Mathematics Standard Articulated by Grade Level

Glossary

Please Note—Changes related to the structure of the Mathematics Glossary

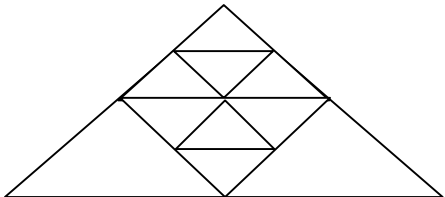Blue text: Definitions or example/explanation are from the 2010 AZ Mathematics Standards (CCSS)

The 2008 AZ Mathematics Standards Glossary is very robust, and for that reason, it is being made available to teachers in conjunction with the 2010 AZ Mathematics Standards Glossary (CCSS) and will continue to be a valuable resource. However, as we transition to 2010 AZ Mathematics Standards (CCSS), if a term contains definitions from both sets of standards, the language of the Common Core Standards Glossary (blue text) should be given priority.

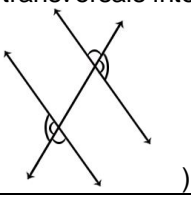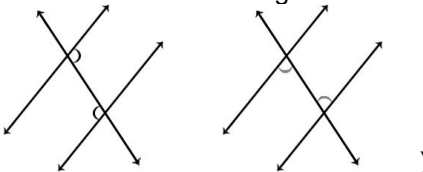# Mathematics Standard Articulated by Grade Level

GLOSSARY

The purpose of this glossary is to help the user better understand and implement the Mathematics Standard. It is not intended to be a study guide for the AIMS and is not a comprehensive list of all mathematics terms.
The definitions in the glossary are general in nature. For specific information about grade level expectations, consult the explanations and examples column in the grade level documents.

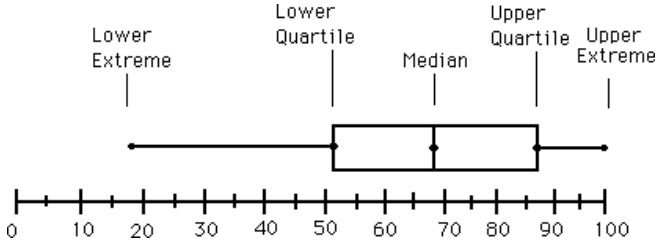|	
absolute value	a number's distance from zero on a number line (e.g., the absolute value of -4 is 4, the absolute value of 4 is 4; symbolically, $\lvert -4 \rvert = 4$ and $\lvert 4 \rvert = 4$)
additive inverses	Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (− 3/4) = (− 3/4) + 3/4 = 0.
absolute value function	a rule that defines a relationship between two sets of numbers that for each value of the independent variable set there is only one value in the dependent value set where $f(x) = \lvert x \rvert$, where $f(x) \geq 0$; for all values of x
accuracy (mathematical)	the extent to which a solution or measurement matches a standard or expected result
acute angle	an angle with measure between zero degrees and ninety degrees
addend	a number used in the mathematical operation of addition (e.g., 6 + 8 = 14, 6 and 8 are addends)
addition	a mathematical operation that combines two or more numbers to calculate a sum
addition principle of counting	a principle that allows for the efficient counting of the total number of ways a task can be accomplished when each part of the task consists of counting items from separate groups that do not overlap. For example, how many triangles are in the figure below? The task is to recognize there are three types of triangles (small, medium, and large) where each group does not overlap with another group; i.e., where each type of triangle appears as a member of one and only one group. elementary school: If you want to count the total number of triangles in the figure above, count the number of small-sized triangles (8), count the number of medium-sized triangles (4), and count the number of large-sized triangles (1) and add them together (8 + 4 + 1 = 13). So there are a total of 13 triangles in the figure. If you have a task that can be accomplished through counting a collection of items among disjoint groups, and you count m

	items in the first group, n items in a second group, and g items in a third group (etc.,) then you can efficiently count the total number of items in the task by using the addition principle of counting. In this example, we would add m plus n plus g or ($m + n + g$). high school: let A_1 and A_2 be separate events that may occur at the same time with n_1 and n_2 possible outcomes for each event, respectively; then the total number of possible outcomes for the two events occurring are $n_1 + n_2$.
adjacency matrix	the arrangement of rows and columns labeled by graph vertices, with a 1 or 0 in position (v_i, v_j) according to whether or not v_i and v_j are adjacent. For a simple graph with no self-loops, the adjacency matrix must have zeros on the diagonal while the adjacency matrix for an undirected graph is symmetrical
adjacent vertices	vertices joined by an edge or neighboring vertices in a vertex-edge graph
algebraic expression	a group of numbers, symbols, and variables that express a single or series of mathematical operations (e.g., $2x + 4 - 16y$)
algebraic form/notation	an algebraic description written in terms of numbers, symbols, and variables
algorithm	a set of step-by-step instructions for completing a task that can be generalized to other tasks, problems, or situations
alternate exterior angles	angles formed by one or more transversals intersecting two lines whose interiors are not between two lines and on different sides of the transversal (e.g.,)
alternate interior angles	angles formed by one or more transversals intersecting two lines whose interiors are between the two lines and on different sides of the transversal (e.g.,)
altitude of a geometric figure	a perpendicular segment from a base to a vertex or between bases
amplitude	a measure of one half the difference between the largest and smallest value of a function
analog clock	a device for the measurement of time that has numbers 1 to 12 around a face, with an hour, minute, and second hand that shows a continuous sweep of time
analyze	a process of dividing a composite into its parts for the purpose of examination

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angle	a geometric figure consisting of the union of two rays that share a common endpoint (vertex)
angle bisector	a line, line segment, or ray that divides an angle into two congruent parts
angle measure	the measure (in degrees or radians) of the arc formed by two rays with a common endpoint (vertex)
annuity	a purchased investment contract between a person and an insurance company that defines payments to the insurer, in lump sum or in a series of payments, in exchange for benefits paid back to the insured at a designated date or series of dates
appropriate	the reasonable use of an attribute, unit, or tool within the context of a problem (e.g., using a ruler to measure dimensions of a tissue box but not the dimensions of a building, using meters as the unit measure for the dimensions of a house but not the dimensions of a picture frame)
appropriate measure of accuracy	the degree of accuracy required for a mathematical task (e.g., approximating the lengths of lumber in framing carpentry requires less accuracy then the lengths of molding in finish carpentry)
approximation	a value or quantity that is close to, but not the same as, the desired value or quantity for a specified purpose
area	a two dimensional space measured by the number of non-overlapping unit squares or parts of unit squares that can fit into the space
arithmetic sequence	an ordered set of items in which the difference between each consecutive item is constant
arrangement	possible order of a set of events or items
array	a rectangular arrangement of objects or elements organized into rows and columns, or a set of objects or elements organized into a specific pattern
associative property	See Table 3. addition: changing the grouping of terms in a sum without changing the sum multiplication: changing the grouping of factors in a product without changing the product
asymptote	a line that a graph approaches
attribute of a figure	a property or common feature of a sets of objects or elements
attribute of a function or graph	a characteristic or distinct feature
average	the result of the sum of all the numbers in a data set divided by the number of elements in that data set
axis (axes: plural) (in two-dimensions)	one of two perpendicular number lines used to form a coordinate system
bar graph	a representation of the length of either vertical or horizontal bars used to enumerate and compare data
base	exponent: a term used to indicate a factor for repeated multiplication (e.g., in 4^7, 4 is the base) logarithm: the quantity a in the equation $x = \log_a y$
base of a polyhedron	the face of a geometric figure that identifies its type
base of a	a side of a polygon that is perpendicular to its height

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polygon	
benchmark	a commonly known point of reference from which measurements may be made (e.g., four quarters make a whole)
benchmark fraction	a commonly known fraction that serves as a meaningful reference point for measurement comparison
bias	sampling: a segment of data that is not representative of the original set of data statistical: an effect which deprives a statistical result of representativeness by systematically distorting it
binomial	an algebraic expression consisting of two terms (e.g., $x + 3$, $4a - 6$)
binomial theorem	a description of the coefficients of the expansion of the binomial $a + b$ raised to the *nth* power
bisect	to divide an object or term into two congruent parts
bisector	a point, segment, line, ray or plane which divides a segment, angle or figure into two parts of equal measure
bivariate data	Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.
box and whisker plot	A box shows the middle 50% of the data. a method for displaying the median, quartiles, and extremes of a data set
brokerage fee	a fee in the form of a commission charged to the buyer by the brokerage firm for acting on behalf of the investor with the bond, commodities, or stock market
calculation	an action, process, or result of a mathematical computation
capacity	the amount of space in units or cubes that can fit into a solid (note: also referred to as volume)
Cartesian coordinate system	a plane containing points identified by their distance from the origin in ordered pairs along two perpendicular lines referred to as axes (note: also referred to as coordinate plane and rectangular coordinate plane)
causation	an agency or action that produces an effect
Celsius	a metric scale for the measurement of temperature based on the properties of water
central angle	an angle whose vertex is the center of a circle and whose sides (rays) are radii
chord of a circle	a segment whose endpoints are on a given circle
chromatic number	fewest number of colors needed to color a vertex-edge graph

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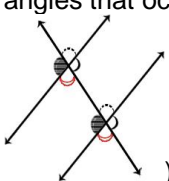|	
circle	a set of points in a plane that are equidistant from a given point called the center
circle graph	a display of data as sections of a circle that represent all the data (note: formerly called pie graph or pie chart)
circuit	a path in a graph that starts and ends at the same vertex
circular arc	a fraction of the circumference of a circle
circumcenter	the point where the three perpendicular bisectors of the sides of a triangle meet
circumference	the total distance around a closed curve like a circle
coefficient	the number part of a term and variable combination (e.g., the coefficient for 7x is 7)
coincident	lines or shapes that have all points in common
collinear	points that lie on the same line
coloring of a graph	assigning colors to the vertices of a vertex-edge graph so that adjacent vertices are assigned different colors
coloring of a picture or map	assigning colors to the regions of a picture or map so that regions that share a common border are assigned different colors
combination	The number of ways of choosing or selecting k unordered outcomes from n possibilities. $$_nC_k \equiv \binom{n}{k} \equiv \frac{n!}{k!\,(n-k)!},$$
common algorithm	a set of step by step instructions that are well known by most practitioners and are frequently used (e.g., borrowing, carrying)
common denominator	a number divisible by all of the denominators in a set of fractions
common factor	a whole number that divides without remainder into two or more non-zero numbers
common irrational numbers	a grouping of well known real numbers that cannot be expressed as a ratio of two integers (e.g., π , $\dfrac{\pi}{2}$, $\dfrac{\pi}{4}$, $\sqrt{3}$, $\sqrt{2}$, e)
common multiple	a whole number multiple of two or more given numbers (e.g., 48 is a common multiple of 2, 3, and 4)
commutative property	See Table 3. addition: the addition of terms in any order obtains the same sum (e.g., $a+b+c=d$, $a+c+b=d$) multiplication: the multiplication of terms in any order obtains the same product (e.g., $a*b*c=d$, $b*c*a=d$)
comparative language	words used to describe the differences in terms and objects (e.g., bigger, smaller, less than, more than, not equal to)
complementary angles	any two angles whose measures have a sum of ninety degrees
complementary	two events whose probabilities of occurring sum to one; mutually exclusive events (e.g., when flipping a coin, getting a head

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events	and getting a tail are complementary events)
complete graph	a vertex-edge graph in which every vertex is adjacent to every other vertex
complex fraction	A fraction A/B where A and/or B are fractions (B nonzero). a fraction that has a fractional numerator, denominator, or both (e.g., $\dfrac{\frac{7}{11x^2}}{31y^5x^6s}$)
complex number	a number that can be written in the form $a + bi$ where *a* and *b* are real numbers and *i* is an imaginary number (e.g., 2+3*i* which is equivalent to $2 + \sqrt{-3}$)
complex solution	a solution to a problem or equation that is <u>not</u> a real number
compose	to create by putting together
composite figure	a geometric figure that is composed of two or more simple polygons
composite number	a number that has factors other than one and itself
composition of function	a function comprised of more than one function arranged such that the output of one function becomes the input of the next function
compound interest	a percentage of an amount that accrues based on the product of the interest rate and the sum of the principal and any previously earned interest
compound probability	the likelihood that an event will occur based on whether another event has occurred
compound probability experiment	an organized process that examines the likelihood of two events occurring simultaneously, or the likelihood of one event occurring, instead of other possible outcomes, in conjunction with another event
computation algorithm	A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.
computational estimation	the method of determining an approximate solution to a numerical problem
computational fluency	the efficient automatic recall of addition, subtraction, multiplication, and division facts; the efficient and automatic recall and use of standard algorithms for addition, subtraction, multiplication, and division
computation strategy	Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.
compute	to determine or calculate by mathematical means
conclusion	the *then* clause in an *if-then* conditional statement; a statement based on a reasonable judgment of two or more proposals
concrete	physical objects and manipulatives used for the purpose of instruction to represent mathematical situations

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materials	
concrete situation	a condition derived from real-world examples and applications (note: also called contextual situation)
conditional probability	the likelihood that an event will occur based on an event that has already occurred
conditional statement	a statement with a hypothesis and conclusion in the form, *if* hypothesis, *then* conclusion (e.g., *if* a closed figure has exactly three sides, *then* the figure is a triangle)
cone	a three-dimensional figure generated by rotating a triangle about one of its legs to form a solid with one circular base
conflict	vertex-edge graphs can be used to model entities which are in conflict
congruent	Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations). having the same shape and exactly the same size
conic section	the intersection of a plane and two right conical surfaces that have the same vertex and whose angles are opposite rays (e.g., ellipse, parabola, hyperbola, circle)
conjecture	an unproven statement based on observations
Conjugate Root Theorem	if $a+bi$ is a root for polynomial P, then $a-bi$ is also a root for polynomial *P*
connected vertex-edge graph	a vertex-edge graph is connected if there is a path between all pairs of vertices (if a path does not exist between all pairs of vertices then the graph is disconnected)
consecutive	to follow in order one after the other
consecutive exterior angles	angles formed when one or more transversals intersect two parallel lines, that are <u>not</u> located between the two parallel lines, and are located on either side of a transversal
consecutive interior angles	angles formed when one or more transversals intersect two parallel lines, that are located between the two parallel lines, and are located on either side of a transversal
consecutive vertices	vertices that share a side of a polygon
constancy	the attribute of being unchanging, consistent, and regular
constant (of an expression)	a term with a degree of zero
constant (rate of change)	a fixed incremental increase or decrease over an interval
construct	arithmetic: the formation of a conclusion or the derivation of a result by joining or organizing forms geometry: to draw a geometric figure using appropriate tools to meet a given set of constraints
contextual situation	real-life scenarios or circumstances that illustrate mathematical problems (note: also called concrete situation)
contrapositive	a conditional statement that is the logical equivalent to the original statement exchanging the hypothesis with the conclusion and negating both of them

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converse	a conditional statement that exchanges the hypothesis (*if*) and conclusion (*then*) components of an *if-then* statement
conversion factor	the ratio of two equal quantities that are measured in different units
convex polygon	a polygon with each interior angle measuring less than 180 degrees and whose diagonals lie inside the polygon
coordinate plane	a plane containing points identified by their distance from the origin in ordered pairs along two perpendicular lines referred to as axes (note: also referred to as Cartesian coordinate system and rectangular coordinate plane)
coordinates of a point	an ordered pair of real numbers that locate a point in a plane
correlation	the relationship between two or more data sets or variables
correlation coefficient	a value between 1 and -1 that determines if two lines have a linear relationship
corresponding angles	a pair of angles that occupy the same location at each intersection when two lines are intersected by one or more transversals (e.g.,)
cosine	in a right triangle, the ratio of the length of the leg adjacent to a given acute angle to the length of the hypotenuse
counterexample	an example used to contradict or disprove a given statement
counting number	a number from the set of numbers consisting of 1, 2, 3, 4, 5, 6, … (note: also referred to as natural numbers)
counting problem	a type of problem that determines the number of arrangements, possibilities, or outcomes of events
counting on	A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."
cross-section	a plane section that intersects a solid
cube	exponents: the third power of a number geometry: a regular 3-dimensional figure having six congruent square faces
cube root	one of only three equal factors of a given number (e.g., the cube root of 27 is 3, 3 x 3 x 3 = 27
cubic function	a rule containing the cube of a variable (e.g., $f(x)=x^3$)
cycle graph	a vertex-edge graph where the vertices can be arranged in a circle so that each vertex is adjacent to the vertices that come before and after it
cylinder	a 3-dimensional figure composed of two congruent and parallel circular regions joined by a curved surface
data	quantitative and/or qualitative information within a context gathered through observation, questioning, and/or measurement

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data set	a defined group of quantitative and/or qualitative information within a context gathered through observations, questioning, and/or measurement
De Moivre's theorem	a method to find the exponential value of an imaginary number; given any nonzero complex number z and any integer n, the nth power of z, rCiS$(\theta)=r(\cos\theta+i\sin\theta)$ is $Z^M = (r(\cos\theta + i\sin\theta))^M = r^M((\cos n\theta + i\sin n\theta) = r^M$ CiS$(n\theta)$
decimal point	a demarcation mark used in a base ten numbering system to designate values that are less than one
decompose	to break down into smaller units to simplify computation
deductive proof	a formal use of deductive reasoning using logical steps in the form of axioms, theorems, and given information
deductive reasoning	a series of logical steps in which a conclusion is drawn directly from a set of statements (premises) that are assumed to be true
degree	algebra: the degree of a term is the sum of the powers of each variable in the term geometry: a unit of measure based on dividing a circle into 360 equal parts, and used to measure angles, arcs and rotations temperature: the unit of measure for temperature
degree of a polynomial	the degree of the highest term of the polynomial
degree of a vertex	the number of edges that meet at a vertex in a vertex-edge graph
degree of accuracy	a standardized mathematic set of rules for rounding using significant figures that allows for the consistent handling of different scales of measurement
denominator	the bottom part of a fraction that indicates the number of equal parts into which the whole is divided (e.g., 4 in the fraction ¾)
density	the ratio of the amount of matter in an object compared to its volume; calculated as mass (m) per unit volume (v)
density property	a statement that says there is always a rational number between any two rational numbers
dependent events	two events such that the likelihood of the outcome of the second event is affected by the outcome of the first event
dependent variable	the output variable in a function which depends on the value of the input or independent variable
Descartes Rule of Signs	a mathematical method for the determination of both positive and negative zeros of a function; let P(x) be a polynomial with real coefficients: the number of positive zeros of P is either equal to the number of variations in sign of P(x) or less than this by an even number, and the number of negative real zeros of P is either equal to the number of variations in sign of P(−x) or less than this by an even number
descending	a sequential organizational method from biggest to smallest, greatest to least, latest to earliest
diagonal	a line segment joining two non-adjacent vertices of a polygon
diameter	a line segment that joins two points on a circle and passes through the center of the circle
difference	the result obtained using the operation of subtraction
digit	the ten symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, used in a base ten numeration system
digital clock	a device for telling time that shifts between discrete states instead of continuous variation
dilation	A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

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	a transformation that either enlarges or reduces a geometric figure proportionally using a scale
dimension	measurement: measure of distance in a specific direction (e.g., length, width, depth) space: the number of coordinates needed to specify a location in space
dimensional analysis	a use of proportional analysis as a problem-solving strategy for the conversion of measurement units
directed graph	a series of items linked by edges that are directed with an initial and terminal vertex (note: also referred to as digraph)
directrix	a fixed line perpendicular to the axis of symmetry and that lies the same distance from the vertex as the focus, but in the opposite direction
discount point	a fee assessed that is equal to 1% of the amount of a loan (e.g., one point on a $100,000 mortgage is equal to $1,000)
discrete	a condition in which the number of possibilities are separated from each other and are distinct
discrete mathematics	a contemporary branch of mathematics that is used in business, industry, and daily life; topics include combinatorics, iteration and recursion, and vertex-edge graphs
distance	the positive value for the length of the shortest line segment joining two points
distance formula	a general method or rule to measure the distance between two points that are identified by ordered pairs (e.g., $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$)
distortion in data display	the use of incorrect proportion, design variation in comparison of data sets, lack of context, or insignificant use of data in direct comparison with meaningful data
distribution of data	the values that a variable has across a spread of data
distributive property	a rule or method that states that every term inside grouping symbols may be multiplied by a term outside grouping symbols to yield an equivalent expression
dividend	the value to be divided in a division problem
divisibility	the ability to divide one whole number by another whole number without a remainder
divisibility rules	a set of general rules that may be used to determine whether or not a number is evenly divisible by another number 2: if the number is even it is divisible by 2 3: if the sum of all of the digits is divisible by three, the number is divisible by 3 4: if the number formed by the last two digits is divisible by 4, the number is divisible by 4 5: if the last digit is a 0 or 5, the number is divisible by 5 6: if a number is divisible by both three and two, it is divisible by 6 7: if the difference of last digit doubled and the rest of the digits is divisible by seven, the number is divisible by 7 (e.g., 343: 34 − 6 = 28) 8: if the last three digits of a number are divisible by 8, the number is divisible by 8 9: if the sum of the digits is divisible by nine, the number is divisible by 9 10: if the last digit of the number is 0 it is divisible by 10
division	the opposite operation of multiplication that separates items or values into equal parts with or without a remainder
divisor	the value by which another quantity is divided in a division problem
documentary stamps	a state tax, in the form of stamps, that must be paid when ownership of a property passes from one owner to another

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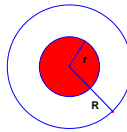|	
domain of a function	the set of values for the independent variable (input value) of a function
dot plot	See: line plot.
dot product	a mathematical operation that calculates a scalar product using two vectors and real numbers (e.g., for vectors A= $<X_a, Y_a>$ and B=$<X_b, Y_b>$ the dot product A·B=$(x_a)(x_b) + (y_a)(y_b)$)
down payment	a portion of the full amount paid at the time of purchase or delivery
durables (consumer)	a product such as an automobile or appliance that has a life expectancy of at least three years
e.g.	the abbreviation for *for example*; precedes a non-exhaustive list of examples provided as options; other examples may be appropriate but not included (compare to i.e.)
edge (vertex-edge graph)	an edge or arc that connects two vertices in a vertex-edge graph or network
edge of a polyhedron	a line segment where two faces of a polyhedron intersect
efficiency (mathematical)	the ability to determine a method for solution quickly and with little effort
elapsed time	the measure of actual time between two distinct events
element	an item or term contained within a set of items or terms
ellipse	the set of all points in which the sum of the distances between focal points is a constant
ellipsis	a series of marks, "…", to indicate the continuance of a pattern or sequence
empty set	a set, signified by the symbol Ø, to indicate that the set contains no items or elements (note: also called the null set)
end behavior	a description of the performance of a function as it increases or decreases without boundaries
endpoint	a point that demarks the beginning and the end of a line segment, the initial point of a ray, or the end of an arc
equal	a term that indicates the same amount, measure, or quantity as another amount, measure, or quantity
equation	a mathematical statement divided by an equal symbol that states the two values or expressions have the same value
equilateral polygon	a polygon in which all sides are congruent
equivalent	two expressions or statements that always have the same truth value
estimate	an approximate and reasonable answer that is close to the exact answer without actually calculating the exact answer
Euclid's 1st Postulate	a line segment may be drawn joining any two points
Euclid's 2nd Postulate	any line segment can be extended indefinitely in a line
Euclid's 3rd Postulate	given a line segment, a circle can be drawn having the segment as a radius and one endpoint as a center
Euclid's 4th Postulate	all right angles are congruent
Euclid's 5th	only one straight line may be drawn between a given line and a point that is not on that line (note: also called the Parallel

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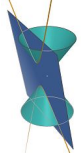|	
Postulate	Postulate)
Euclidean geometry	the study of geometry based on definitions, undefined terms (point, line and plane), and the assumptions of Euclid
Euler circuit	a path in a vertex-edge graph that starts and ends at the same vertex and does not retrace any edges
Euler path	a path in a vertex-edge graph that travels every edge exactly once and the starting vertex differs from the ending vertex
evaluate	the use of one or more mathematical operations to calculate the value of an expression for a given input
even function	a function that meets the mathematical rule $f(x) = f(-x)$
even number	a natural number that is divisible by two without a remainder
even vertex	a vertex in a vertex-edge graph whose degree is even
event	outcomes during a probability activity
expanded notation/ form	A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3. elementary: the display of digits to show the place value of each digit secondary: the display of an expression without parentheses
expected value	For a random variable, the weighted average of its possible values, with weights given by their respective probabilities. the average value distribution for a random variable
experimental (empirical) probability	a ratio formed by the comparison of the number of times an event occurs in an experiment to the number of times the experiment is completed
explicit	a statement that is expressed without ambiguity
explicit formula	an equation in which the dependent variable is written in terms of the independent variable (e.g., y=2x+3, f(x)=x^5-7, or I=Prt)
explicit sequence	a group of terms arranged in a predictable way (pattern) with a rule that is used to generate the nth term of the pattern
exponent	a number placed to the right and above (superscript) a non-zero base that indicates the operation of repeated multiplication
exponential form	a mathematical representation of a term raised to a power or terms grouped and raised to a power (e.g., 5x^3 or (5x+7)$^{2/5}$)
exponential function	an equation format written as $f(x) = a^x$ where the base, a, is a constant real number greater than zero but not equal to one.
exponential growth	the increase in a quantity over time represented by $y = a \cdot b^x$ where a > 0 and b > 1 (e.g., $y = 5(2)^x$; each time x is increased by 1, y increases by a factor of 2)
expression	a mathematical phrase containing one or more terms linked by operation symbols
extraneous	any data or information in a problem that is not necessary to determine a solution or to answer a question
extrapolation	to infer a value for an unknown variable in an interval using known values in a defined interval
extreme value	a maximum or minimum value of a function on a given interval

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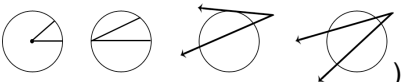|	
face of a polyhedron	each polygon that combines to construct a three-dimensional solid
fact family	a collection of related addition and subtraction facts, or multiplication and division facts, made from the same numbers (e.g., [7+2=9, 2+7=9, 9-7=2, 9-2=7} and {7X2=14, 2X7=14, 14÷7=2, 14÷2=7}
factor	noun: the value that can be divided into another value with no remainder verb: rewrite a number or polynomial as a product of numbers, simpler polynomials, or of polynomials and monomials
factorial	the product of all integers from a given number down to the number one
factorial notation	the format and symbol (!) used to represent a factorial
factoring	decomposing, through division, a complicated expression into the most simple expressions possible, that when multiplied yields the original expression
Fahrenheit	the U.S. customary or standard scale measure of temperature
fair sharing	the equal opportunity for the occurrence of all possible events or being equally divided
Fibonacci sequence	a recursive sequence in which every number is the sum of the two preceding numbers
financing	extending credit or purchasing on contract
finite set	a set of items or values that is limited to a countable number of elements
first quartile	For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 See also: median, third quartile, interquartile range.
flexibility (mathematical)	a student's ability to recognize strategies necessary to complete a mathematical task, and a student's ability to apply learned strategies to alternative mathematical tasks
fluency	the efficient automatic recall of addition, subtraction, multiplication, and division facts; the efficient and automatic recall and use of standard algorithms for addition, subtraction, multiplication, and division
foci (of an ellipse)	two fixed points on an ellipse from which the sum of the distances of all other points on the ellipse is a constant
focus	a fixed point from which all other points are equidistant
formula	a general mathematical equation that relates two or more terms or values
Four Color Theorem	given any plane or spherical surface separated into regions, such as a political map of the states of a country, the regions may be colored using no more than four colors in such a way that no two adjacent regions receive the same color
fractal	a rough or fragmented geometric shape that can be subdivided into parts, each of which is (approximately) a reduced-size copy of the whole
fraction	A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number. a number written in the form of a ratio where the top number is referred to as the numerator and the bottom number is referred to as the denominator
fractional part	a part of a whole or a part of a group
frequency	the number of occurrences of an event within a specified interval
frequency table	a collection of data organized to display the number of events in a specified interval or multiple intervals

frieze pattern	a classification of patterns on two-dimensional surfaces that repeat in one direction
front-end estimation	using the leading, or left-most, digits to make an estimate quickly and easily (e.g., when asked to estimate the sum of 594, 32, and 221 an original estimate would be 5+0+2 hundreds or 700)
function (algebraic)	a rule that defines a relationship between two sets of numbers in that for each value of the independent variable set there is only one value in the dependent variable set
function notation	an equation in the form of $f(x) =$ to show the output value of a function, f, for an input value x
Fundamental Theorem of Algebra	an nth degree polynomial has n solution(s), real or complex
generalize	the ability to apply a solution method to many different problems and situations
geometric model	a representation of a geometric figure or concept
geometric pattern	a design representation of nonfigurative shapes including, lines, rectangles, and polygons
geometric probability	the likelihood of an event occurring based on geometric relationships such as area, surface area, or volume (e.g., if an arrow hits the target, the probability of hitting the red (shaded) bulls eye is $\dfrac{\pi r^2}{\pi R^2}$
geometric sequence	a finite or infinite progression of real numbers where each element is equal to the previous term multiplied by a constant referred to as the common ratio
geometric solid	a 3-dimensional shape bounded by surfaces (e.g., rectangular prism, pyramid, cylinder, cone, and sphere)
graph	a representation of an algebraic equation applied to a coordinate grid
graphic organizer	a visual tool designed to represent data in a format that improves understanding (e.g., Venn diagram, concept web, K-W-L chart)
greatest common factor	the largest natural number or monomial that divides into different natural numbers or terms without a remainder
grouping symbols	a variety of symbols of inclusion; parentheses, brackets, braces, or bars (i.e., ( ) , [ ] , {} , ─ , | |)
growing pattern	patterns that show an arithmetic or geometric change between pairs of elements in the pattern (e.g., numbers in decreasing order; buildings in decreasing size; or 3, 5, 8, 12, ……)
Hamilton circuit	a path in a vertex-edge graph that begins at a vertex, passes through every vertex exactly once, and returns to the original vertex
Hamilton path	a path in a vertex-edge graph that starts as some vertex in the graph and visits every other vertex of the graph exactly once.
height	a perpendicular segment from a base to a vertex or between bases (note: also called altitude)
hexagon	a polygon with six sides
higher order	an expression with a degree equal to or greater than two

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polynomial	
histogram	a vertical bar graph with each bar representing a certain interval of data
horizontal	parallel to or in the plane of the horizon; in a coordinate grid, the x-axis is a horizontal line
hyperbola	a set of all points on a plane such that the difference between the distances from the plane to the foci is a constant, and is created by the intersection of the plane and the cone
hypotenuse	the longest of the three sides of a right triangle; the side opposite the right angle in a right triangle
hypothesis	the *if* clause in an *if-then* conditional statement
i.e.	abbreviation for *that is*; precedes a specific list of items in which all of the items should be used (compare to e.g.)
identity element	a number when used in an operation with a given number leaves the given number unchanged
identity property of	See Table 3. addition: the rule that recognizes that a given number remains unchanged after the addition of a zero multiplication: the rule that recognizes that a given number remains unchanged after multiplication with the number one
image	a figure produced as the result of one or more transformations
imaginary numbers	the square root of a negative number expressed using i ($\sqrt{-1} = i$)
implicit	assumed or indirectly stated; inferred
implicit formula	an equation in which the dependent variable and independent variable are not separated by the equal sign, or in which the dependent variable is written in terms of the independent variable (e.g., 2x+y=3,= or $\dfrac{I}{P} = rt$)
improper fraction	a fraction in which the numerator is greater than the denominator
income tax	a monetary charge levied by an authority for public purposes that is based on monies made from employment, business, or capital gains
independent events	two events in which the outcome of the second event is not affected by the outcome of the first event
independent variable	the input value for a function
indirect measurement	a measurement determined without the direct application of measurement tools
indirect proof	a deductive reasoning strategy that uses contradiction or elimination to rule out all possible conclusions except the original statement which must be true
Individual retirement	an account that allows the holder to delay paying income tax and reduces the amount of taxes owed on the funds deposited

account (IRA)	
inductive reasoning	a reasoning process in which a conclusion is drawn from several observations
inequality	Properties of inequality. See Table 5 a statement relating two or more quantities or values that are not equal using words or symbols ($\neq$, $<$, $>$, $\leq$, $\geq$)
inference	a conclusion drawn from given information, many times in the form of data
infinite set	the set in which the number of elements cannot be counted
input/output machine	a method used to build functions by applying a rule to an input value which generates an output value
inscribed angles	an angle with its vertex on the circle and with sides (rays) that are chords of the circle
integers	A number expressible in the form a or –a for some whole number a. the set of real numbers consisting of the whole numbers and their opposites … -2, -1, 0, 1, 2 …
integral	general: an integer calculus: a function used for the calculation of the area under a curve
intercept	the point at which a line or curve crosses a given axis
intercepted arc	that part of a circle that lies between two segments, rays, or secants that intersect the circle (e.g.,)
interpolation	a method for the estimation of the value of a function using the known values of a number above and below the unknown value
interquartile range	A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile. a measure of variability, that is resistant to outliers, determined by the difference between the first and third quartiles
interval	a set of numbers or values between, and in some cases including, two given values
inverse function	a function $f(y) = x$, denoted by $f^{-1}(x)$ such that the domain of the function $f(x)$ becomes the range of the inverse function $f^{-1}(x)$, and the range of $f(x)$ becomes the domain of $f^{-1}(x)$; the function will only have an inverse function if it is a one to one relation
inverse matrix	a rectangular array of values with columns and rows which when multiplied by the original array of values results in an array of values with a one for every diagonal element from the top left to the bottom right and a zero for all other elements in the array
inverse of a statement	a conditional statement obtained by negating both the hypothesis and the conclusion of a given conditional statement
inverse operation	a related but opposite process (i.e., multiplication is the inverse of division)
inverse relationship	additive: a number when added to a given number results in a sum of zero (note: also called identity property of addition) Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of oneanother because 3/4 × 4/3 = 4/3 × 3/4 = 1.

	multiplicative: a number when multiplied to a given number results in a product of one (note: also referred to as the identity property of multiplication)
irrational numbers	a set of real numbers that cannot be expressed as a ratio of two integers (i.e., $\pi, \sqrt{2}$)
irregular polygon	a polygon whose interior angles are not equal and/or its sides are not equal in length
isosceles triangle	a triangle that has two or more congruent sides (note: equilateral triangles are a subset of isosceles triangles)
iteration	the repetition of a pattern or sequence
iterative pattern/ sequence	a pattern/sequence generated by using an initial value and repeatedly applying the same rule
justify	to prove or show to be true or valid using logic and/or evidence
kite	a quadrilateral with two distinct pairs of congruent adjacent sides and no congruent opposite sides
lateral face	a 2-dimensional surface that is not a base of a 3-dimensional figure
lateral surface	the sum of the lateral faces of a three-dimensional figure
Law of cosines	a law that allows for the calculation of the measurement of a side or angle of a triangle given other values for the triangle; for $$a^2 = b^2 + c^2 - 2bc\cos A$$ any ΔABC: $b^2 = a^2 + c^2 - 2ac\cos B$ $$c^2 = a^2 + b^2 - 2ab\cos C$$
Law of Large Numbers	the larger the sample the closer the experimental probability will approximate the theoretical probability
Law of sines	a description of the relationship between the angles of a triangle and the opposite sides of the same triangle; for any ΔABC: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
least common multiple (LCM)	the smallest value for which two or more values are factors (e.g., the LCM of 3, 4, and 6 is 12, the LCM of $x^2 - 1$ and $x^2 - 3x - 4$ is $(x+1)(x-1)(x-4)$)
limit	the value, if one exists, that the dependent variable approaches as the independent variable approaches a given value
line	a straight set of points that extends infinitely in opposite directions (note: this is an undefined term in Euclidean geometry)
line graph	a representation used to show change over an interval, with the data points connected by line segments
line of best fit	a line drawn on a scatter plot to estimate the linear relationship among the data
line of reflection	the line that behaves as a mirror such that after a figure is reflected across the line all the points on the line are left unchanged by the reflection (transformation)
line of symmetry	a line that divides a figure into two congruent parts that are mirror images of each other
line plots	A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number

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	line. Also known as a dot plot. a sketch of data in which check marks, or other marks above a number line, shows the frequency of each value
line segment	two points or endpoints and all the points on the line between the endpoints
linear equation	an equation containing one or more terms in which the variable(s) is/are raised to the power of one but no higher
linear expression	a collection of numbers, symbols, operations, and two or fewer variables with a degree of one
linear function	a function that has a constant rate of change and can be modeled by a straight line
linear growth	a model for growth that adds a fixed amount to each time period
liter (L)	a metric unit of capacity that is equal to the volume of a cube that measures ten centimeters on a side
logarithm	a power to which a positive number base greater than one must be raised to generate a given number
logarithmic function	functions that involve logarithms and are the opposite of the exponential function
logic	a system of reasoning used to validate arguments
logic problem	a rational and varied systematic series of steps based on sound mathematical procedures in order to arrive at the solution
lowest common denominator (LCD)	the least common multiple of the denominators of every fraction in a given collection of fractions
magnitude	size or quantity
manipulatives	a wide variety of physical materials or objects that students use to foster the learning of abstract ideas in mathematics (note: also referred to as concrete materials)
mass	the amount of matter a body contains
mathematical argument	the justification of a particular solution, algorithm, or method using logic, evidence, and mathematically sound reasoning
mathematical fluency	the use of mathematical strategies with efficiency, accuracy, and flexibility
matrix	a rectangular array of numbers or letters arranged in rows and columns
maximum	the number with the greatest value in a set of numbers; the greatest vertical value in a graph
mean	A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21. a measure of center where the sum of a set of numbers is divided by the number of elements in the set (also referred to as the average)
mean absolute deviation	A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.
meaningful context	the real world application of a mathematical concept

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measurable attribute	a common feature of a set of objects or numbers that can be measured
measures of center	numbers that communicate the "center" or "middle" of a set of data (i.e., mean, median, and mode)
measures of spread	an indication of the dispersion or variation of data values including range, quartiles, interquartile range, standard deviation, and variance
median	A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11. a measure of center that identifies a value such that half the data is above the value and half the data is below the value when the data is listed in order
metric system of measurement	a measurement system based on the base-ten numeration system (e.g., meter, liter, gram)
midline	In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.
midpoint	a point on a line segment halfway between the two endpoints
mid-spread	the difference between the upper and lower quartiles
minimum	the number with the smallest value in a set of numbers; the least vertical value in a graph
minuend	the number from which you are subtracting
mitigate	to cause to become less severe
mixed number	a number represented by a whole number next to a fraction, and is equal to the sum of the whole number and the fraction
mode	a measure of center that is the value or values that occur(s) most frequently in a given set of numbers
model (noun)	an object, drawing, graph, expression, or equation that represents a given context
Model (verb)	algebra and functions: choice of an equation or function to represent a given context geometry: use of physical objects or manipulatives to show a geometric situation
monomial/ monomial expression	an algebraic expression consisting of a single term that does not require any addition or subtraction (e.g., 5y)
multi-line graph	a representation consisting of two or more line graphs that correspond to discrete data sets
multiple of a number	a number into which a given number may be divided with no remainder
multiplication	the operation of repeated addition

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multiplication principle of counting	a principle that allows for the efficient counting of the total number of ways a task is accomplished when some number of parts follows a first part of the task. For example, how many outfits can you make using three shirts, two pants, and four shoes? The task is to make an outfit with three parts (a shirt selection, a pants selection, and a shoes selection). elementary school: If you want to count the total number of ways a task can be completed that is accomplished through a series of parts, and you can select m ways to complete the first part, n ways to complete the second part, and g ways to complete the third part (etc.,) then you can efficiently count the total number of ways to accomplish the task by using the multiplication principle of counting. In this example, we would multiply m times n times g or $(m \cdot n \cdot g)$ In the example above, we can count the total number of outfits by 3 x 2 x 4 or 24 outfits. high school: let A_1 and A_2 be events with n_1 and n_2 possible outcomes, respectively; then the total number of outcomes for the sequence of the two events is $n_1 \cdot n_2$high school: let A_1 and A_2 be events with n_1 and n_2 possible outcomes, respectively; then the total number of outcomes for the sequence of the two events is $n_1 \cdot n_2$
multi-variable equation	an equation with three or more variables that can be graphed in three or more dimensions
natural numbers	the set of real numbers consisting of 1, 2, 3, 4, 5, 6, … (note: also referred to as counting numbers)
necessary information	the values and statements required to find the solution to a problem
negation	statements meaning not or the opposite of; for any given statement p, its negation is the statement , $\sim p$ (not p) whose truth value is the opposite of the truth value of p
negative number	a real number that is less than zero
neighboring vertices (of a vertex-edge graph)	vertices that share an edge (note: also referred to as adjacent vertices)
net of a polyhedron	a two-dimensional representation of the surface of a three-dimensional figure
network	A network or vertex-edge graph consists of a collection of vertices and edges where each edge connects two of the vertices
non-contextual problem	a problem given without an application/story
non-Euclidean geometry	a geometry that contains an axiom which is equivalent to the negation of the Euclidean parallel postulate *(e.g., R*iemannian geometry is a non-Euclidean geometry using the statement, "If l is any line and P is any point not on l, then there are no lines through P that are parallel to l" as its parallel postulate (also called elliptic geometry); and Hyperbolic geometry is a non-Euclidean geometry using the statement, "If l is any line and P is any point not on l, then there exists at least two lines through P that are parallel to l" as its parallel postulate
non-random	a sample selected using a biased method

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sample	
non-routine problem	word problems that include a model of a real life situation, focus on higher levels of interpretation, are organized with no obvious solution, and that may require multiple problem solving strategies
non-standard shapes	geometric figures that are not in common usage but fulfill a given definition (e.g., ▯▵◇▷▸)
non-standard units of measurement	measurement units that are not commonly accepted as standard but are applied uniformly when measuring (e.g., paperclips, pencils, a tennis shoe, and cubes)
normal curve	the symmetric statistical distribution of data evenly spread along a bell-shaped curve that reaches its maximum height at the mean
normal distribution	the spread of data that is symmetric in a given interval, has a median and mean that are equal, and can be fit with a normal curve
number line	Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity. a model that represents real numbers as points on a line with a uniform scale
numerator	the number of equal parts of a total number of parts in a fraction; it is found above the fraction bar (e.g., 4 in the fraction $\frac{4}{7}$)
numerical expression	any combination of constants, operators, and/or words that result in a number (note: also referred to as an arithmetic expression)
observable attribute	a common feature of a set of objects or numbers that is noticeable (can be observed)
observational study	a study attempting to infer the effects of an action in which the assignment of subjects to the group receiving the action and the group not receiving the action is outside the control of the observer
obtuse angle	an angle whose measure is greater than 90 degrees and less than 180 degrees
octagon	a polygon with eight sides
odd function	a function that meets the mathematical rule $f(-x) = -f(x)$
odd number	an integer that is <u>not</u> divisible by two
odd vertex	a vertex in a vertex-edge graph whose degree is odd
one-to-one correspondence	a relationship that pairs each element in a set with one element in another set
one-variable data	the data generated by one input cell used with a formula
operation	the process or execution of a specific rule on a set of numbers
order of operations	the sequence in which specific rules of mathematics are performed when evaluating an expression or equation
ordered pair	a pair of numbers used to locate and describe points in the coordinate plane in the form (x, y)

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ordinal number	a whole number that names the position of an object in a set
ordinal position	numbers used to specify position in a sequence (e.g., first, second, third, fourth)
organized list	an orderly table of numeric or descriptive data used to solve a problem or an ordered plan to solve a problem
orientation	the arrangement of the points or objects, relative to one another, after a transformation; the direction traversed (clockwise or counterclockwise) when traveling around a geometric figure
origin	the intersection of the axes in a coordinate grid, often defined as (0, 0) in two-dimensions
origination fee	a charge levied by a lending institution for setting up a loan
outcome	a possible result for a probability experiment or simulation
outcome set	a set of all possible results for a probability experiment or simulation
outliers	numerical data that are significantly larger or smaller than the rest of the data in a set
parabola	the set of all points equidistant from the focus and the directrix
parallel lines	lines in the same plane that never intersect and are always equidistant
parallelism	a parallel relationship; the relation of opposition between things that will never intersect
parallelogram	a quadrilateral in which both pairs of opposite sides are parallel
parameter	algebraic/geometric: a quantity or constant whose value varies with the circumstances of its application statistical: a single number that describes some aspect of an entire population
Pascal's triangle	a triangular arrangement of numbers in which each row starts and ends with 1, and each other number is the sum of the two numbers above it 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1
path (vertex-edge graph)	a connected sequence of edges that starts at a vertex and ends at a vertex
pattern	a set or sequence of figures or numbers that are repeated in a predictable manner
pentagon	a polygon with five sides
percent	a ratio that calculates the parts per hundred (e.g; 20% is 20 parts of 100)
percent rate of change	A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.
perfect square	a whole number whose square root is a whole number
perimeter	the sum of all lengths of a polygon
period	the repeating interval of a periodic function
periodic	a function that repeats itself at regular intervals

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function	
permutation	an ordered arrangement of a set of events or items
perpendicular lines	two lines that intersect to form right angles
perpendicularity	a perpendicular relationship; the relation of opposition between things at right angles
phase shift	the horizontal translation of a periodic graph
pi (π)	the ratio of the circumference of a circle to its diameter
pictograph	a representation that uses pictures or symbols to represent data
piece-wise defined function	a function that uses different rules for the number x depending on the element of the domain
place value	the value of a numeral based on the position of each digit in the number
plane	a 2-dimensional surface that extends infinitely in all directions (note: this is an undefined term in Euclidean geometry)
plane figure	a two-dimensional figure or shape formed by straight lines or a curve
point	a location in space that has no dimension (note: this is an undefined term in Euclidean geometry)
point of rotation	the point about which a figure is rotated or turned
points of discontinuity	a point where a function is not continuous, noted by an open circle on the graph of the function
polar coordinate system	a system in which a point on a coordinate plane is identified using its distance from the origin (r) and the positive angle (q) required to reach the point from 0° [e.g., (2, 40°)]
polygon	a closed two-dimensional figure made up of segments which intersect only at the segment endpoints
polyhedron	a closed three-dimensional figure or shape in which all the surfaces are polygons
polynomial/ polynomial expression	an expression containing more than one monomial connected by addition or subtraction
population	an entire set of objects that have something in common (e.g; animals with four legs, quadrilaterals, male students in Mr. R's class)
postulate	a mathematical statement that is accepted as true without proof
power	a quantity with a base and an exponent (e.g; x^5, where x is the base and 5 is the exponent)
precision	an indicator of how finely a measurement is made; it is related to the unit of measurement and the calibration of the tool
predictions	the use of base information to produce an approximation of change or result
pre-image	an object before it undergoes a transformation
premise	a statement that is given to be true
prime factor	all the factors of a quantity that are only divisible by the number one and itself (e.g; the prime factors of 42 are 7, 3, and 2; the prime factors of $6x^2y$ are 2, 3, x, x, and y)
prime factorization	the representation of the prime factors of a quantity

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prime number	a number that has exactly two different factors, one and itself
prism	a three-dimensional figure made up of two parallel congruent faces and lateral faces that are parallelograms
probability	A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition). the measure of the likelihood of the occurrence of an event
probability distribution	The set of possible values of a random variable with a probability assigned to each.
Probability model	A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.
product	the result obtained when two or more quantities are multiplied
proof	a sequence of logical arguments that prove a conjecture to be true
proper fraction	a fraction whose numerator is smaller than its denominator
properties of equality	Properties of equality. See Table 4. rules for producing equivalent expressions (e.g., identity, transitive, reflexive, addition property of equality, to name a few)
properties of: operations, real number operations, real number system	Properties of operations. See Table 3. mathematical principles that are always true (e.g., commutative, associative, distributive, identity, and inverse, to name a few)
proportion	the statement of equality between two ratios
proportional relationship	a relationship between two variables in which one is a constant (the constant of proportionality) times the other
proportionality	the concept of having equivalent ratios
proposition	a statement of truth that has yet to be proven
proximity	distance from an object
pyramid	a three-dimensional figure whose base is a polygon and whose lateral faces are triangles that share a common vertex
Pythagorean theorem	the statement that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse ($a^2+b^2=c^2$)
quadrant	one of the four sections into which the coordinate plane is divided by the x- and y-axes
quadratic equation	a polynomial equation containing one or more terms in which the variable is raised to the second power but no higher
quadratic formula	the formula used to find the roots (solutions) of a quadratic equation (i.e., $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)
quadratic function	a function in the form: $f(x) = ax^2 + bx + c,\ a \neq 0$
quadrilateral	a polygon with four sides
~~**quartiles**~~	~~the four equally sized groups of data set~~

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quotient	the answer to a division problem
radian/ radian measure	the measure of an angle formed by taking the radius of a circle and wrapping it along the circumference of the circle, the measure of an entire circle is 2π radians
radical	a symbol used to refer to the root of a number or term
radical expression	a mathematical expression containing one or more radicals
radical form	a term, expression, or equation that uses a radical instead of a fractional exponent(s) (e.g., writing the expression $\left(\sqrt[4]{2x}\right)^7$ instead of $(2x)^{\frac{7}{4}}$)
radius of a circle	the distance from the center of a circle to a point on the circle (plural: radii)
random sample	a sample in which each item or element of the population has an equal chance of being chosen as part of a sample of the population
random variable	An assignment of a numerical value to each outcome in a sample space.
randomized experiment	an experiment attempting to infer the effects of an action in which subjects are randomly assigned either to a group receiving the action or a group not receiving the action
range	the set of all possible output values for a function
range (of a data set)	the difference between the greatest and least value in a set of data
rate	a ratio comparing different types of measures (e.g., miles per gallon)
rate of change	the amount the function's output increases or decreases for each unit of change in the input
ratio	a comparison of two quantities by division that can be expressed as a to b, $\frac{a}{b}$, or $a{:}b$
rational expression	A quotient of two polynomials with a non-zero denominator. the quotient of two polynomials in the form $\frac{A}{B}$, where A and B are polynomials (e.g., $\frac{2x+1}{3x^2-9}$, $3x^2-9 \neq 0$)
rational number	A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers. a number that can be expressed as a quotient of two integers
Rational Root Theorem	for a polynomial with integer coefficients, the only possible rational numbers that can be roots of the polynomial are ones of the form a/b, where a is a factor of the constant term and b is a factor of the leading coefficient
ray	a line segment that extends infinitely in one direction from one of its endpoints
real numbers	the set of rational and irrational numbers
re-allotment of square units	the application of the idea that what is changed in one place must be made up elsewhere in measurement problems involving square units

reasonable	within likely or sensible boundaries
reasonable estimations	approximations based on mathematical reasoning that are within the desired degree of accuracy (e.g., in the problem 35+43 a reasonable estimation would be 75 or 80)
reasoning (mathematical)	the justification of a particular solution, algorithm, or solution method using logical and mathematically sound arguments
reciprocal function	the function f(x) = 1/ax, where *a* is a constant and *a*≠0
reciprocals	two numbers whose product is equal to one (note also referred to as multiplicative inverses)
rectangle/ rectilinear figure	A polygon all angles of which are right angles. a quadrilateral with two pairs of congruent parallel sides and four right angles
rectangular coordinate plane	a plane containing two perpendicular lines referred to as axes (note: also referred to as Cartesian coordinate system and coordinate system)
recursion	an inherently repetitive process by which the terms of a sequence can be computed from some or all of the preceding terms by an algorithmic procedure
recursive formula	a formula used to determine the next term in a sequence by using an algorithm with one or more of the preceding terms
recursive pattern	a pattern that uses the solution from previous steps to generate the solution to the next step (i.e., 2, 2, 4, 6, 10, 16…)
reflection	a transformation creating a mirror image of the original figure on the opposite side of the line of reflection
reflex angle	an angle that is greater than 180° and less than 360°
reflexive property	a property that states a quantity or figure is equal or congruent to itself
refute	to prove false by argument or evidence
regression equation	the equation for the line of best fit to a set of data points in the plane
regular polygon	a convex polygon which is equiangular and equilateral
relative magnitude	the value of numbers relative to a given value or number
Remainder Theorem	a theorem stating, "If f(x) is a polynomial, then the remainder obtained by dividing f(x) by x-r equals f(r)
repeating decimal	The decimal form of a rational number. See also: terminating decimal. a decimal in which one or more digits repeats in a pattern without termination
repeating pattern	a sequence of figures or numbers that repeat in a predictable manner
representation	verb: the act of capturing a mathematical concept in some form noun: the form expressing a mathematical concept (e.g., equation, graph, model, written description, sketch, table, construction, manipulative)

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revise	to change or modify based on evaluation
rhombus	a quadrilateral with four congruent sides (plural: rhombi)
right angle	an angle whose measure is 90°
rigid motion	A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.
right triangle	a triangle that contains a right angle
root	the solution (zeros) of a function
rotation	a transformation in which a figure is turned a given degree and direction around a point (the point of rotation)
ROTH account	an individual retirement arrangement that can be an account or annuity whose contributions are not tax deferred
round	to approximate the value of a number to a specified place value
sample	a part of the total population used in statistics to make predictions about the characteristics of the entire group
sample space	Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered. a list of all possible outcomes of an activity
scalar	a constant used in operations on matrices and vectors, distinguished from a vector or matrix in that it has size but not direction
scale	measuring: a tool or system used for the determination of weight graphing: a system of marks at fixed intervals
scale drawing	a reduced or enlarged drawing which is mathematically similar to the object which it represents
scale factor	the ratio between the lengths of corresponding sides of two similar figures
scalene triangle	a triangle with no congruent sides
scatterplot	A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot. a graph of the points representing a collection of data
scientific notation	a representation of a very large or very small number expressed as the product of a power of ten and a decimal number greater than or equal to one and less than ten
secant	a line that intersects a circle or some other curve at two points
sector of a circle	a region bounded by a central angle and its arc
sequence	a set of numbers in a defined order
series	the sum or difference of a sequence of numbers
shortest path	the path in a weighted vertex-edge graph from one vertex to another that has the least total weight
side	the segment joining two adjacent vertices in a figure
side length	the measure of the segment joining two adjacent vertices in a figure
sigma notation (Σ)	the Greek letter sigma used to indicate summation
similar figures	two or more figures that have the same shape and are related in size by a scale factor

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similarity transformation	A rigid motion followed by a dilation.
simple interest	a fixed percent calculated on a principal amount without regard to accrued interest
simple polygon	a closed 2-dimensional figure that cannot be decomposed into closed figures with fewer sides without adding segments; a 2-dimensional figure whose sides do not cross through the interior of the figure
simplest form	fractional: a fraction that has no common factor for the numerator and denominator polynomial: an expression that has no common factors for all terms and no like terms radical: there are no perfect square factors contained in the radicand and there are no like terms
simplify	the act of writing a quantity in simplest form
simulation	an experiment to model a real-life situation for the purpose of examining a problem
sine	in a right triangle, the ratio of the length of the side opposite the given acute angle to the length of the hypotenuse
single event	one occurrence that can take place during a probability simulation that is not in conjunction with another occurrence
skip counting	the method of counting by equal intervals
slant height	pyramid: the altitude of a lateral face of a pyramid cone: the length of a line segment drawn on the lateral surface of a cone from its vertex to a point on its circular base
slope of a line	the measure of steepness of a line calculated as the change in y divided by the change in x (the rise over the run)
solid	a closed 3-dimensional figure
solution	the value or values for a variable that makes an equation or inequality true
solution methods	the strategy or set of strategies employed to solve a contextual or non-contextual problem
solution set	all the values that make an equation or inequality true
solve	to find a solution for a problem
space	the set of all points in three or more dimensions
spanning tree	a subgraph of a vertex-edge graph that is a tree and includes every vertex of the graph
sphere	a three-dimensional figure made up of all points in space equidistant from a given point called the center
spherical geometry	geometry applied to the surface of a sphere (note: this is a type of non-Euclidean geometry)
square	geometry: a parallelogram with four congruent sides and four right angles exponent: the result of multiplying a number by itself
square root	one of the two equal factors of a number
standard deviation	a statistical calculation of the dispersion of the data
standard notation	a number written with one digit for each place value in a base ten numeric system
statistics	the collection, organization, description, and analysis of quantitative data

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stem-and-leaf plot	a display of data in which digits with larger place values (10's or greater) are "stems" and digits with smaller place values (1's) are "leaves" (e.g., 0 | 0015888 1 | 23445569 2 | 3 | 014 represents {0,0,1,5,8,8,8,12,13,14,14,15,15,16,19,30,31,34})
straight angle	an angle whose measure is 180°
subdivide	to decompose into smaller parts
subgraph	a portion of a vertex-edge graph that includes some of its vertices and some (or all) of its edges that connect those vertices
subscript	a number written to the right of and slightly below a term, usually used for indexing
subsets of a population	organizational groupings within a population
subsets of the real number system	organizational groupings of real numbers (e.g., rational numbers, irrational numbers, integers, whole numbers, natural numbers)
substitution property	the mathematical rule that allows equal values to replace each other
subtraction	a mathematical operation that calculates the difference between two numbers
subtrahend	the number being subtracted in a subtraction problem
sum	the result of addition
summary statistics	statistics used to summarize a set of observations, in order to communicate as much as possible as simply as possible; statisticians commonly try to describe the observations in three ways: • a measure of center, such as the arithmetic mean, median, mode, or interquartile mean; • a measure of statistical dispersion like standard deviation, variance, range, or interquartile range; and • a measure of the shape of the distribution like a normal curve
summation	the process of adding terms in a sequence for a given interval
supplementary angles	two angles whose measures have a sum of 180 degrees
surface area	a measure of the amount of area in a three-dimensional solid
symbol	shorthand marks that represent math concepts (e.g., $\leq, 4, \pm, \in, \angle, \pi$)
symmetric property	the mathematical rule that states for real numbers a and b, if $a = b$, then $b = a$
symmetry	a one-to-one correspondence in size, form, and arrangement of parts, related to a plane, line, or point
synthesize	the use of reasoning to combine sometimes diverse concepts or statements
system of equations	a set of two or more equations that must all be true for the same value(s) (note: also referred to as simultaneous equations)
systematic lists	an orderly listing of all possibilities for a given situation

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table of values	a chart that organizes data (values) in rows and columns to illustrate facts and figures
tallies	a method of counting using marks usually in groups of five
tally chart	a method for recording occurrences of an event and for the development of frequency distribution tables
tangent	geometry: a line in the plane of a circle that intersects a circle at exactly one point trigonometry: in a right triangle, the ratio of the length of the leg opposite a given acute angle to the leg adjacent to the same angle
tape diagram	A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.
t-chart	a two column organizational tool used to display and record data, patterns, and functions/rules
term	a product or quotient of numerals, variables, or both; often separated by addition or subtraction operations in an expression
terminating decimal	A decimal is called terminating if its repeating digit is 0. a decimal that contains a finite number of digits
tessellation	one or more types of congruent figures that completely cover a plane without overlapping
theorem	a mathematical statement or proposition proven using previously accepted results
theoretical probability	the likelihood an event will occur under ideal circumstances divided by the total possible outcomes
third quartile	For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: median, first quartile, interquartile range.
tolerance	the allowable error in a given measurement
transformation	an operation that creates an image from a pre-image (e.g., translation, reflection, rotation, dilation, and glide-reflection)
transitive property	the rule stating that for real numbers *a, b,* and *c:* • if $a = b$ and $b = c$, then $a = c$; • if $a > b$ and $b > c$ then, $a>c$; and • if $a < b$ and $b < c$, then $a < c$
transitivity principle for indirect measurement	If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.
translate	the act of moving a figure in the coordinate plane preserving shape, size, and orientation
translation	a transformation that moves every point on a figure a given distance in a given direction
transversal	in a plane, a line that intersects two or more lines at different points
trapezoid	a quadrilateral that has exactly one pair of parallel sides
tree diagram	a representation used to find all the possible permutations for a set of items or the prime factorization of a number
trend	the general drift, tendency, or direction of data
triangle	a polygon with three sides
triangle inequality property	a property stating that, in a triangle, the sum of the lengths of two sides is greater than the length of the third side

trigonometric form	the form r(cos(θ) + I sin(θ)), where r is the magnitude of the complex number and θ is the angle it makes with the positive real axis
trigonometric functions	the functions sine, cosine, tangent, cotangent, secant and cosecant
trigonometric identities	equalities that are helpful for the simplification of complex trigonometric functions and that are true for every value of the variables (e.g., $\sin^2\theta + \cos^2\theta = 1$)
trigonometric ratios	the ratios of the lengths of pairs of sides in a right triangle (e.g., sine, cosine and tangent)
truth value	a value indicating whether a statement is true or false (note: typically written as sometimes true, always true, never true)
two-variable data	the data generated by two input cells used with one formula
uniform probability model	A probability model, which assigns equal probability to all outcomes. See also: probability model.
unimodality	a function with one maximum during a defined interval
unit circle	the circle with a radius of one and center at the origin
unit fraction	a fraction with a numerator of one
unit rate	the ratio of a quantity to one unit of another quantity (e.g., unit price)
unnecessary information	information that does not assist with the solution to a problem
U.S. Customary system of measurement	a measuring system used most often in the United States (e.g., inches, pounds, gallons) (note: also called the standard system of measurement)
valid argument	an argument that is correctly inferred or deduced from a premise
variable	a symbol that represents a quantity
variance	population: a measure of variability given by the average of squared deviations if the data is taken from an entire population (i.e., $V_P = \dfrac{\sum(x_i - \overline{x})^2}{n}$) sample: a measure of variability given by the average of squared deviations if data is taken from a sample instead of an entire population(i.e., $V_S = \dfrac{\sum(x_i - \overline{x})^2}{n-1}$)
vector	a quantity that has magnitude (length) and direction in the plane or in space, defined by an ordered pair or triple of real numbers.
Venn diagram	a representation that uses circles to show relationships between two or more sets
verify	the process of demonstrating or proving that a response is correct
vertex	geometry: the point at which the rays of an angle, two sides of a polygon, or the edges of a polyhedron meet (plural: vertices) vertex-edge graph: vertices (singular 'vertex') are elements or nodes of a graph or network that may or may not be joined by

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	edges
vertex-edge graph	a graph or network that consists of a collection of vertices and edges where each edge connects two of the vertices
vertical	at right angles to the plane of the horizon or to a horizontal axis
vertical angles	the opposite angles formed when two lines intersect
visual fraction model	A tape diagram, number line diagram, or area model.
volume	the measure of the capacity of a three-dimensional figure (measured in cubic units)
weight	a measure of the heaviness of, or the force of gravity on, an object
weight on an edge	value (or some number of objects) placed along an edge in a vertex-edge graph to represent some quantity such as distance, time, cost, or number of traffic lights
whole	the entire object, collection of objects, or quantity being considered
whole numbers	The numbers 0, 1, 2, 3, …. the set of numbers consisting of the natural numbers and zero
***x*-intercept**	the coordinate at which the graph of a line intersects the x-axis
***y*-intercept**	the coordinate at which the graph of a line intersects the y-axis
zero property	addition: the mathematical rule stating that the sum of a term and zero is equal to the original term subtraction: the mathematical rule stating that the difference of a term and zero is equal to the original term multiplication: the mathematical rule stating that the product of a term and zero is zero division: the mathematical rule stating that division of a term by zero is undefined
zeros (of a function)	the points at which the value of a function is zero (note: also called the roots of a function and the solutions for a function)

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Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \square c = b \square c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \square c = b \square c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

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Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, then b < a.
If a > b, then –a < –b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.