengths of 18, 14, and 15 feet. At what angles are they standing from each other? Is it possible to know two sides of a triangle and the included angle and not be able to solve for the third side? **Proving Law of Sines:** Consider the following triangle:  Construct an altitude through C. Label it h. Write an equation for h in terms of angle B. Also write an equation for h in terms of angle A. Write a new equation relating the two equations together. Now construct another altitude through B. Label it j. Write an equation for j in terms of angle A. Also write an equation for j in terms of angle C.

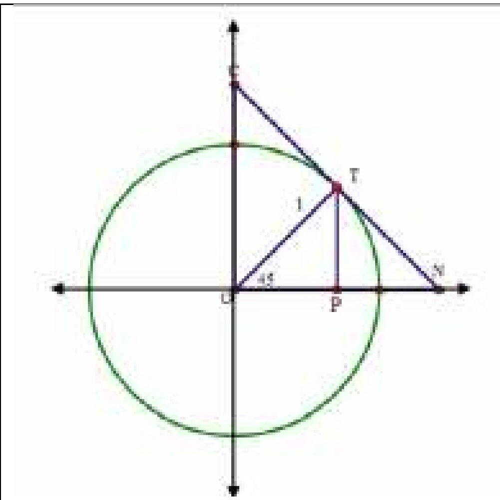
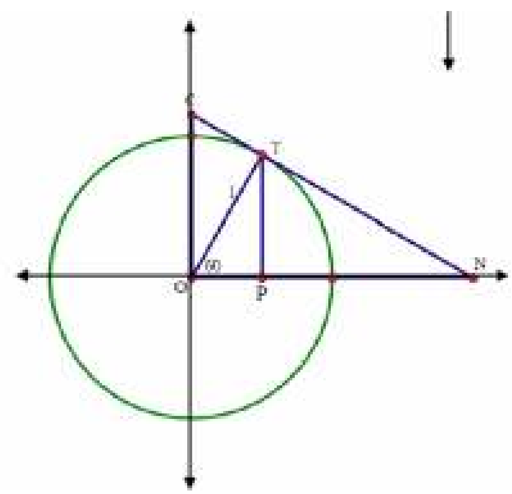
		Write a new equation relating the two equations together. **Applying Law of Sines:** 1. A surveyor is near a river and wants to calculate the distance across the river. He measures the angle between his observations of two points on the shore, one on his side and one on the other side, to be 28º. The distance between him and the point on his side of the river can be measured and is 300 feet. The angle formed by him, the point on his side of the river, and the point on the opposite side of the river is 128º. What is the distance across the river? Remember that the distance across the river should be the shortest distance.

Section 5.2 – Trigonometric Functions   Suggested number of days: 15

In this section students will explore the meaning of radians as a measure of arc length, developing this understanding through an investigation of circles of varied diameters. Students will explore the relationship between sine and cosine of angles on the unit circle to develop an understanding of the relationship between these values. Students should be able to reconstruct their understanding of the unit circle accurately. Students willgraph trig functions, identify properties of trig functions, and explain the meaning of trig functions (amplitude, period…). Through experimentation, students will transform trig functions, symbolically and graphically, explaining the meaning of the changes in terms of the impact on period, amplitude, phase shift, and displacement … connecting these meanings to real world context. Students will learn to solve trig equations, write complex numbers in trig form, and graph using polar coordinates.

Arizona College and Career Ready Standards for Mathematical Content	Comments	Explanations and Examples
Throughout this section students will build new functions from existing functions in support of their developing sense of trigonometric functions. **F.BF.4 – Find inverse functions** **d. Produce and invertible function from a non-invertible function by restricting the domain.** • Explore the meaning of radians as a measure of arc length. • Explore the relationship between sine and cosine of angles on the unit circle to develop an understanding of the relationship between these values. Students should be able to reconstruct their understanding the unit circle accurately. ○ **F.TF.3 – Use special triangles to determine geometrically the**	**Arizona College and Career Ready Standards for Mathematical Practice** 2. Reason abstractly and quantitatively 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning	**HS.N-CN.B.4** • Students will represent complex numbers using rectangular and polar coordinates. $a + bi = r(\cos \vartheta + \sin \vartheta)$   **Examples:** • Plot the points corresponding to $3 – 2i$ and $1 + 4i$. Add these complex numbers and plot the result. How is this point related to the two others?

values of sine, cosine, tangent for π/3, π/4, and π/6, and use the unit circle to express the values of sine, cosine and tangent for π + x, π – x, and 2π – x in terms of their values for x, where x is any real number. ○ **F.TF.4 – Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.** • Graph trig functions, identify properties of trig functions, explain the meaning of trig functions (amplitude, period…). • Transform trig functions, symbolically and graphically. Explain the meaning of the changes in terms of the impact on period, amplitude, phase shift, and displacement … in real world context. ○ **F.TF.6 – Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.** ○ **F.TF.7 – Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.**		• Write the complex number with modulus (absolute value) 2 and argument π/3 in rectangular form. • Find the modulus and argument $(0 < \theta < 2\pi)$ of the number $\sqrt{6} + \sqrt{-6}$ **F.TF.3** In the figures below segment TP is perpendicular to segment ON. Line CN is tangent to circle O at T. N is the point where the line intersects the x-axis and C is the where the line intersects the y-axis. (Only segment CN is shown.) Use your knowledge of sine and cosine to determine the length of segment TN. Use exact answers, no decimal approximations. 

• Solve trig equations, write complex numbers in trig form, and graph using polar coordinates. • **HS.N-CN.B.4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.** • **HS.N-CN.B.5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.** ***For example,*** ***(-1 + √3 i)³ = 8 because*** • ***(-1 + √3 i) has modulus 2 and argument 120°.*** **<u>ENDURING UNDERSTANDINGS</u>** • Understand the relationship between right triangle trigonometry and unit circle trigonometry • Use the unit circle to define trigonometric functions • There are many instances of periodic data in the world around us and trigonometric functions can be used to model real world data that is periodic in nature. • The inverses of sine, cosine and tangent functions are not functions unless the domains are limited.		  Using your understanding of the unit circle and $tangent\ \theta = si$, to complete the chart below for the indicated angles.

θ	Sin θ	Cos θ	Tan θ
30°			
45°			
60°			

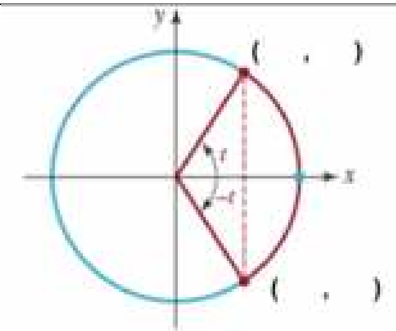
What would happen to the length of TN if the angle was changed to 0°?

What would happen to the length of TN if the angle was changed to 90°?

Based on the chart above fill in the sine, cosine, and tangent values for all the angles with the indicated reference angle.

	θ	Sin θ	Cos θ	Tan θ
x	30°			
$\pi - x$				
$\pi + x$				
$2\pi - x$				

The symmetry of the unit circle is useful is generating the trigonometric values for an infinite number of angles. It is also useful in illustrating whether the sine and cosine functions are even or odd. Let's take 30° as an example. In the picture below $t = 30^{\circ}$. Fill in the missing information.



Average monthly temperatures are periodic in nature and can be modeled by sine and/or cosine functions. The function below models the average monthly temperatures for Asheville, NC. (The average monthly temperature is an average of the daily highs and daily lows.)
$f(t) = 18.5\sin(\frac{\pi}{6}t - 4) + 54.5$ where t=1 represents January.
How can you use this model to find the month that has a specific average temperature?

Students should use what they know about inverse relationships to find the month in terms of a given average temperature.

1. Graph $(\theta)=\sin\theta$ and the line y = ½ .
   a. How many times do these functions intersect between -2π and 2π?
   b. How is this graph related to finding the solution to ½ = sin θ?
   c. If the domain is not limited, how many solutions exist to the equation ½ = sin θ?
   d. Would this be true for the other trigonometric functions? Explain.

2. What happens to the axes and coordinates of a function when you reflect it over the line y = x?
   a. Sketch a graph of $(\theta)=\cos\theta$ and its reflection over the line y = x.
   b. How many times does the line x= ½ intersect the reflection of cos x?

3. Sketch the inverse of $(\theta)=\tan\theta$ and determine if it is a function.

4. We use the names $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ or Arcsin, Arccos, and Arctan to represent the inverse of these functions on the limited domains you explored above. The values in the limited domains of sine, cosine and tangent are called **principal values**. (Similar to the principal values of the square root function.) Calculators give principal values when reporting $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

Function	Domain	Range
$f(\theta)=\sin^{-1}\theta$		
$f(\theta)=\cos^{-1}\theta$		
$f(\theta)=\tan^{-1}\theta$		

The inverse functions do not have ranges that include all 4 domains. Add a column to your chart that indicates the quadrants included in the range of the function. This will be important to remember when you are determining values of the inverse functions.

Function	Domain	Range	Quadrant for Range
$f(\theta) = \sin^{-1}\theta$			
$f(\theta) = \cos^{-1}\theta$			
$f(\theta) = \tan^{-1}\theta$			

Section 5.3 – Trigonometric Identities Suggested number of days: 10		
In this section students will demonstrate an understanding of the basic trigonometric functions and their inverses. Apply the basic trigonometric ratios and identities both in right triangles and on the unit circle. Students will apply trigonometry in various problem solving contexts.		
Arizona College and Career Ready Standards for Mathematical Content	**Comments**	**Explanations and Examples**
• Demonstrate an understanding of the basic trig functions and their inverses. Apply the basic trig ratios and identities both in right triangles and on the unit circle. • Apply trig in various problem solving contexts. ○ **F.TF.9 – Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.**	**Arizona College and Career Ready Standards for Mathematical Practice** 2. Reason abstractly and quantitatively 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning	**Addition Identity for Cosine**: $\cos(x+y)=\cos x\cos y-\sin x\sin y$ **Addition Identity for Sine**: $\sin(x+y)=\sin x\cos y+\cos x\sin y$ **Addition Identity for Tangent**: $\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x\tan y}$ **Even Function**: a function with symmetry about the y-axis that satisfies the relationship $f(x)=f(-x)$ **Identity**: an identity is a relation that is always true, no matter the value of the variable. **Odd Function**: a function with symmetry about the origin that satisfies the relationship $-f(x)=f(-x)$ **Subtraction Identity for Cosine**: $\cos(x-y)=\cos x\cos y+\sin x\sin y$ **Subtraction Identity for Sine**: $\sin(x-y)=\sin x\cos y-\cos x\sin y$ **Subtraction Identity for Tangent**: $\tan(x-y)=\dfrac{\tan x-\tan y}{1+\tan x\tan y}$

Unit 6: Conic Sections. Suggested number of days: 20

Finding the distance between points, midpoint of a line segment and determine the equation of a line given two points are skills students will use to find the center between two endpoints on a diameter of a circle or ellipse. Students will explore applications of graphs of ellipses and circles to generate an understanding of how these graphs are used in real world contexts. First through context and later without context, students will create graphs of conic sections to model relationships between inputs and outputs.

<table>
<tr><th>Arizona College and Career Ready Standards for Mathematical Content</th><th>Comments</th><th>Explanations and Examples</th></tr>
<tr><td>

- Find distance between points, midpoint of a line segment and determine the equation of a line given two points are skills students will use to find the center between two endpoints on a diameter of a circle or ellipse.
- Explore applications of graphs of ellipses and circles to generate an understanding of how these graphs are used in real world contexts.
- First through context and later without context, create graphs of conic sections to model inputs and outputs of relationships.

HS.G-GPE.A.3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

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Arizona College and Career Ready Standards for Mathematical Practice

2. Reason abstractly and quantitatively
6. Attend to precision
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

</td><td>

Examples for Circles:

1. Write the equation for the circle with a diameter containing the endpoints $(-3,0)$ and $(3,0)$.
2. Write the equation for the entire set of points that are 4 units away from $(1,-5)$.
3. Write the equation of the circle with a radius from the center at $(2,7)$ to an endpoint at $(6,5)$.

Put the following equations into standard form.

1. $x^2+y^2-4x+12y-6=0$
2. $x^2-6x=y-y^2+7$
3. $\frac{7x^2}{3}+\frac{7y^2}{3}=1$

Examples for Parabolas:

1. Given a graph of a parabola, ask students to label the vertex, focus, axis of symmetry, directrix, and directed difference, p, labeled two different places
   (a) What relationship does the locus of points forming a parabola have with the focus and directrix?
   (b) What relationship does the vertex have with the focus and the directrix?
   (c) What relationship does the directed distance p have with the focus and the directrix?

With vertex $(h,)$ and directed distance from the vertex to the focus p:

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Enduring understandings:

- Conic sections are quadratic relations that can be expressed generally by the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ and the comparison of the coefficients A and C reveal the specific type of conic.
- All conic sections are defined by the relationship of their locus of points to fixed points known as foci.
- Ellipses arise from a locus of points that represent a constant sum of distances from two fixed points (foci).
- Hyperbolas arise from a locus of points that represent a constant absolute value of difference of distances from two fixed points (foci).

- $Vertical\ Axis\ of\ Symmetry: (x-h)^2=4p(y-k)$

If p is positive, the parabola opens up; if p is negative, the parabola opens down.

- $Horizontal\ Axis\ of\ Symmetry: (y-k)^2=4p(x-h)$

If p is positive, the parabola opens to the right; if p is negative, the parabola opens to the left.

2. Students should be able to
   a. Write the equation of the parabola with a vertex at the origin and a focus at $(5,0)$.
   b. Write the equation of the parabola with focus at $(-3,3)$ and directrix at $y=9$.
   c. Write the equation of the parabola that opens to the left, contains a distance of 5 between the focus and the directrix, and contains a vertex at $(9,6)$.
3. Convert the following equations into standard form and graph each.
   a. $x^2+x-y=5$
   b. $2y^2+16y=-x-27$
   c. $x=-y^2+6y-5$
4. If you want to build a parabolic dish where the focus is 200 mm above the surface, what measurements do you need? Assume the parabolic dish points directly upwards.

Examples for Ellipses:

1. Students should experiment with manipulatives or graphing software to realize the following:
   a. The sum of the distances from each focus to any point on an ellipse remains constant
   b. The sum of the distances from each focus to any point on the ellipse is congruent to the length of the major axis.
2. Students should be able to label the parts of an ellipse, including the major and minor axes, the foci, and develop a geometric definition of an ellipse: The set of all

		points in a plane such that the sum of the distances from two fixed points (foci) is constant. 3. The path of Earth around the sun forms an ellipse with the sun at one focus. If the distance from the Earth to the sun at its most extreme point is 94.5 million miles, a. find the distance from the Earth to the sun when they are closest, b. calculate the length of the Earth's orbit around the sun c. create an equation to represent this ellipse
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Unit 7: Statistics and Probability. Suggested number of days: 15

Students will use the rules of probability to compute probabilities of compound events in a uniform probability model. As students develop their understanding of probabilities, they will use analyze possible outcomes, calculate expected values and use them to solve problems. Students will use their understanding of probability to evaluate outcomes of decisions.

Arizona College and Career Ready Standards for Mathematical Content	Comments	Explanations and Examples
- Students will use the rules of probability to compute probabilities of compound events in a uniform probability model. **HS.S-CP.B.8. Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.** **HS.S-CP.B.9. Use permutations and combinations to compute probabilities of compound events and solve problems.** - Students will calculate expected values and use them to solve problems. **HS.S-MD.A.1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same**	**Arizona College and Career Ready Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 4. Model with mathematics 5. Use appropriate tools strategically 7. Look for and make use of structure.	**Fundamental Counting Principle** a) A deli has a lunch special which consists of a sandwich, soup, and a dessert for $4.99. They offer the following choices: **Sandwich** – chicken salad, turkey, ham, or roast beef **Soup** – tomato, chicken noodle, or broccoli cheddar **Dessert** – cookie or pie Use a diagram to determine the number of different lunch combinations. Then, use the Fundamental Counting Principle to determine the number of different lunch combinations. Diagram: Fundamental Counting Principle: b) Karl has 5 shirts, 3 pairs of pants, and 2 sweaters in his closet. How many different outfits that consist of a shirt, pair of pants, and sweater can he make? **Permutations and Combinations** **Permutations Examples** Definition: A **permutation** is an ordered arrangement of *n* objects (people, numbers, letters, etc.) The **order** of the objects matters – **a different order creates a different outcome.** a) There are 8 people running a race. How many different outcomes for the race are there?

graphical displays as for data distributions. ○ **HS.S-MD.A.2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.** ○ **HS.S-MD.A.3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.** ***For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*** ○ **HS.S-MD.A.4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.** ***For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you***		b) There are 8 people competing in a race. In how many different ways can first, second, and third place medals be awarded? In our race example, there are 8 people to choose from which would represent *n* and we are choosing 3 of them to win first, second, and third place which would represent *r:* ${}_8P_3=P(8,3)=8!/(8-3)!=8!/5!=(8*7*6*5*4*3*2*1)/(5*4*3*2*1)=336$ **Combinations** Definition: A **combination** is an arrangement of objects in which **order does NOT matter**. a) A pizza shop offers twelve different toppings. How many different three-topping pizzas can be formed with the twelve toppings? **Using your calculator:** To compute a combination using your calculator, do the following: b) Your English teacher has asked you to select 3 novels from a list of 10 to read as an independent project. In how many ways can you choose which books to read? Using the Binomial Probability Formula, $P(X)=\left(\frac{n!}{(n-X)!X!}\right)(p)^X(1-p)^{n-X}$

expect to find in 100 randomly selected households? • Students will use probability to evaluate outcomes of decisions. ○ **HS.S-MD.B.5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.** **a. Find the expected payoff for a game of chance.** ***For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*** **b. Evaluate and compare strategies on the basis of expected values.** ***For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*** • **HS.S-MD.B.6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).** • **HS.S-MD.B.7. Analyze decisions and strategies using probability concepts (e.g., product testing, medical**		Where X has the binomial distribution with n observations and probability p of success on each observation, the possible values of X are 0, 1, 2, …, n.

testing, pulling a hockey goalie at the end of a game).		