Arizona Mathematic Standards
Mathematics Curriculum Map

Kindergarten

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS
### Mathematics – Kindergarten

#### Chandler Unified School District Standards

**Kindergarten – At a Glance**

**Curriculum Map**

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<td><strong>Counting and Cardinality</strong></td>
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<td>K.CC.A.1 (Count by ones to 50)</td>
<td>K.CC.A.1 (Count by ones to 100)</td>
<td>K.CC.A.1 (Count by ones to 100)</td>
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<td>K.CC.A.2 (Count by ones to 20 starting with a number other than 1)</td>
<td>K.CC.A.2 (Count by ones to 50 starting with a number other than 1)</td>
<td>K.CC.A.2 (Count by ones to 100 starting with a number other than 1)</td>
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<td>K.CC.B.4 - 5 (Revisit)</td>
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<td>K.CC.C.5 (Go Math)</td>
<td>K.CC.A.6 (Go Math)</td>
<td>K.CC.C.6 (Go Math)</td>
<td>K.OA.A.5 (Fluency Standard)</td>
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<td>K.CC.C.7 (Go Math)</td>
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<td>K.CC.C.7 (Go Math)</td>
<td></td>
</tr>
<tr>
<td>K.OA.A.5 (Fluency Standard)</td>
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</table>

#### Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**NOTE:** Mathematical practices are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

**Kindergarten Key:** CC = Counting and Cardinality, OA = Operations and Algebraic Thinking, NBT = Number and Operations in Base Ten, MD = Measurement and Data, G = Geometry
Kindergarten Overview

Counting and Cardinality (CC)
- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking (OA)
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten (NBT)
- Work with numbers 11–19 to gain foundations for place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data (MD)
- Describe and compare measurable attributes.
- Classify objects and count the number of objects in each category.

Geometry (G)
- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Standards for Mathematical Practices (MP)
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
## Kindergarten Content Emphasis

### Counting and Cardinality (CC)
- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

### Operations and Algebraic Thinking (OA)
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

### Number and Operations in Base Ten (NBT)
- Work with numbers 11-19 to gain foundations for place value.
- Use place value understanding and properties of operations to add and subtract.

### Measurement and Data (MD)
- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

### Geometry (G)
- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

- **- Major Content**
- **△ -Supporting Content**

Major content (●) from the content emphasis section should account for approximately 70% of instructional time.
# Quarter 1

## Essential Question(s):
- How do I show how many?
- What do numbers tell me?
- How can I show numbers beyond 10?

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Arizona Mathematics Standards</th>
<th>Resources</th>
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</thead>
<tbody>
<tr>
<td><strong>Fluency Standard</strong></td>
<td>To be taught throughout the year: K.OA.A.5 Fluently add and subtract within 5.</td>
<td>My Math Chapter 1 Additional Resources: EngageNY Module 1 Topic B Illustrative Mathematics K.CC.A.3 Tasks K.CC.B.4 Tasks K.CC.C.6 Tasks Georgia Standards – Unit 1</td>
</tr>
<tr>
<td><strong>Numbers 0 - 5</strong></td>
<td>K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 to 20 (with 0 representing a count of no objects). K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (one to one correspondence). b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted (cardinality). c. Understand that each successive number name refers to a quantity that is one larger (hierarchical inclusion). K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. (Include groups with up to ten objects.)</td>
<td>My Math Chapter 2 Additional Resources: EngageNY Module 1 Topic C Topic D Topic E Topic F Topic G Topic H Illustrative Mathematics K.CC.A.3 Tasks K.CC.B.4 Tasks K.CC.C.6 Tasks Georgia Standards – Unit 1</td>
</tr>
<tr>
<td><strong>Numbers 0 - 10</strong></td>
<td>K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 to 20 (with 0 representing a count of no objects). K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (one to one correspondence). c. Understand that each successive number name refers to a quantity that is one larger (hierarchical inclusion). K.CC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. (Include groups with up to ten objects.)</td>
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</table>
**Essential Question(s):**
- How do I show how many?
- What do numbers tell me?
- How can I show numbers beyond 10?

**Numbers Beyond 10**

<table>
<thead>
<tr>
<th>Standards to Supplement</th>
<th>K.CC.A.1 Count to 100 by ones and by tens.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 to 20 (with 0 representing a count of no objects).</td>
</tr>
</tbody>
</table>

**My Math Chapter 3**

**Additional Resources:**
- EngageNY Module 5
  - Topic D
- Illustrative Mathematics
  - K.CC.A.1 Tasks
  - K.CC.A.3 Tasks
- Georgia Standards – Unit 1

**Supplemental Resources:**
- Illustrative Mathematics
  - K.CC.B.5 Tasks
  - K.CC.C.7 Tasks
- Inside Mathematics
  - Kinder CC Tasks

*The Standards for Mathematical Practices are to be embedded throughout every chapter.*
# Quarter 2

**Essential Question(s):**
- How can I show a number in other ways?
- How can I use objects to add?
- How can I use objects to subtract?

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

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<td>Fluency Standard</td>
<td>To be taught throughout the year: K.OA.A.5 Fluently add and subtract within 5.</td>
<td></td>
</tr>
<tr>
<td>&quot;Standard to Supplement throughout the quarter</td>
<td>*Standard to Supplement throughout the quarter</td>
<td>Supplemental Resources: Illustrative Mathematics K.OA.A.2 Tasks K.CC.A.2 Tasks</td>
</tr>
<tr>
<td>Compose and Decompose Number to 10</td>
<td>K.CC.A.1 Count to 100 by ones and by tens.</td>
<td>My Math Chapter 4</td>
</tr>
<tr>
<td></td>
<td>K.OA.A.1 Represent addition and subtraction concretely. See Table 1.</td>
<td>Additional Resources: EngageNY Module 4 Topic A Topic B Topic E</td>
</tr>
<tr>
<td></td>
<td>K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., using fingers, objects, symbols, tally marks, drawings, expressions).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given number (e.g., using fingers, objects, symbols, tally marks, drawings, or equations).</td>
<td>Georgia Standards – Unit 5 Georgia Standards – Unit 6</td>
</tr>
<tr>
<td></td>
<td>K.NBT.B.2 Demonstrate understanding of addition and subtraction within 10 using place value. See Table 1.</td>
<td>Inside Mathematics Kinder OA Tasks</td>
</tr>
</tbody>
</table>
Mathematics – Kindergarten  My Math

Quarter 2

Essential Question(s):
- How can I show a number in other ways?
- How can I use objects to add?
- How can I use objects to subtract?

<table>
<thead>
<tr>
<th>Addition</th>
<th>K.OA.A.1 Represent addition and subtraction concretely. See Table 1.</th>
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<tr>
<td></td>
<td>K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given number (e.g., using fingers, objects, symbols, tally marks, drawings, or equation).</td>
</tr>
<tr>
<td></td>
<td>K.OA.A.5 Fluently add and subtract within 5.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subtraction</th>
<th>K.CC.A.1 Count to 100 by ones and by tens.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K.OA.A.1 Represent addition and subtraction concretely. See Table 1.</td>
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<td>K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., using fingers, objects, symbols, tally marks, drawings, expressions).</td>
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<tr>
<td></td>
<td>K.OA.A.5 Fluently add and subtract within 5.</td>
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</tbody>
</table>

My Math Chapter 5
Additional Resources:
- EngageNY Module 4
  - Topic C
  - Topic F
- Illustrative Mathematics
  - K.OA.A.1 Tasks
  - K.OA.A.5 Tasks
- Georgia Standards – Unit 5
- Georgia Standards – Unit 6
- Inside Mathematics Kinder OA Tasks

My Math Chapter 6
Additional Resources:
- EngageNY Module 4
  - Topic D
  - Topic G
- Illustrative Mathematics
  - K.OA.A.1 Tasks
  - K.OA.A.3 Tasks
  - K.OA.A.5 Tasks
- Georgia Standards – Unit 5
- Georgia Standards – Unit 6
- Inside Mathematics Kinder OA Tasks

*The Standards for Mathematical Practices are to be embedded throughout every chapter.*
Quarter 3

Essential Question(s):
- How do we show numbers 11 to 19 in another way?
- How do I describe objects by length, height, and weight?
- How do I classify objects?

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

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<td>Fluency Standard</td>
<td>To be taught throughout the year:</td>
<td>My Math Chapter 7</td>
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<tr>
<td></td>
<td>K.OA.A.5 Fluently add and subtract within 5.</td>
<td>Additional Resources:</td>
</tr>
<tr>
<td>Compose and Decompose Numbers 11 to 19</td>
<td>K.CC.A.1 Count to 100 by ones and by tens.</td>
<td>EngageNY Module 5</td>
</tr>
<tr>
<td></td>
<td>K.NBT.A.1 Compose and decompose numbers from 11 to 19 into ten ones and additional ones by using objects, drawings and/or equations. Understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones (e.g., 18 = 10 + 8).</td>
<td>Topic A</td>
</tr>
<tr>
<td></td>
<td>K.OA.A.5 Fluently add and subtract within 5.</td>
<td>Topic B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Topic C</td>
</tr>
<tr>
<td>Measurement</td>
<td>K.MD.A.1 Describe measurable attributes of a single object (e.g., length and weight).</td>
<td>Illustrative Mathematics</td>
</tr>
<tr>
<td></td>
<td>K.MD.A.2 Directly compare two objects with a measurable attribute in common to see which object has “more of” or “less of” the attribute, and describe the difference (e.g., directly compare the length of 10 cubes to a pencil and describe one as longer or shorter).</td>
<td>K.NBT.A.1 Tasks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>K.OA.A.5 Tasks</td>
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<td>Georgia Standards – Unit 2</td>
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<td>My Math Chapter 8</td>
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<td>Additional Resources:</td>
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<td></td>
<td>EngageNY Module 3</td>
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<tr>
<td></td>
<td></td>
<td>Topic A</td>
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<td></td>
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<td>Topic B</td>
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<td></td>
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<td>Topic C</td>
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<td></td>
<td></td>
<td>Illustrative Mathematics</td>
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<tr>
<td></td>
<td></td>
<td>K.MD.A.1 Tasks</td>
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<td></td>
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<td>K.MD.A.2 Tasks</td>
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<td></td>
<td></td>
<td>Georgia Standards – Unit 4</td>
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<td></td>
<td></td>
<td>Inside Mathematics</td>
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<td></td>
<td></td>
<td>Kinder MD Tasks</td>
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## Quarter 3

### Essential Question(s):
- How do we show numbers 11 to 19 in another way?
- How do I describe objects by length, height, and weight?
- How do I classify objects?

<table>
<thead>
<tr>
<th>Classify Objects</th>
<th>K.MD.B.3 Classify objects into given categories; count the number in each category and sort the categories by count. (Note: limit category counts to be less than or equal to 10.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>My Math Chapter 9</td>
</tr>
</tbody>
</table>

### Additional Resources:
- EngageNY Module 3  
  Topic E  
  Topic F  
- Illustrative Mathematics  
  K.MD.B.3 Tasks  
- Georgia Standards – Unit 4

*The Standards for Mathematical Practices are to be embedded throughout every chapter.*
Essential Question(s):
- How do I identify positions?
- How can I compare shapes?
- How do I identify and compare three-dimensional shapes?

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

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<tr>
<th>Topic</th>
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<tr>
<td>Fluency and Standard Position</td>
<td>To be taught throughout the year: K.OA.A.5 Fluently add and subtract within 5. K.G.A.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.</td>
<td>My Math Chapter 10</td>
</tr>
<tr>
<td>Two-Dimensional Shapes</td>
<td>K.G.A.2 Correctly name shapes regardless of their orientation or overall size (e.g., circle, triangle, square, rectangle, rhombus, trapezoid, hexagon, cube, cone, cylinder, sphere). K.G.A.3 Identify shapes as two-dimensional (lying in a plane, flat) or three-dimensional (solid). K.G.B.4 Analyze and compare two-dimensional and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/corners), and other attributes (e.g., having sides of equal length). K.G.B.5 Model shapes in the world by building shapes from components (e.g., use sticks and clay balls) and drawing shapes. K.G.B.6 Use simple shapes to form composite shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”</td>
<td>My Math Chapter 11</td>
</tr>
</tbody>
</table>

Additional Resources:
- Georgia Standards – [Unit 3](#)
- Inside Mathematics [Kinder G Tasks](#)
- EngageNY Module 2 [Topic A](#) [Topic C](#) Module 6 [Topic A](#) [Topic B](#)
- Illustrative Mathematics [K.G.B.4 Tasks](#)
- Georgia Standards – [Unit 3](#)
- Inside Mathematics [Kinder G Tasks](#)
### Essential Question(s):
- How do I identify positions?
- How can I compare shapes?
- How do I identify and compare three-dimensional shapes?

<table>
<thead>
<tr>
<th>Three-Dimensional Shapes</th>
<th>K.G.A.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.</th>
</tr>
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<tr>
<td></td>
<td>K.G.A.2 Correctly name shapes regardless of their orientation or overall size (e.g., circle, triangle, square, rectangle, rhombus, trapezoid, hexagon, cube, cone, cylinder, sphere).</td>
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<td>K.G.A.3 Identify shapes as two-dimensional (lying in a plane, flat) or three-dimensional (solid).</td>
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<td>K.G.B.4 Analyze and compare two-dimensional and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/corners), and other attributes (e.g., having sides of equal length).</td>
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<tr>
<th>Mathematics Practices</th>
<th>Narratives</th>
<th>Related Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.MP.1 Make sense of</td>
<td>Mathematically proficient students explain to themselves the meaning of a</td>
<td>• How would you describe the problem in your own words?</td>
</tr>
<tr>
<td>problems and</td>
<td>problem, look for entry points to begin work on the problem, and plan</td>
<td>• How would you describe what you are trying to find?</td>
</tr>
<tr>
<td>persevere in solving</td>
<td>and choose a solution pathway. While engaging in productive struggle to</td>
<td>• What do you notice about...?</td>
</tr>
<tr>
<td>them</td>
<td>solve a problem, they continually ask themselves, “Does this make sense?” to</td>
<td>• What information is given in the problem?</td>
</tr>
<tr>
<td></td>
<td>monitor and evaluate their progress and change course if necessary. Once</td>
<td>• Describe the relationship between the quantities.</td>
</tr>
<tr>
<td></td>
<td>they have a solution, they look back at the problem to determine if the</td>
<td>• Describe what you have already tried. What might you change?</td>
</tr>
<tr>
<td></td>
<td>solution is reasonable and accurate. Mathematically proficient students</td>
<td>• Talk me through the steps you’ve used to this point.</td>
</tr>
<tr>
<td></td>
<td>check their solutions to problems using different methods, approaches,</td>
<td>• What steps in the process are you most confident about?</td>
</tr>
<tr>
<td></td>
<td>or representations. They also compare and understand different</td>
<td>• What are some other strategies you might try?</td>
</tr>
<tr>
<td></td>
<td>representations of problems and different solution pathways, both their</td>
<td>• What are some other problems that are similar to this one?</td>
</tr>
<tr>
<td></td>
<td>own and those of others.</td>
<td>• How might you use one of your previous problems to help you begin?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• How else might you organize...represent... show...?</td>
</tr>
<tr>
<td>4.MP.6 Attend to</td>
<td>Mathematically proficient students clearly communicate to others using</td>
<td>• What mathematical terms apply in this situation?</td>
</tr>
<tr>
<td>precision</td>
<td>appropriate mathematical terminology, and craft explanations that convey</td>
<td>• How did you know your solution was reasonable?</td>
</tr>
<tr>
<td></td>
<td>their reasoning. When making mathematical arguments about a solution,</td>
<td>• Explain how you might show that your solution answers the problem.</td>
</tr>
<tr>
<td></td>
<td>strategy, or conjecture, they describe mathematical relationships and</td>
<td>• What would be a more efficient strategy?</td>
</tr>
<tr>
<td></td>
<td>connect their words clearly to their representations. Mathematically</td>
<td>• How are you showing the meaning of the quantities?</td>
</tr>
<tr>
<td></td>
<td>proficient students understand meanings of symbols used in mathematics,</td>
<td>• What symbols or mathematical notations are important in this problem?</td>
</tr>
<tr>
<td></td>
<td>calculate accurately and efficiently, label quantities appropriately, and</td>
<td>• What mathematical language...,definitions..., properties can you use to explain...?</td>
</tr>
<tr>
<td></td>
<td>record their work clearly and concisely.</td>
<td>• How could you test your solution to see if it answers the problem?</td>
</tr>
</tbody>
</table>

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book
<table>
<thead>
<tr>
<th>Mathematics Practices</th>
<th>Narratives</th>
<th>Related Questions</th>
</tr>
</thead>
</table>
| 4.MP.2 Reason abstractly and quantitatively | Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context. | • What do the numbers used in the problem represent?  
• What is the relationship of the quantities?  
• How is _______ related to ________?  
• What is the relationship between _______and _______?  
• What does_______mean to you? (e.g. symbol, quantity, diagram)  
• What properties might we use to find a solution?  
• How did you decide in this task that you needed to use...?  
• Could we have used another operation or property to solve this task? Why or why not? |

| Reasoning and Explaining | 4.MP.3 Construct viable arguments and critique the reasoning of others | Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others. | • What mathematical evidence would support your solution?  
• How can we be sure that...? / How could you prove that...?  
• Will it still work if...?  
• What were you considering when...?  
• How did you decide to try that strategy?  
• How did you test whether your approach worked?  
• How did you decide what the problem was asking you to find?  
• Did you try a method that did not work? Why didn’t it work? Could it work?  
• What is the same and what is different about...?  
• How could you demonstrate a counter-example? |
# The Mathematical Practices: Narratives and Questions

<table>
<thead>
<tr>
<th>Mathematics Practices</th>
<th>Narratives</th>
<th>Related Questions</th>
</tr>
</thead>
</table>
| **4.MP.4** Model with mathematics | Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. | • What number model could you construct to represent the problem?  
• What are some ways to represent the quantities?  
• What is an equation or expression that matches the diagram, number line, chart, table, and your actions with the manipulatives?  
• Where did you see one of the quantities in the task in your equation or expression? What does each number in the equation mean?  
• How would it help to create a diagram, graph, table...?  
• What are some ways to visually represent...?  
• What formula might apply in this situation? |
| **4.MP.5** Use appropriate tools strategically | Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others. | • What mathematical tools can we use to visualize and represent the situation?  
• Which tool is more efficient? Why do you think so?  
• What information do you have?  
• What do you know that is not stated in the problem?  
• What approach are you considering trying first?  
• What estimate did you make for the solution?  
• In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative?  
• Why was it helpful to use...?  
• What can using a _____ show us that _____ may not?  
• In what situations might it be more informative or helpful to use...? |

*Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011*

*Most questions from all Grades Common Core State Standards Flip Book*
### Mathematics Practices: Narratives and Questions

<table>
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<th>Mathematics Practices</th>
<th>Narratives</th>
<th>Related Questions</th>
</tr>
</thead>
</table>
| 4.MP.7                | Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed. | • What observations do you make about...?  
• What do you notice when...?  
• What parts of the problem might you eliminate..., simplify...?  
• What patterns do you find in...?  
• How do you know if something is a pattern?  
• What ideas that we have learned before were useful in solving this problem?  
• What are some other problems that are similar to this one?  
• How does this relate to...?  
• In what ways does this problem connect to other mathematical concepts?                                                                 |
| 4.MP.8                | Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency. | • Explain how this strategy works in other situations?  
• Is this always true, sometimes true or never true?  
• How would we prove that...?  
• What do you notice about...?  
• What is happening in this situation?  
• What would happen if...?  
• Is there a mathematical rule for...?  
• What predictions or generalizations can this pattern support?  
• What mathematical consistencies do you notice?                                                                 |

*Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011*  
*Most questions from all Grades Common Core State Standards Flip Book*
### Table 1. Common Addition and Subtraction Problem Types/Situations. ¹

<table>
<thead>
<tr>
<th>Result Unknown</th>
<th>Change Unknown</th>
<th>Start Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Add to</strong></td>
<td><strong>Change Unknown</strong></td>
<td><strong>Start Unknown</strong></td>
</tr>
<tr>
<td>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?</td>
<td>Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5</td>
<td>Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5</td>
</tr>
<tr>
<td><strong>Take from</strong></td>
<td><strong>Addend Unknown</strong></td>
<td><strong>Both Addends Unknown²</strong></td>
</tr>
<tr>
<td>Five apples were on the table. I ate two apples. How many apples are on the table now? 5 − 2 = ?</td>
<td>Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3</td>
<td>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? − 2 = 3</td>
</tr>
<tr>
<td><strong>Total Unknown</strong></td>
<td><strong>Addend Unknown</strong></td>
<td><strong>Both Addends Unknown²</strong></td>
</tr>
<tr>
<td>Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?</td>
<td>Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 − 3 = ?</td>
<td>Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2</td>
</tr>
<tr>
<td><strong>Put together/Take Apart³</strong></td>
<td><strong>Bigger Unknown</strong></td>
<td><strong>Smaller Unknown</strong></td>
</tr>
<tr>
<td>(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? 2 + ? = 5, 5 − 2 = ?</td>
<td>(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?</td>
<td>(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?</td>
</tr>
<tr>
<td>(“How many fewer?”): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 − 2 = ?</td>
<td>(Version with “fewer”): Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?</td>
<td>(Version with “fewer”): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 − 3 = ?, ? + 3 = 5</td>
</tr>
</tbody>
</table>

¹ Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

² These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children that the = sign does not always mean makes or results in but always does mean is the same quantity as.

³ Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.
Table 2. Common Multiplication and Division Situations.

<table>
<thead>
<tr>
<th>Unknown Product</th>
<th>Group Size Unknown</th>
<th>Number of Groups Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 x 6 = ?</td>
<td>3 x ? = 18, and 18 ÷ 3 = ?</td>
<td>? x 6 = 18, and 18 ÷ 6 = ?</td>
</tr>
</tbody>
</table>

**Equal Groups**
- There are 3 bags with 6 plums in each bag. How many plums are there in all?
- Measurements example:
- You need 3 lengths of string, each 6 inches long. How much string will you need altogether?
- Measurement example:
- You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?
- Measurement example:
- You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?

**Arrays**
- There are 3 rows of apples with 6 apples in each row. How many apples are there?
- Area example:
- What is the area of a 3 cm by 6 cm rectangle?
- Area example:
- A rectangle has the area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?
- Area example:
- A rectangle has the area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?

**Compare (Grade 4 ONLY)**
- A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?
- Measurement example:
- A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?
- Measurement example:
- A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?
- Measurement example:
- A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?
- Measurement example:
- A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?
- Measurement example:
- A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?

**General**
- a x b = ?
- a x ? = p, and p ÷ a = ?
- ? x b = p, and p ÷ b = ?

---

1 The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

4 The language is the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.