



# Arizona Mathematics Standards

---

## Fourth Grade

ARIZONA DEPARTMENT OF EDUCATION  
HIGH ACADEMIC STANDARDS FOR STUDENTS  
December, 2016

# Arizona Mathematics Standards 4<sup>th</sup> Grade

## Fourth Grade Overview

---

### Operations and Algebraic Thinking (OA)

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

### Number and Operations in Base Ten (NBT)

*Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.*

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

### Number and Operations—Fractions (NF)

*Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.*

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

### Measurement and Data (MD)

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: Understand concepts of angle and measure angles.

### Geometry (G)

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

### Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Arizona Mathematics Standards 4<sup>th</sup> Grade

### Fourth Grade: Critical Areas

---

*In fourth grade, instructional time should focus on three critical areas:*

- 1. Extend understanding of place value to multi-digit numbers and fluently add and subtract multi-digit numbers.**
- 2. Develop competency with multi-digit multiplication, and develop understanding of dividing to find quotients involving multi-digit dividends.**
- 3. Develop understanding of fraction equivalence, addition, and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**

(1) Students generalize their understanding of place value through 1,000,000, and the relative size of numbers in each place. They use their understanding of properties of operations to perform multi-digit arithmetic with multi-digit whole number less than or equal to 1,000,000. They round multi-digit numbers and fluently add and subtract multi-digit whole numbers within 1,000,000.

(2) Students apply their understanding of models for multiplication, place value, and properties of operations, in particular the distributive property, to compute products of multi-digit whole numbers. They develop fluency with efficient strategies for multiplying multi-digit whole numbers through 1,000,000; understand and explain why the strategies work; and use them to solve problems (*Table 2*). Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication to find quotients involving multi-digit dividends.

(3) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,  $15/9 = 5/3$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

*The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.*

## Arizona Mathematics Standards 4<sup>th</sup> Grade

### Operations and Algebraic Thinking (OA)

<b>4.OA.A</b> <b>Use the four operations with whole numbers to solve problems.</b>	<b>4.OA.A.1</b>	Represent verbal statements of multiplicative comparisons as multiplication equations. Interpret a multiplication equation as a comparison (e.g., 35 is the number of objects in 5 groups, each containing 7 objects, and is also the number of objects in 7 groups, each containing 5 objects).
	<b>4.OA.A.2</b>	Multiply or divide within 1000 to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison). <i>See Table 2.</i>
	<b>4.OA.A.3</b>	Solve multistep word problems using the four operations, including problems in which remainders must be interpreted. Understand how the remainder is a fraction of the divisor. Represent these problems using equations with a letter standing for the unknown quantity.
<b>4.OA.B</b> <b>Gain familiarity with factors and multiples.</b>	<b>4.OA.B.4</b>	Find all factor pairs for a whole number in the range 1 to 100 and understand that a whole number is a multiple of each of its factors.
<b>4.OA.C</b> <b>Generate and analyze patterns.</b>	<b>4.OA.C.5</b>	Generate a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself and explain the pattern informally (e.g., given the rule “add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers).
	<b>4.OA.C.6</b>	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.

### Number and Operations in Base Ten (NBT)

*Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.*

<b>4.NBT.A</b> <b>Generalize place value understanding for multi-digit whole numbers.</b>	<b>4.NBT.A.1</b>	Apply concepts of place value, multiplication, and division to understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
	<b>4.NBT.A.2</b>	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$ , $=$ , and $<$ symbols to record the results of comparisons.
	<b>4.NBT.A.3</b>	Use place value understanding to round multi-digit whole numbers to any place.

## Arizona Mathematics Standards 4<sup>th</sup> Grade

<b>4.NBT.B</b> Use place value understanding and properties of operations to perform multi-digit arithmetic.	<b>4.NBT.B.4</b>	Fluently add and subtract multi-digit whole numbers using a standard algorithm.
	<b>4.NBT.B.5</b>	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
	<b>4.NBT.B.6</b>	Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
<b>Number and Operations – Fractions (NF)</b> <i>Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</i>		
<b>4.NF.A</b> Extend understanding of fraction equivalence and ordering.	<b>4.NF.A.1</b>	Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.
	<b>4.NF.A.2</b>	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators and by comparing to a benchmark fraction). a. Understand that comparisons are valid only when the two fractions refer to the same size whole. b. Record the results of comparisons with symbols $>$ , $=$ , or $<$ , and justify the conclusions.
<b>4.NF.B</b> Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.	<b>4.NF.B.3</b>	Understand a fraction $a/b$ with $a > 1$ as a sum of unit fractions ( $1/b$ ). a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., $3/8 = 1/8 + 1/8 + 1/8$ ; $3/8 = 2/8 + 1/8$ ; $2 \frac{1}{8} = 1 + 1 + 1/8$ or $2 \frac{1}{8} = 8/8 + 8/8 + 1/8$ ). c. Add and subtract mixed numbers with like denominators (e.g., by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.
	<b>4.NF.B.4</b>	Build fractions from unit fractions. a. Understand a fraction $\frac{a}{b}$ as a multiple of a unit fraction $\frac{1}{b}$ . In general, $\frac{a}{b} = a \times \frac{1}{b}$ . b. Understand a multiple of $\frac{a}{b}$ as a multiple of a unit fraction $\frac{1}{b}$ , and use this understanding to multiply a whole number by a fraction. In general, $n \times \frac{a}{b} = \frac{n \times a}{b}$ . c. Solve word problems involving multiplication of a whole number by a fraction. <i>For example, if each person at a party will eat <math>3/8</math> of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i>

## Arizona Mathematics Standards 4<sup>th</sup> Grade

<b>4.NF.C</b> <b>Understand decimal notation for fractions, and compare decimal fractions.</b>	<b>4.NF.C.5</b>	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). <i>For example, express <math>3/10</math> as <math>30/100</math>, and <math>3/10 + 4/100 = 34/100</math>.</i> (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators, in general, is not a requirement at this grade.)
	<b>4.NF.C.6</b>	Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line.
	<b>4.NF.C.7</b>	Compare two decimals to hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$ , $=$ , or $<$ .
<b>Measurement and Data (MD)</b>		
<b>4.MD.A</b> <b>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</b>	<b>4.MD.A.1</b>	Know relative sizes of measurement units within one system of units which could include km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit and in a smaller unit in terms of a larger unit. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1,12), (2,24), (3,36).</i>
	<b>4.MD.A.2</b>	Use the four operations to solve word problems and problems in real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations, including number lines that feature a measurement scale.
	<b>4.MD.A.3</b>	Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world contexts including problems with unknown side lengths. <i>See Table 2.</i>
<b>4.MD.B</b> <b>Represent and interpret data.</b>	<b>4.MD.B.4</b>	Make a line plot to display a data set of measurements in fractions of a unit ( $1/2$ , $1/4$ , $1/8$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
<b>4.MD.C</b> <b>Geometric measurement: Understand concepts of angle and measure angles.</b>	<b>4.MD.C.5</b>	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:  a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a “one-degree angle,” and can be used to measure angles.  b. An angle that turns through $n$ one-degree angles is said to have an angle measure of $n$ degrees.
	<b>4.MD.C.6</b>	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

## Arizona Mathematics Standards 4<sup>th</sup> Grade

4.MD.C (cont.)	4.MD.C.7	Understand angle measures as additive. (When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.) Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts.
<b>Geometry (G)</b>		
4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	4.G.A.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
	4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size (e.g., understand right triangles as a category, and identify right triangles).
	4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

## Arizona Mathematics Standards 4<sup>th</sup> Grade

### Standards for Mathematical Practice

<b>4.MP.1</b>	<b>Make sense of problems and persevere in solving them.</b> Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
<b>4.MP.2</b>	<b>Reason abstractly and quantitatively.</b> Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
<b>4.MP.3</b>	<b>Construct viable arguments and critique the reasoning of others.</b> Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
<b>4.MP.4</b>	<b>Model with mathematics.</b> Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.



## Arizona Mathematics Standards 4<sup>th</sup> Grade

<b>4.MP.5</b>	<b>Use appropriate tools strategically.</b> Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
<b>4.MP.6</b>	<b>Attend to precision.</b> Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
<b>4.MP.7</b>	<b>Look for and make use of structure.</b> Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
<b>4.MP.8</b>	<b>Look for and express regularity in repeated reasoning.</b> Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

## Arizona Mathematics Standards 4<sup>th</sup> Grade

**Table 2. Common Multiplication and Division Problem Types/Situations.<sup>1</sup>**

	Unknown Product	Group Size Unknown (“How many in each group?” Division)	Number of Groups Unknown (“How many groups?” Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
<b>Equal Groups</b>	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><b>Measurement example.</b></p> <p>You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><b>Measurement example.</b></p> <p>You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><b>Measurement example.</b></p> <p>You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
<b>Arrays,<sup>2</sup> Area<sup>3</sup></b>	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><b>Area example.</b></p> <p>What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><b>Area example.</b></p> <p>A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><b>Area example.</b></p> <p>A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
<b>Compare</b>	<p>A straw hat costs \$6. A baseball hat costs 3 times as much as the straw hat. How much does the baseball hat cost?</p> <p><b>Measurement example.</b></p> <p>A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A baseball hat costs \$18 and that is 3 times as much as a straw hat costs. How much does a blue straw cost?</p> <p><b>Measurement example.</b></p> <p>A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A baseball hat costs \$18 and a straw hat costs \$6. How many times as much does the baseball hat cost as the straw hat?</p> <p><b>Measurement example.</b></p> <p>A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
<b>General</b>	$a \times b = ?$	$a \times ? = p$ , and $p \div a = ?$	$? \times b = p$ , and $p \div b = ?$

<sup>1</sup>The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

<sup>2</sup>The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>3</sup>Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.