LESSON 2.1

Graphing Absolute Value Functions

Common Core Math Standards

The student is expected to:

COMMON CORE **F-IF.C.7b**

Graph … piecewise-defined functions, including … absolute value functions. Also A-CED.A.2, F-IF.B.4, F-BF.B.3

Mathematical Practices

COMMON CORE **MP.4 Modeling**

Language Objective

Identify the vertex, slope, and direction of the opening for a variety of absolute value functions by describing them to a partner.

ENGAGE

Essential Question: How can you identify the features of the graph of an absolute value function?

Possible answer: The domain consists of x values for which the function is defined or on which the real-world situation is based. The range consists of the corresponding $f(x)$ values. The end behavior describes what happens to the $f(x)$ values as the x values increase without bound or decrease without bound.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo, how a musician might make an instrument play louder or softer, and how a graph might show an increase and then a decrease in loudness. Then preview the Lesson Performance Task.

Name ______________________ Class ____________ Date ________

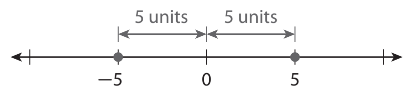
2.1 Graphing Absolute Value Functions

Essential Question: How can you identify the features of the graph of an absolute value function?

Resource Locker

Explore 1 Graphing and Analyzing the Parent Absolute Value Function

Absolute value, written as $|x|$, represents the distance between x and 0 on a number line. As a distance, absolute value is always positive. For every point on a number line, there is another point on the opposite side of 0 that is the same distance from 0. For example, both 5 and –5 are five units away from 0. Thus, $|-5| = 5$ and $|5| = 5$.



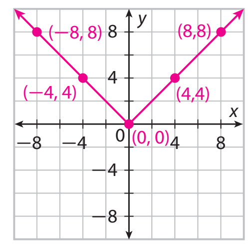
The absolute value function $|x|$, can be defined piecewise as $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$. When x is nonnegative, the function simply returns the number. When x is negative, the function returns the opposite of x.

(A) Complete the input-output table for $f(x)$.

$$f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

x	f(x)
–8	**8**
–4	**4**
0	**0**
4	**4**
8	**8**

(B) Plot the points you found on the coordinate grid. Use the points to complete the graph of the function.

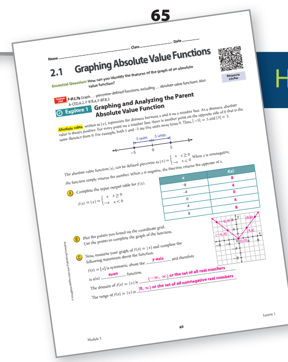


(C) Now, examine your graph of $f(x) = |x|$ and complete the following statements about the function.

$f(x) = |x|$ is symmetric about the **y-axis** and therefore is a(n) **even** function.

The domain of $f(x) = |x|$ is **$(-\infty, \infty)$ or the set of all real numbers**.

The range of $f(x) = |x|$ is **$[0, \infty)$ or the set of all nonnegative real numbers**.



HARDCOVER PAGES 49–56

Turn to these pages to find this lesson in the hardcover student edition.

Reflect

1. Use the definition of the absolute value function to show that $f(x) = |x|$ is an even function.

In an even function $f(x) = f(-x)$ for all x. $f(x) = |x|$ is defined piecewise, so there are two cases to examine: $x \geq 0$ and $x < 0$. If $x \geq 0$ then $-x \leq 0$. You know $f(x) = x$ when $x \geq 0$. To find $f(-x)$, recall that $f(x) = -x$ when $x \leq 0$. In this case, $-x \leq 0$, so $f(-x) = -(-x)$ or $f(-x) = x$. This shows that $f(x) = f(-x)$ when $x \geq 0$. If $x < 0$ then $-x > 0$. You know $f(x) = -x$ when $x < 0$. To find $f(-x)$, recall that $f(x) = x$ when $x > 0$. In this case, $-x > 0$, so $f(-x) = -x$. This shows that $f(x) = f(-x)$ when $x < 0$.

Explain 1 Graphing Absolute Value Functions

You can apply general transformations to absolute value functions by changing parameters in the equation $g(x) = a\left|\frac{1}{b}(x - h)\right| + k$.

Example 1 **Given the function $g(x) = a\left|\frac{1}{b}(x - h)\right| + k$, find the vertex of the function. Use the vertex and two other points to help you graph $g(x)$.**

Ⓐ $g(x) = 4|x - 5| - 2$

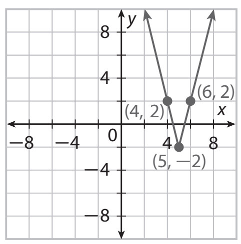
The vertex of the parent absolute value function is at (0, 0).

The vertex of $g(x)$ will be the point to which (0, 0) is mapped by $g(x)$.

$g(x)$ involves a translation of $f(x)$ 5 units to the right and 2 units down.

The vertex of $g(x)$ will therefore be at (5, –2).

Next, determine the location to which each of the points (1, 1) and (–1, 1) on $f(x)$ will be mapped.



Since $a > 1$, then $g(x)$, in addition to being a translation, is also a vertical stretch of $f(x)$ by a factor of 4. The x-coordinate of each point will be shifted 5 units to the right while the y-coordinate will be stretched by a factor of 4 and then moved down 2 units. So, (1, 1) moves to $(1 + 5, 4 \cdot |1| - 2) = (6, 2)$, and (–1, 1) moves to $(-1 + 5, 4 \cdot |1| - 2) = (4, 2)$. Now plot the three points and graph $g(x)$.

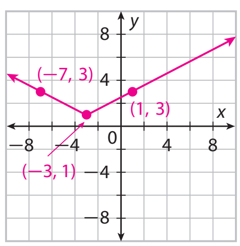
Ⓑ $g(x) = \left|-\frac{1}{2}(x + 3)\right| + 1$

The vertex of the parent absolute value function is at (0, 0).

$g(x)$ is a translation of $f(x)$ **3** units to the **left** and **1** unit **up**.

The vertex of $g(x)$ will therefore be at $(\boxed{-3}, \boxed{1})$.

Next, determine to where the points (2, 2) and (–2, 2) on $f(x)$ will be mapped.



PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.4**, which calls for students to "model with mathematics." Students learn the meaning of the parameters *a, b, h,* and *k* in an absolute value function, and use those parameters to graph and draw conclusions about absolute value functions.

EXPLORE 1

Graphing and Analyzing the Parent Absolute Value Function

INTEGRATE TECHNOLOGY

Using calculators to graph the parent absolute value function can illustrate that other absolute value functions are transformations of the parent function.

CONNECT VOCABULARY EL

Students should recognize that the graph of the parent function relates to the definition of *absolute value*. For each coordinate point, the y–value tells how far each x–value is from 0.

EXPLAIN 1

Graphing Absolute Value Functions

AVOID COMMON ERRORS

Students who recognize that (0, 0) is the vertex for the parent absolute value function may try to find the vertex for a transformation function by substituting 0 for x. Remind students that the vertex cannot be determined by substitution.

QUESTIONING STRATEGIES

In a function in the form $g(x) = a\left|\frac{1}{b}(x-h)\right| + k$ which parameters can be used to find the vertex of the function? Explain. **h and k; the vertex of the function, will be at the coordinates (h, k).**

Why do some graphs of absolute value functions extend higher in one direction than in the other? **When one half of the function extends higher than the other half, that graph's vertex is not in the center of the portion of the coordinate plane shown.**

INTEGRATE TECHNOLOGY

Students can use a graphing calculator to check their graphs of absolute value functions by verifying that the points they found are correct.

CONNECT VOCABULARY EL

Relate *absolute value function* graphs to the graphs of other *linear functions* by showing that all of them can be stretched, compressed, and reflected. Encourage students to describe the shapes and slopes of absolute value functions in their own words: for example, *upside–down V–shaped, composed of two lines or linear pieces*, and so on.

EXPLAIN 2

Writing Absolute Value Functions from a Graph

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 In order to verify that expressions are equivalent, students can substitute values in equivalent forms of absolute value expressions. For example, students can show that $a\left|\frac{1}{b}(x-h)\right| = \frac{a}{b}\left|(x-h)\right|$ by substituting values for a, b, x, and h and simplifying the resulting expressions.

Since $|b| = 2$, $g(x)$ is also a **horizontal stretch** of $f(x)$ and since b is negative, a **reflection across the y-axis**.

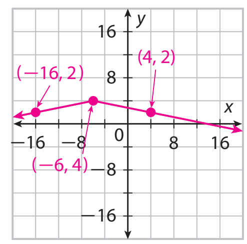
The x-coordinate will be reflected in the y-axis and **stretched** by a factor of **2**, then moved **3** units to the **left**.

The y-coordinate will move **up 1** unit.

So, $(2, 2)$ becomes $\left(\boxed{-2(2)-3}, \boxed{2+1}\right) = \left(\boxed{-7}, \boxed{3}\right)$, and $(-2, 2)$ becomes $\left(\boxed{1}, \boxed{3}\right)$. Now plot the three points and use them to sketch $g(x)$.

Your Turn

2. Given $g(x) = -\frac{1}{5}\left|(x+6)\right| + 4$, find the vertex and two other points and use them to help you graph $g(x)$.



Vertex: $(h, k) = (-6, 4)$

$-1 < a < 0$ so $g(x)$ is a reflection across the x-axis; vertical compression by $\frac{1}{5}$.

$(10, 10) \rightarrow \left(10-6, -\frac{1}{5}\cdot|10|+4\right) = (4, 2)$

$(-10, 10) \rightarrow (-16, 2)$

Explain 2 Writing Absolute Value Functions from a Graph

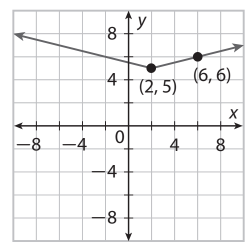
If an absolute value function in the form $g(x) = a\left|\frac{1}{b}(x-h)\right| + k$ has values other than 1 for both a and b, you can rewrite that function so that the value of at least one of a or b is 1.

When a and b are positive: $a\left|\frac{1}{b}(x-h)\right| = \left|\frac{a}{b}(x-h)\right| = \frac{a}{b}\left|(x-h)\right|$.

When a is negative and b is positive, you can move the opposite of a inside the absolute value expression. This leaves −1 outside the absolute value symbol: $-2\left|\frac{1}{b}\right| = -1(2)\left|\frac{1}{b}\right| = -1\left|\frac{2}{b}\right|$.

When b is negative, you can rewrite the equation without a negative sign, because of the properties of absolute value: $a\left|\frac{1}{b}(x-h)\right| = a\left|\frac{1}{-b}(x-h)\right|$. This case has now been reduced to one of the other two cases.

Example 2 **Given the graph of an absolute value function, write the function in the form $g(x) = a\left|\frac{1}{b}(x-h)\right| + k$.**



(A) Let $a = 1$.

The vertex of $g(x)$ is at $(2, 5)$. This means that $h = 2$ and $k = 5$.
The value of a is given: $a = 1$.

Substitute these values into $g(x)$, giving $g(x) = \left|\frac{1}{b}(x-2)\right| + 5$.

Choose a point on $g(x)$ like $(6, 6)$, Substitute these values into $g(x)$, and solve for b.

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs to construct graphs with three parameters the same and one parameter different. Instruct one student to choose which parameter $(a, b, h, \text{or } k)$ will be different. Both roll number cubes to determine the similar and different values. Have them create two graphs, then write a paragraph explaining how the different parameter affected the shape of each graph.

Substitute. $6 = \left|\frac{1}{b}(6-2)\right| + 5$

Simplify. $6 = \left|\frac{1}{b}(4)\right| + 5$

Subtract 5 from each side. $1 = \left|\frac{4}{b}\right|$

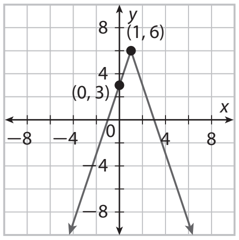
Rewrite the absolute value as two equations. $1 = \frac{4}{b}$ or $1 = -\frac{4}{b}$

Solve for b. $b = 4$ or $b = -4$

Based on the problem conditions, only consider $b = 4$. Substitute into $g(x)$ to find the equation for the graph.

$g(x) = \left|\frac{1}{4}(x-2)\right| + 5$

(B) Let $b = 1$.



The vertex of $g(x)$ is at **(1, 6)**. This means that $h = \boxed{1}$ and $k = \boxed{6}$ The value of b is given: $b = 1$.

Substitute these values into $g(x)$, giving $g(x) = a\left|x - \boxed{1}\right| + \boxed{6}$.

Now, choose a point on $g(x)$ with integer coordinates, $\left(0, \boxed{3}\right)$.

Substitute these values into $g(x)$ and solve for a.

$$g(x) = a\left|x - \boxed{1}\right| + \boxed{6}$$

Substitute. $\boxed{3} = a|0-1| + 6$

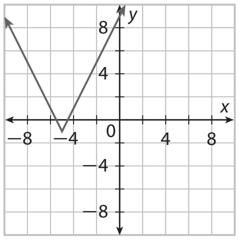
Simplify. $\boxed{3} = a|-1| + 6$

Solve for a. $\boxed{-3} = a$

Therefore $g(x) = \boxed{-3|x-1|+6}$.

Your Turn

3. Given the graph of an absolute value function, write the function in the form $g(x) = a\left|\frac{1}{b}(x-h)\right| + k$.



$a = 1$, vertex $= (-5, -1) = (h, k)$

$g(x) = \left|\frac{1}{b}(x-(-5))\right| - 1$

Choose (0, 9).

$9 = \left|\frac{1}{b}(0+5)\right| - 1$ **$b = \frac{1}{2}$ or $b = -\frac{1}{2}$**

$10 = \left|\frac{5}{b}\right|$ **$g(x) = |2(x+5)| - 1$**

QUESTIONING STRATEGIES

When writing an absolute function from a graph, how can you use the direction in which an absolute value function opens to check your work? **An absolute value function which opens upward will have a positive value for *a*, and an absolute value function which opens downward will have a negative value for *a*.**

DIFFERENTIATE INSTRUCTION

Critical Thinking

Discuss with students ways to determine if a graph of a function represents an absolute value function. Students should realize that an absolute value function has symmetry about a vertical line through the vertex, so the two pieces of the function will have equal but opposite slopes. Challenge students to show that the slopes of these two lines are opposites.

EXPLAIN 3

Modeling with Absolute Value Functions

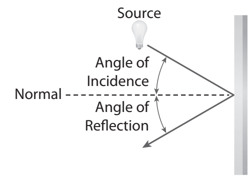
INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 When writing equations to solve real-world problems, discuss with students how to choose the part of the description that describes the origin. Students should understand that they can select an origin that will make the problem easy to solve.

Explain 3 Modeling with Absolute Value Functions

Light travels in a straight line and can be modeled by a linear function. When light is reflected off a mirror, it travels in a straight line in a different direction. From physics, the angle at which the light ray comes in is equal to the angle at which it is reflected away: the angle of incidence is equal to the angle of reflection. You can use an absolute value function to model this situation.



Law of Reflection

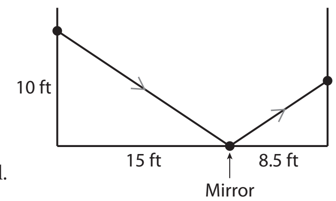
Example 3 **Solve the problem by modeling the situation with an absolute value function.**

At a science museum exhibit, a beam of light originates at a point 10 feet off the floor. It is reflected off a mirror on the floor that is 15 feet from the wall the light originates from. How high off the floor on the opposite wall does the light hit if the other wall is 8.5 feet from the mirror?

Analyze Information

Identify the important information.

- The model will be of the form $g(x) =$ $a\left|\frac{1}{b}(x-h)\right|+k$.
- The vertex of $g(x)$ is **(15, 0)**.
- Another point on $g(x)$ is **(0, 10)**.
- The opposite wall is **23.5** feet from the first wall.



Formulate a Plan

Let the base of the first wall be the origin. You want to find the value of $g(x)$ at $x =$ **23.5**, which will give the height of the beam on the opposite wall. To do so, find the value of the parameters in the transformation of the parent function.

In this situation, let $b = 1$. The vertex of $g(x)$ will give you the values of ***h* and *k***.

Use a second point to solve for a. Evaluate $g(23.5)$.

Solve

The vertex of $g(x)$ is at $(15, 0)$. Substitute, giving $g(x) = a|x - 15| + 0$.

Evaluate $g(x)$ at **(0,10)** and solve for a.

Substitute. $\quad 10 = a|0 - 15| + 0$

Simplify. $\quad 10 = a|-15|$

Simplify. $\quad 10 = 15a$

Solve for a. $\quad a = \frac{2}{3}$

Therefore $g(x) = \frac{2}{3}|x - 15|$. Find $g(23.5)$. $g(23.5) = \frac{17}{3} \approx 5.67$

LANGUAGE SUPPORT EL

Connect Context

Discuss how the term *parent function* relates to the common use of the word *parent*. Students should understand that transformations of the parent absolute value function will always have certain characteristics in common with the parent function.

Justify and Evaluate

The answer of **5.67** makes sense because the function is symmetric with respect to the line **$x = 15$**. The distance from this line to the second wall is a little more than **half** the distance from the line to the beam's origin. Since the beam originates at a height of **10 feet**, it should hit the second wall at a height of a little over **5 feet**.

Your Turn

4. Two students are passing a ball back and forth, allowing it to bounce once between them. If one student bounce-passes the ball from a height of 1.4 m and it bounces 3 m away from the student, where should the second student stand to catch the ball at a height of 1.2 m? Assume the path of the ball is linear over this short distance.

Let $a = 1$. **vertex $= (3,0) = (h,k)$**

Use the point $(0, 1.4)$ or $\left(0, \frac{7}{5}\right)$.

$$g(x) = \left|\frac{1}{b}(x-3)\right|$$
$$\frac{7}{5} = \left|\frac{1}{b}(0-3)\right|$$
$$\frac{7}{5} = \left|\frac{3}{b}\right|$$
$$\frac{7}{5} = \frac{3}{b} \quad \text{or} \quad \frac{7}{5} = -\frac{3}{b}$$
$$b = \frac{15}{7} \quad \text{or} \quad b = -\frac{15}{7}$$
$$g(x) = \left|\frac{7}{15}(x-3)\right|$$

Now, replace $g(x)$ with 1.2 or $\frac{6}{5}$ and solve for x. $\frac{6}{5} = \left|\frac{7}{15}(x-3)\right|$

$$\frac{6}{5} = \frac{7}{15}(x-3) \quad \text{or} \quad -\frac{6}{5} = \frac{7}{15}(x-3)$$
$$x = \frac{39}{7} \approx 5.57 \quad \text{or} \quad x = \frac{3}{7} \approx 0.43$$

Only $x = \frac{39}{7}$ makes sense (the second student has to be on the other side of the vertex from the first student). Therefore, the second student should stand 5.57 meters away from the first student.

Elaborate

5. In the general form of the absolute value function, what does each parameter represent?
h is horizontal translation, k is vertical translation, a is vertical stretch/compression and b is horizontal stretch/compression.

6. **Discussion** Explain why the vertex of $f(x) = |x|$ remains the same when $f(x)$ is stretched or compressed but not when it is translated.
The vertex of $f(x) = |x|$ is at $(0, 0)$. When $f(x)$ is stretched or compressed, one of the coordinates is multiplied by a or b. In either case, the product is 0 so the coordinates remain $(0, 0)$. But when $f(x)$ is translated, h or k is added to a coordinate, which changes the vertex.

7. **Essential Question Check-In** What are the features of the graph of an absolute value function?
The features are the vertex, the direction of opening, and the slope of each ray.

QUESTIONING STRATEGIES

The points A and B are both on the same absolute value function. If the x–value for Point A is greater than the x–value for Point B, can you determine which point has the greater y–value? Explain. **No; depending on the values for a, b, h, and k, either point could have a greater y–value.**

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 The general form of the absolute value function is similar to the quadratic function in vertex form. Students can use the similarities between the forms to remember what each variable represents.

QUESTIONING STRATEGIES

What are the values for h, k, a, and b in the parent absolute value function? **In the parent absolute value function, $h = 0$, $k = 0$, $a = 1$, and $b = 1$.**

SUMMARIZE THE LESSON

How can you use the parameters of an absolute value function in general form to predict the shape of the function? **The parameters h and k will tell you the coordinates for the vertex, (h, k). The sign of a will tell you whether the function opens upward or downward. The values used for a and b will tell you how much the function is stretched or compressed.**

EVALUATE

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explain 1 Graphing Absolute Value Functions	Exercises 6–11
Explain 2 Writing Absolute Value Functions from a Graph	Exercises 12–13
Explain 3 Modeling with Absolute Value Functions	Exercises 14–17

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

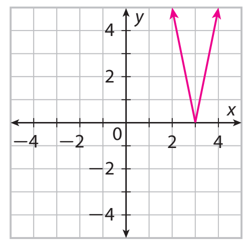
MP.3 When interpreting graphs of real-world absolute value functions, discuss with students what data is represented on the x–axis and what data is represented on the y–axis.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

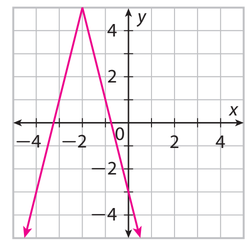
Predict what the graph of each given function will look like. Verify your prediction using a graphing calculator. Then sketch the graph of the function.

1. $g(x) = 5|x - 3|$



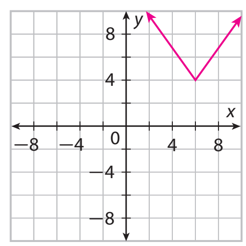
$g(x)$ is the graph of $f(x) = |x|$ vertically stretched by a factor of 5 and shifted 3 units to the right.

2. $g(x) = -4|x + 2| + 5$



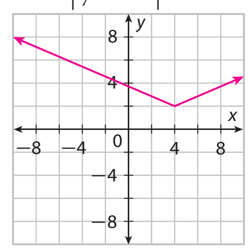
$g(x)$ is the graph of $f(x) = |x|$ vertically stretched by a factor of 4, shifted 2 units to the left, reflected across the x-axis and shifted 5 units up.

3. $g(x) = \left|\frac{7}{5}(x - 6)\right| + 4$



$g(x)$ is the graph of $f(x) = |x|$ horizontally compressed by a factor of $\frac{5}{7}$, shifted 6 units to the right and 4 units up.

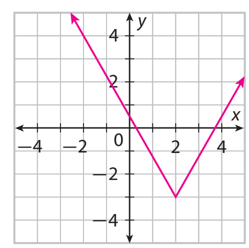
4. $g(x) = \left|\frac{3}{7}(x - 4)\right| + 2$



$g(x)$ is the graph of $f(x) = |x|$ horizontally stretched by a factor of $\frac{7}{3}$, shifted 4 units to the right, and 2 units up.

5. $g(x) = \frac{7}{4}|(x - 2)| - 3$

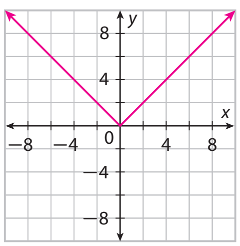
$g(x)$ is the graph of $f(x) = |x|$ vertically stretched by a factor of $\frac{7}{4}$, shifted 2 units to the right, and 3 units down.



Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–5	**1** Recall of Information	**MP.2** Reasoning
6–11	**1** Recall of Information	**MP.5** Using Tools
12–13	**1** Recall of Information	**MP.2** Reasoning
14–17	**2** Skills/Concepts	**MP.4** Modeling
18	**2** Skills/Concepts H.O.T.	**MP.5** Using Tools

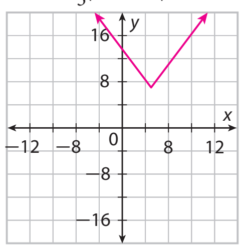
Graph the given function and identify the domain and range.

6. $g(x) = |x|$



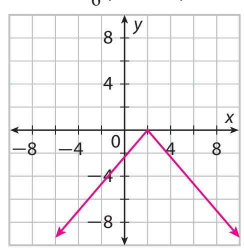
D: all real numbers; R: $y \geq 0$

7. $g(x) = \frac{4}{3}\left|(x-5)\right| + 7$



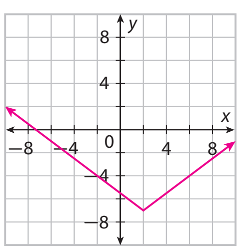
D: all real numbers; R: $y \geq 7$

8. $g(x) = -\frac{7}{6}\left|(x-2)\right|$



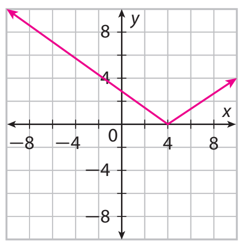
D: all real numbers; R: $y \leq 0$

9. $g(x) = \left|\frac{3}{4}(x-2)\right| - 7$



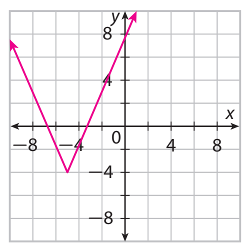
D: all real numbers; R: $y \geq -7$

10. $g(x) = \left|\frac{5}{7}(x-4)\right|$



D: all real numbers; R: $y \geq 0$

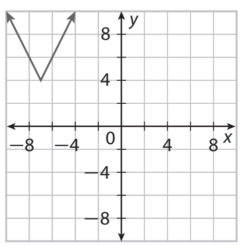
11. $g(x) = \left|-\frac{7}{3}(x+5)\right| - 4$



D: all real numbers; R: $y \geq -4$

Write the absolute value function in standard form for the given graph. Use a or b as directed, $b > 0$.

12. Let $a = 1$.



Vertex: $(-7, 4)$; equation $g(x) = \left|\frac{1}{b}(x+7)\right| + 4$

Use the point $(-6, 6)$: $6 = \left|\frac{1}{b}(-6+7)\right| + 4$

$$6 = \left|\frac{1}{b}(1)\right| + 4$$

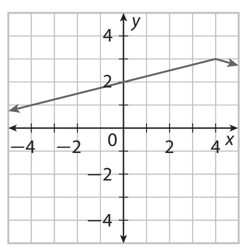
$$2 = \left|\frac{1}{b}\right|$$

$$\frac{1}{b} = 2 \text{ or } \frac{1}{b} = -2$$

$$b = \frac{1}{2} \text{ or } b = -\frac{1}{2}$$

So the equation is $g(x) = \left|2(x+7)\right| + 4$.

13. Let $b = 1$.



Vertex: $(4, 3)$; equation $g(x) = a\left|(x-4)\right| + 3$

Use the point $(0, 2)$: $2 = a\left|(0-4)\right| + 3$

$$2 = a\left|-4\right| + 3$$

$$2 = 4a + 3$$

$$-\frac{1}{4} = a$$

So the equation is $g(x) = -\frac{1}{4}\left|(x-4)\right| + 3$.

AVOID COMMON ERRORS

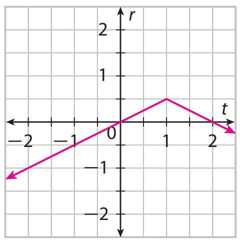
Students should recognize how to use the sign between x and h to correctly to translate the function in a negative or positive direction along the x-axis. Remind students that a negative sign in front of h refers to a translation to the right, and a positive sign refers to a translation to the left.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
19	2 Skills/Concepts H.O.T.	MP.1 Problem Solving
20	3 Strategic Thinking H.O.T.	MP.2 Reasoning

MULTIPLE REPRESENTATIONS

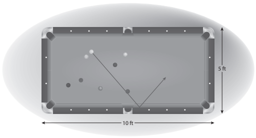
Challenge students to check their answers to real-world absolute value problems by using different methods. Since the graphs of absolute value functions have symmetry about the vertical line that contains the vertex, many real-world problems can be solved by drawing similar or congruent right triangles using segments from the graph of the function.

14. A rainstorm begins as a drizzle, builds up to a heavy rain, and then drops back to a drizzle. The rate r (in inches per hour) at which it rains is given by the function $r = -0.5|t - 1| + 0.5$, where t is the time (in hours). Graph the function. Determine for how long it rains and when it rains the hardest.



Since there can't be negative rainfall, the negative values can be discarded. Therefore, it rains for a total of 2 hours. The vertex of the graph is at (1, 0.5), so it rains the hardest at 1 hour.

15. While playing pool, a player tries to shoot the eight ball into the corner pocket as shown. Imagine that a coordinate plane is placed over the pool table. The eight ball is at $\left(5, \frac{5}{4}\right)$ and the pocket they are aiming for is at (10, 5). The player is going to bank the ball off the side at (6, 0).



a. Write an equation for the path of the ball.

The vertex of the path of the ball is (6, 0), so the equation has the form $y = a|x - 6|$. Substitute the coordinates of the point $\left(5, \frac{5}{4}\right)$ into the equation and solve for a.

$$y = a|x - 6|$$

$$\frac{5}{4} = a|5 - 6|$$

$$\frac{5}{4} = a|-1|$$

$$\frac{5}{4} = a$$

An equation for the path of the ball is $y = \frac{5}{4}|x - 6|$.

b. Did the player make the shot? How do you know?

The player will make the shot if the point (10, 5) lies on the path of the ball.

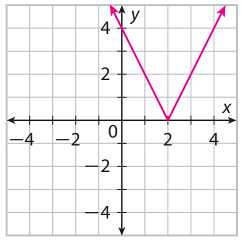
$$5 \stackrel{?}{=} \frac{5}{4}|10 - 6|$$

$$5 \stackrel{?}{=} \frac{5}{4}|4|$$

$$5 = 5$$

The point (10, 5) satisfies the equation, so the player does make the shot.

16. Sam is sitting in a boat on a lake. She can get burned by the sunlight that hits her directly and by sunlight that reflects off the water. Sunlight reflects off the water at the point (2, 0) and hits Sam at the point (3.5, 3). Write and graph the function that shows the path of the sunlight.



The vertex is (2, 0), so the equation has the form $y = a|x - 2|$.

Substitute (3.5, 3) into the equation and solve for a.

$3 = a|3.5 - 2|$

$3 = a|1.5|$

$2 = a$; An equation for the path of the sunlight is $y = 2|x - 2|$.

17. The Transamerica Pyramid is an office building in San Francisco. It stands 853 feet tall and is 145 feet wide at its base. Imagine that a coordinate plane is placed over a side of the building. In the coordinate plane, each unit represents one foot. Write an absolute value function whose graph is the V-shaped outline of the sides of the building, ignoring the "shoulders" of the building.

The vertex of the building is going to be the top, which will be at the point $(72.5, 853)$, so the equation has the form $y = a|x - 72.5| + 853$. Substitute the coordinates of the point $(145, 0)$ into the equation and solve for a.

$0 = a|145 - 72.5| + 853$

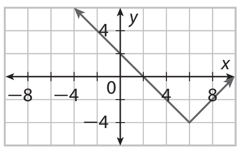
$0 = a|72.5| + 853$

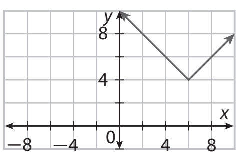
$-\frac{1706}{145} = a$

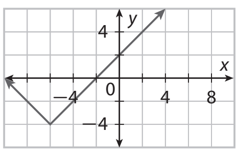
So, the equation has the form $y = -\frac{1706}{145}|x - 72.5| + 853$.

18. Match each graph with its function.

C $y = |x + 6| - 4$ **A** $y = |x - 6| - 4$ **B** $y = |x - 6| + 4$

A


B


C


CRITICAL THINKING

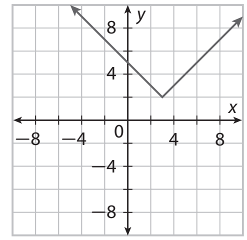
Have students consider the significance of h in determining values of the domain for which an absolute value function is increasing and for which it is decreasing, and how the value of a is useful for refining this information.

JOURNAL

Have students describe what can be determined about the shape of an absolute value function by examining the values of *a, b, h,* and *k*.

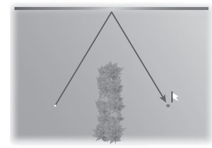
H.O.T. Focus on Higher Order Thinking

19. Explain the Error Explain why the graph shown is not the graph of $y = |x + 3| + 2$. What is the correct equation shown in the graph?



The graph shown cannot be the graph of $y = |x + 3| + 2$ because the $+3$ inside of the absolute value symbols means the parent graph should be shifted to the left 3 units. The graph shown is shifted to the right 3 units, so it represents $y = |x - 3| + 2$.

20. Multi-Step A golf player is trying to make a hole-in-one on the miniature golf green shown. Imagine that a coordinate plane is placed over the golf green. The golf ball is at (2.5, 2) and the hole is at (9.5, 2). The player is going to bank the ball off the side wall of the green at (6, 8).



a. Write an equation for the path of the ball.

The vertex of the path of the ball is (6, 8), so the equation has the form $y = a|x - 6| + 8$. Substitute the coordinates of the point (2.5, 2) into the equation and solve for a.

$$y = a|x - 6| + 8$$
$$2 = a|2.5 - 6| + 8$$
$$2 = a|-3.5| + 8$$
$$2 = 3.5a + 8$$
$$-6 = 3.5a$$
$$-\frac{12}{7} = a$$

An equation for the path of the ball is $y = -\frac{12}{7}|x - 6| + 8$.

b. Use the equation in part a to determine if the player makes the shot.

The player will make the shot if the point (9.5, 2) lies on the path of the ball.

$$2 \stackrel{?}{=} \frac{12}{7}|9.5 - 6| + 8$$
$$2 \stackrel{?}{=} -\frac{12}{7}|3.5| + 8$$
$$2 \stackrel{?}{=} -6 + 8$$
$$2 = 2$$

The point (9.5, 2) satisfies the equation, so the player does make the shot.

Lesson Performance Task

Suppose a musical piece calls for an orchestra to start at *fortissimo* (about 90 decibels), decrease steadily in loudness to *pianissimo* (about 50 decibels) in four measures, and then increase steadily back to *fortissimo* in another four measures.

a. Write a function to represent the sound level s in decibels as a function of the number of measures m.

b. After how many measures should the orchestra be at the loudness of *mezzo forte* (about 70 decibels)?

c. Describe what the graph of this function would look like.

a. $s = a|m - 4| + 50$

$90 = a|0 - 4| + 50$

$90 = a|-4| + 50$

$90 = 4a + 50$

$40 = 4a$

$10 = a$

So, the equation is $s = 10|m - 4| + 50$.

b. Substitute 70 for s.

$70 = 10|m - 4| + 50$

$20 = 10|m - 4|$

$2 = |m - 4|$

$2 = m - 4$ OR $-2 = m - 4$

$6 = m$ $\quad$ $2 = m$

So, at both measures 2 and 6, the orchestra will be at the loudness of *mezzo forte*.

The graph will be the graph of $f(x) = |x|$ shifted 4 units to the right, 50 units up, and vertically stretched by a factor of 10.

EXTENSION ACTIVITY

Have students consider these two situations:

- A note is played on an instrument, and then the instrument becomes silent.
- An empty room is noisier than a room furnished with rugs, drapes, and furniture.

Have students research where sound energy goes, and how sound vibrations dissipate, resulting in silence. Students should discover that sound energy is transmitted through the air and absorbed by many materials, especially those that are soft.

LANGUAGE SUPPORT EL

A *measure* or *bar* (*bar* is more common in British English, while *measure* is more common in American English) in musical notation is a segment of time defined by a given number of beats. Dividing music into measures helps a musician keep place in the written music and also keep time, or, stay in the proper rhythm.

CONNECT VOCABULARY EL

A *decibel*, dB, is a unit of how loud a sound is. On the decibel scale, 0 dB is used for a barely heard sound; normal conversation is about 60 dB; and 130 dB sound may cause pain. The word *decibel* is made up of two parts, *deci*, meaning one-tenth, and *bel*, named after Alexander Graham Bell.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 In this lesson, students have studied absolute value denoted by two vertical bars. In technology, the term *abs()* is often used. So, abs $(-2) = 2$ is the same as $|-2| = 2$.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Discuss with students which values of s have no associated value of m $(s < 50)$, one associated value of m $(s = 50)$, and more than one associated value of m $(s > 50.)$

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.