Arizona's Common Core Standards

Mathematics Curriculum Map

Geometry

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS
State Board Approved June 2010

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Mathematics – Geometry

Overview of the Common Core Standards Structure - HS

Conceptual Category	**Number & Quantity (N)**	**Algebra (A)**	**Functions (F)**	**Geometry (G)**	**Statistics & Probability (S)**	**Modeling**
Domains	The Real Number System (N-RN)	Seeing Structure in Expressions (A-SSE)	Interpreting Functions (F-IF)	Congruence (G-CO)	Interpreting Categorical & Quantitative Data (S-ID)	Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (☆)
	Quantities (N-Q)	Arithmetic with Polynomials & Rational Expressions (A-APR)	Building Functions (F-BF)	Similarity, Right Triangles, & Trigonometry (G-SRT)	Making Inferences & Justifying Conclusions (S-IC)	
	The Complex Number System (N-CN)	Creating Equations (A-CED)	Linear, Quadratic, & Exponential Models (F-LE)	Expressing Geometric Properties with Equations (G-GPE)	Conditional Probability & the Rules of Probability (S-CP)	
	Vector & Matrix Quantities (N-VM)	Reasoning with Equations & Inequalities (A-REI)	Trigonometric Functions (F-TF)	Geometric Measurement & Dimension (G-GMD)	Using Probability to Make Decisions (S-MD)	

The Geometry course outlined in this document begins with developing the tools of geometry, including transformations, proof, and constructions. These tools are used throughout the course as students formalize geometric concepts studied in earlier courses and extend those ideas to new concepts presented in the high school standards. There is a focus on modeling, problem solving, and proof throughout the course.

The units are sequenced to best develop and connect the mathematical content described in the CCSSM; however, the order of standards in any unit does not imply a sequence of content within the unit. Some standards may be revisited several times during the geometry course; others may be only partially addressed in different units, depending on the focus of the unit. Strikethroughs in the text indicate that only part of the standard is addressed in the unit.

The Mathematical Practices should become a natural way in which students come to understand and do mathematics. While, depending on the content to be understood or on the problem to be solved, any practice might be brought to bear, some practices may prove more useful than others. In a high school geometry course, reasoning and developing viable argument are particularly important, as are use of strategic tools and precision of language. Opportunities for highlighting certain practices are indicated in different units in this document, but this highlighting should not be interpreted to mean that other practices should be neglected in those units.

	Units In Order of Instruction	Topic	Domains Addressed*	Mathematical Practices (Address all. Highlight bold.)	Suggested Unit Length (in days)
Q1	Unit 1	Geometric transformations	G-CO	1, 2, 3, **4, 5, 6**, 7, 8	15
	Unit 2	Angles and lines	G-CO G-GPE	1, 2, **3**, 4, 5, **6, 7**, 8	15
	Unit 3	Triangles	G-CO G-C ★G-MG	1, 2, **3**, 4, **5**, 6, 7, **8**	15
Q2	Unit 4	Triangle congruence	G-CO	1, 2, **3**, 4, 5, **6, 7**, 8	20
	Unit 5	Similarity transformations	G-CO G-SRT G-GPE ★G-MG	**1**, 2, **3**, 4, 5, 6, 7, **8**	15
Q3	Unit 6	Right triangle relationships and trigonometry	★G-SRT ★G-GPE ★G-MG	**1**, **2**, 3, **4**, 5, 6, 7, 8	20
	Unit 7	Quadrilaterals	G-CO G-GPE	1, **2, 3**, 4, **5**, 6, 7, 8	15
Q4	Unit 8	Circles	G-CO G-C G-PE G-MD ★G-MG	1, **2, 3, 4**, 5, 6, 7, 8	15
	Unit 9	Geometric modeling in two dimensions	★G-MG	**1, 2,** 3, **4**, 5, 6, 7, 8	10
	Unit 10	Understanding and modeling with three-dimensional figures	G-MD ★G-MG	**1, 2,** 3, **4**, 5, 6, 7, 8	20

* Some standards in the domain are addressed in this unit.

★ Specific modeling standards are in this unit.

Geometry Overview

Congruence (CO)

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry (SRT)

- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Apply trigonometry to general triangles.

Circles (C)

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.

Expressing Geometric Properties with Equations (GPE)

- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems Algebraically.

Geometric Measurement and Dimension (GMD)

- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry (MG)

- Apply geometric concepts in modeling situations.

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics – Geometry

The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Overarching habits of mind of a productive math thinker	**1. Make sense of problems and persevere in solving them**	• Have or value sense-making • Use patience and persistence to listen to others • Be able to use strategies • Use self-evaluation and redirections • Be able to show or use multiple representations • Communicate both verbally and in written format • Be able to deduce what is a reasonable solution	• Provide open-ended and rich problems • Ask probing questions • Model multiple problem-solving strategies through Think- Alouds • Promotes and values discourse and collaboration • Cross-curricular integrations • Probe student responses (correct or incorrect) for understanding and multiple approaches • Provide solutions	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about...? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize...represent... show...?
	6. Attend to precision	• Communicate with precision-orally & written • Use mathematics concepts and vocabulary appropriately. • State meaning of symbols and use appropriately • Attend to units/labeling/tools accurately • Carefully formulate explanations • Calculate accurately and efficiently • Express answers in terms of context • Formulate and make use of definitions with others and their own reasoning.	• Think aloud/Talk aloud • Explicit instruction given through use of think aloud/talk aloud • Guided Inquiry including teacher gives problem, students work together to solve problems, and debriefing time for sharing and comparing strategies • Probing questions targeting content of study	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language...,definitions..., properties can you use to explain...? • How could you test your solution to see if it answers the problem?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book

Adapted from Agile Mind CCSS Geometry Scope and Sequence 2011, Agile Mind, Inc. and Charles A. Dana Center

Mathematics – Geometry

The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Reasoning and Explaining	**2.Reason abstractly and quantitatively**	• Create multiple representations • Interpret problems in contexts • Estimate first/answer reasonable • Make connections • Represent symbolically • Visualize problems • Talk about problems, real life situations • Attending to units • Using context to think about a problem	• Develop opportunities for problem solving • Provide opportunities for students to listen to the reasoning of other students • Give time for processing and discussing • Tie content areas together to help make connections • Give real world situations • Think aloud for student benefit • Value invented strategies and representations • Less emphasis on the answer	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _______ related to ________? • What is the relationship between ______and ______? • What does_______mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use...? • Could we have used another operation or property to solve this task? Why or why not?
	3.Construct viable arguments and critique the reasoning of others	• Ask questions • Use examples and non-examples • Analyze data • Use objects, drawings, diagrams, and actions • Students develop ideas about mathematics and support their reasoning • Listen and respond to others • Encourage the use of mathematics vocabulary	• Create a safe environment for risk-taking and critiquing with respect • Model each key student disposition • Provide complex, rigorous tasks that foster deep thinking • Provide time for student discourse • Plan effective questions and student grouping	• What mathematical evidence would support your solution? • How can we be sure that...? / How could you prove that...? • Will it still work if...? • What were you considering when...? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? • Did you try a method that did not work? Why didn't it work? Could it work? • What is the same and what is different about...? • How could you demonstrate a counter-example?

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The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

Mathematics Practices		Student Dispositions	Teacher Actions	Related Questions
Modeling and Using Tools	**4. Model with mathematics**	• Realize they use mathematics (numbers and symbols) to solve/work out real-life situations • When approached with several factors in everyday situations, be able to pull out important information needed to solve a problem. • Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable. If not, go back and look for more information • Make sense of the mathematics	• Allow time for the process to take place (model, make graphs, etc.) • Model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) • Make appropriate tools available • Create an emotionally safe environment where risk taking is valued • Provide meaningful, real world, authentic, performance-based tasks (non-traditional work problems)	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the diagram, number line, chart, table, and your actions with the manipulatives? • Where did you see one of the quantities in the task in your equation or expression? What does each number in the equation mean? • How would it help to create a diagram, graph, table...? • What are some ways to visually represent...? • What formula might apply in this situation?
	5. Use appropriate tools strategically	• Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base 10 blocks, compass, protractor) • Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools)	• Maintain appropriate knowledge of appropriate tools • Effective modeling of the tools available, their benefits and limitations • Model a situation where the decision needs to be made as to which tool should be used	• What mathematical tools can we use to visualize and represent the situation? • Which tool is more efficient? Why do you think so? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? • Why was it helpful to use...? • What can using a ______ show us that _____may not? • In what situations might it be more informative or helpful to use...?

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Mathematics – Geometry

The Mathematical Practices: Student Dispositions and Related Teacher Actions and Questions

	Mathematics Practices	Student Dispositions	Teacher Actions	Related Questions
Seeing structure and generalizing	7. **Look for and make use of structure**	• Look for, interpret, and identify patterns and structures • Make connections to skills and strategies previously learned to solve new problems/tasks • Reflect and recognize various structures in mathematics • Breakdown complex problems into simpler, more manageable chunks	• Be quiet and allow students to think aloud • Facilitate learning by using open-ended questioning to assist students in exploration • Careful selection of tasks that allow for students to make connections • Allow time for student discussion and processing • Foster persistence/stamina in problem solving • Provide graphic organizers or record student responses strategically to allow students to discover patters	• What observations do you make about...? • What do you notice when...? • What parts of the problem might you eliminate..., simplify...? • What patterns do you find in...? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to...? • In what ways does this problem connect to other mathematical concepts?
	8. **Look for and express regularity in repeated reasoning**	• Identify patterns and make generalizations • Continually evaluate reasonableness of intermediate results • Maintain oversight of the process	• Provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge • Provide adequate time for exploration • Provide time for dialogue and reflection • Ask deliberate questions that enable students to reflect on their own thinking • Create strategic and intentional check in points during student work time.	• Explain how this strategy works in other situations? • Is this always true, sometimes true or never true? • How would we prove that...? • What do you notice about...? • What is happening in this situation? • What would happen if...? • Is there a mathematical rule for...? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

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Geometry

Unit 1: Geometric transformations. Suggested number of days: 15

In this unit, students formalize much of the geometric exploration from middle school. In this unit, students develop rigorous definition of three familiar congruence transformations: reflections, translations, and rotations and use these transformations to discover and prove geometric properties. Throughout the course, students will use transformations as a tool to analyze and describe relationships between geometric figures.

Common Core State Standards for Mathematical Content

Congruence—G-CO

A. Experiment with transformations in the plane.

1. Know precise definitions of angle, ~~circle~~, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, ~~and distance around a circular arc.~~
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

B. Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Comments

As students begin to build a geometric system, precise use of language is key. IN this unit, the focus of **G-CO.A.1** is on definitions not related to a circle. (Definitions related to a circle will be addressed in unit 8.) Students begin to extend their understanding of rigid transformations to define congruence (**G-CO.B.6**). (Dilations will be addressed in unit 5.) This definition lays the foundation for work students will do throughout the course around congruence.

Common Core State Standards for Mathematical Practice

4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision

Students can use transformations to model the world in which they live, attending to **MP.4**, as they consider symmetry in nature. Students should strategically use tools, including tracing paper or dynamic geometry software, to perform transformations (**MP.5**). As they describe motion, students will need to attend to **MP.6**, using precise language. Allowing students to critique non-precise definitions and make them better definitions can help students understand the importance of the language used in writing a precise definition.

Explanations and Examples

HS.G-CO.2 and –CO.3.
Students may use geometry software and/or manipulatives to model and compare transformations.

HS.G-CO.4.
Students may observe patterns and develop definitions of rotations, reflections, and translations.

HS.G-CO.5.
Students may use geometry software and/or manipulatives to model transformations and demonstrate a sequence of transformations that will carry a given figure onto another.

HS.G-CO.6.
A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.
Students may use geometric software to explore the effects of rigid motion on a figure(s).

Unit 2: Angles and lines Suggested number of days: 15		
This unit gives students the foundational tools for developing viable geometric arguments using relationships students studied in middle school related to lines, transversals, and special angles associated with them. Students learn how to combine true statements within a mathematical system to deductively prove other statements. Students should begin to see the structure of a mathematical system as they make conjectures and then prove statements involving lines and angles.		
Common Core State Standards for Mathematical Content **Congruence—G-CO** A. Experiment with transformations in the plane 1. Know precise definitions of angle, ~~circle~~, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, ~~and distance around a circular arc~~. C. Prove geometric theorems 9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **Expressing Geometric Properties with Equations—G-GPE** B. Use coordinates to prove simple geometric theorems algebraically 5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	**Comments** Students build on their work with **G-CO.A.1** from the previous unit as they solidify their understanding and use of definitions related to angles and lines. These definitions will become core vocabulary and will be used throughout the rest of the course. Precise definitions are important as students begin to formulate proofs about lines and angles as described in **G-CO.C.9**. (Definitions related to a circle will be addressed in unit 8). **Common Core State Standards for Mathematical Practice** 3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision 7. Look for and make use of structure Students build proficiency with **MP.3** and **MP.7** as they build a mathematical system with structured statements, including postulates and proven theorems. Students should be exposed to a variety of proof styles, including flow-chart proofs, two-column proofs, and	**Explanations and Examples** **HS.G-CO.9.** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about lines and angles. **HS.G-GPE.5.** Lines can be horizontal, vertical, or neither. Students may use a variety of different methods to construct a parallel or perpendicular line to a given line and calculate the slopes to compare the relationships.

	paragraph proofs, as they begin to build viable logical arguments. Again, the use of precise language, **MP.6**, is critical to building a logical argument.	

Unit 3: Triangles   Suggested number of days: 15

This unit explores basic theorems and conjectures about triangles, including the triangle inequality conjecture, the Triangle Sum Theorem, and theorems regarding centers of a triangle. Students explored some of these relationships in middle school but will build on their work in unit 2 with deductive reasoning and proof related to triangles in this unit. Students make and verify conjectures related to isosceles triangles and explore physical properties of the centroid of a triangle. In this unit, students also learn basic construction techniques and use these as they explore triangle properties. Throughout this unit, students will use the precise definitions developed in **G-CO.A.1**.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Congruence—G-CO C. Prove geometric theorems 10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; ~~the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length~~; the medians of a triangle meet at a point.* D. Make geometric constructions 12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* 13. Construct an equilateral triangle, ~~a square, and a regular hexagon inscribed in a circle~~. **Circles—G-C** A. Understand and apply theorems about circles 3. Construct the inscribed and circumscribed circles of a triangle, ~~and prove properties of angles for a quadrilateral inscribed in a circle~~. **Modeling with Geometry—G-MG** A. Apply geometric concepts in modeling situations 1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	In this unit, the focus of **G-CO.D.13** and **G-C.A.3** should be on triangles, leaving the midsegments proof (**G-CO.C.10**) until unit 5. At that time, students can apply properties of similar triangles to midsegments. (Constructions of inscribed squares and regular hexagons and the properties of angles for inscribed quadrilaterals will be addressed in unit 8.) **Common Core State Standards for Mathematical Practice** 3. Construct viable arguments and critique the reasoning of others. 5. Use appropriate tools strategically. 8. Look for and express regularity in repeated reasoning. As students explore properties of triangles, they will attend to **MP.5**, strategically choosing tools such as tracing paper, compass and straightedge, flow charts, and dynamic geometry software for a given situation. As students use the tools to look for patterns, they will make conjectures about properties of triangles. Students gain proficiency in **MP.3** as they continue to write simple proofs using a variety of styles.	**HS.G-CO.10.** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles. **HS.G-CO.12.** Students may use geometric software to make geometric constructions. **Examples:** • Construct a triangle given the lengths of two sides and the measure of the angle between the two sides. • Construct the circumcenter of a given triangle. **HS.G-CO.13.** Students may use geometric software to make geometric constructions. **HS.G-C.3.** Students may use geometric simulation software to make geometric constructions. **HS.G-MG.1.** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

Unit 4: Triangle congruence   Suggested number of days: 20

This unit builds on students' work with transformations in unit 1 and properties of triangles in unit 3 to formalize the definition of congruent triangles. Students reason to identify criteria for triangle congruence and use precise notation to describe the correspondence in congruent triangles.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Congruence—G-CO B. Understand congruence in terms of rigid motions 6. ~~Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure~~; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. 7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. 8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	In unit 1, students began to use rigid motion transformations to decide if two figures were congruent (G-CO.B.6). In this unit, students will return to this idea as they develop shortcuts for proving two triangles are congruent (G-CO.B.8). **Common Core State Standards for Mathematical Practice** 3. Construct viable arguments and critique the reasoning of others 6. Attend to precision 7. Look for and make use of structure Students build proficiency with **MP.3** and **MP.7** as they create congruent triangle proofs. Allowing students to critique proofs of other students, whether the work of classmates or fictional student work, will help them develop their own skill in writing proofs. Students continue to build understanding of the structure of a mathematical system and recognize the importance of precise language (**MP.6**).	**HS.G-CO.6.** A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures. Students may use geometric software to explore the effects of rigid motion on a figure(s). **HS.G-CO.7.** A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures. Congruence of triangles Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.

Unit 5: Similarity transformations   Suggested number of days: 15

This unit moves away from rigid motion and focuses on dilations and similarity. Students prove theorems involving similarity and apply dilations and similarity to model situations in the real world.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Congruence—G-CO C. Prove geometric theorems 10. Prove theorems about triangles. *Theorems include: ~~measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent~~; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; ~~the medians of a triangle meet at a point.~~* **Similarity, Right Triangles, and Trigonometry—G-SRT** A. Understand similarity in terms of similarity transformations 1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. 2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding	In this unit, students return to G-CO.C.10 to address midsegments of a triangle, making a connection to G-SRT.B.4, treating midsegments as a special case of triangle proportionality. Students revisit the Pythagorean Theorem, which they studied in grade 8, but look at a proof based on similarity (G-SRT.B.4). The focus is on developing logical arguments to prove a known theorem in a different way. G-GPE.B.6 provides a unique way to think about finding the midpoint of a line segment. **Common Core State Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others. 8. Look for and express regularity in repeated reasoning. Similarity and proportional reasoning provide powerful tools in representing and solving real-world problems,	**HS.G-CO.10.** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles. **HS.G-SRT.1.** Dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. Students may use geometric simulation software to model transformations. Students may observe patterns and verify experimentally the properties of dilations. **HS.G-SRT.2.** A similarity transformation is a rigid motion followed by dilation. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. **HS.G-SRT.4.** Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. **HS.G-SRT.5.** Similarity postulates include SSS, SAS, and AA. Congruence postulates include SSS, SAS, ASA, AAS, and H-L. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

pairs of sides. 3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. B. Prove theorems involving similarity 4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* 5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **Expressing Geometric Properties with Equations—G-GPE** B. Use coordinates to prove simple geometric theorems algebraically 6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **Modeling with Geometry—G-MG** A. Apply geometric concepts in modeling situations 3. Apply geometric methods to solve	allowing students to develop proficiency with **MP.1**. As students investigate design problems (**G-MG.A.3**), they will often need to model the problem with scaled images. Much of **MP.1** has to do with understanding how to represent a situation and what mathematical tools can be applied to the situation. In this unit, students continue to build their mathematical system, attending to **MP.8** as they look for patterns in geometric relationships, and then prove their conjectures, attending to **MP.3**.	**HS.G-GPE.6** Students may use geometric simulation software to model figures or line segments. **Examples:** Given A(3, 2) and B(6, 11), • Find the point that divides the line segment AB two-thirds of the way from A to B. ○ The point two-thirds of the way from A to B has *x*-coordinate two-thirds of the way from 3 to 6 and *y* coordinate two-thirds of the way from 2 to 11. So, (5, 8) is the point that is two-thirds from point A to point B. ○ Find the midpoint of line segment AB **HS.G-MG.3.** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).		

Unit 6: Right triangle relationships and trigonometry   Suggested number of days: 20

This unit extends the idea of similarity to indirect measurements. Students develop properties of special right triangles, and use properties of similar triangles to develop trigonometric ratios. Students apply these ideas as they model real-world situations and solve problems involving unknown side lengths and angle measures.

Common Core State Standards for Mathematical Content

Similarity, Right Triangles, and Trigonometry—G-SRT

B. Prove theorems involving similarity

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

C. Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Comments

In the previous unit, students looked at another proof of the Pythagorean Theorem. In this unit, they continue to solve problems involving right triangles (**G-SRT.C.8**), but can combine the Pythagorean Theorem and trigonometric ratios in their solutions. **G-GPE.B.7** presents an opportunity for students to apply the Pythagorean Theorem to develop the distance formula and use the formula to compute area and perimeter.

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.

Although area and perimeter were fully covered in middle school, the standards in this unit give students an opportunity to consolidate old learning and new learning as they solve more complex problems with an array of mathematical tools to choose

Explanations and Examples

HS.G-SRT.5.

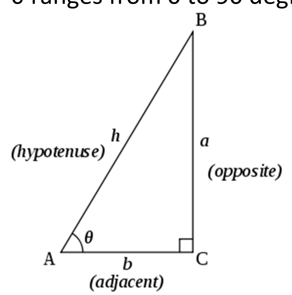
Similarity postulates include SSS, SAS, and AA.

Congruence postulates include SSS, SAS, ASA, AAS, and H-L.

Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

HS.G-SRT.6.

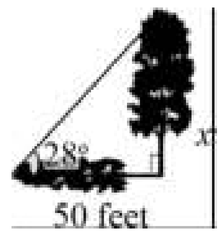
Students may use applets to explore the range of values of the trigonometric ratios as θ ranges from 0 to 90 degrees.



Name	Ratio	Notation
Sine	opposite/hypotenuse	Sin(θ)
Cosine	adjacent/hypotenuse	Cos(θ)
Tangent	opposite/adjacent	Tan(θ)
Cosecant (1/Sine)	hypotenuse/opposite	Cosec(θ) or csc(θ)
Secant (1/Cosine)	hypotenuse/adjacent	Sec(θ)
Cotangent (1/Tangent)	adjacent/opposite	Cot(θ)

HS.G-SRT.7.

Geometric simulation software, applets, and graphing calculators can be used to explore the relationship between sine and cosine.

7. Explain and use the relationship between the sine and cosine of complementary angles. 8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **Expressing Geometric Properties with Equations—G-GPE** B. Use coordinates to prove simple geometric theorems algebraically 7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. **Modeling with Geometry—G-MG** A. Apply geometric concepts in modeling situations 1. Use geometric shapes, their measures, and their properties to	from. These problem-solving experiences attend to **MP.1** and **MP.4**. As students decide how to model situations geometrically and apply properties to the situations, they are attending to **MP.2**.	**HS.G-SRT.8.** Students may use graphing calculators or programs, tables, spreadsheets, or computer algebra systems to solve right triangle problems. **Example:** Find the height of a tree to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the tree is 50 ft.  **HS.G-GPE.7.** Students may use geometric simulation software to model figures. **HS.G-MG.1.** Students may use simulation software and modeling software to explore which model best describes a set of data or situation. **HS.G-MG.3** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).		

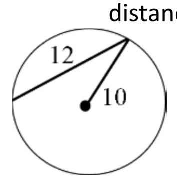
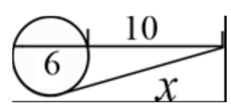
Unit 7: Quadrilaterals Suggested number of days: 15

Prior units of this course have focused on triangles. This unit extends that work to the study of quadrilaterals. Students use triangle congruence as they prove theorems about parallelograms. This unit also provides an opportunity for students to become proficient with coordinate proofs.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Congruence—G-CO C. Prove geometric theorems 11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **Expressing Geometric Properties with Equations—G-GPE** B. Use coordinates to prove simple geometric theorems algebraically 4. Use coordinates to prove simple geometric theorems algebraically*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle;* ~~*prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*~~	Many of the properties of special quadrilaterals lend themselves to coordinate proofs, making a nice connection between **G-CO.C.11** and **G-GPE.B.4**. (Coordinate proofs related to circles are addressed in unit 8.) Because students have studied and applied the Pythagorean Theorem and the distance formula in previous units, they are ready to use them, along with the midpoint formula and slope relationships from unit 2, to prove theorems about quadrilaterals. **Common Core State Standards for Mathematical Practice** 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 5. Use appropriate tools strategically. As students become more proficient in geometric reasoning, they begin to consolidate all of their learning to solve problems and prove theorems (**MP.2**, **MP.3**). Coordinate proofs are a strategic tool students can use (**MP.5**).	**HS.G-CO.11.** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms. **HS.G-GPE.4.** Students may use geometric simulation software to model figures and prove simple geometric theorems. **Example:** • Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram.

Unit 8: Circles   Suggested number of days: 15

This unit explores properties of circles. Students draw on geometric relationships involving lines, angles, triangles and quadrilaterals as they derive the equation of a circle and explore properties of chords, arcs, and angles on circles.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Congruence—G-CO D. Make geometric constructions 13. Construct ~~an equilateral triangle~~, a square, and a regular hexagon inscribed in a circle. **Circles—G-C** A. Understand and apply theorems about circles 1. Prove that all circles are similar. 2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle*.	As students explore relationships involving circles, they can return to quadrilaterals to prove an additional property of inscribed quadrilaterals (**G-C.A.3**). They will also use properties of quadrilaterals and triangles, combined with inscribed angles and trigonometry, to construct a square and hexagon inscribed in a circle (**G-CO.D.13**). This unit provides an opportunity to reinforce many concepts addressed earlier in the course as they are applied within circles. **G-C.B.5** and **G-GMD.A.1** work together to present an opportunity to revisit the concept of area and perimeter in a more complex manner than in middle school. (Volume of a cylinder, pyramid and cone is addressed in unit 10.) **G-C.B.5** also extends the idea of arc length to introduce the idea of the radian measure of an angle. The focus in this course should be on proportional relationships, as radian measure can be reinforced in fourth-year course. Students use coordinates to	**HS.G-CO.13** Students may use geometric software to make geometric constructions. **HS.G-C.1.** Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. **HS.G-C.2** **Examples:** • Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.  • Find the unknown length in the picture below.  **HS.G-C.3 and G-C.4.** Students may use geometric simulation software to make geometric constructions.

3. ~~Construct the inscribed and circumscribed circles of a triangle, and~~ prove properties of angles for a quadrilateral inscribed in a circle. 4. (+) Construct a tangent line from a point outside a given circle to the circle. B. Find arc lengths and areas of sectors of circles 5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **Expressing Geometric Properties with Equations—G-GPE** A. Translate between the geometric description and the equation for a conic section 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	describe and analyze circles (**G-GPE.A.1** and **G-GPE.B.4**).	**HS.G-C.5.** Students can use geometric simulation software to explore angle and radian measures and derive the formula for the area of a sector. **HS.G-GPE.1.** Students may use geometric simulation software to explore the connection between circles and the Pythagorean Theorem. **Examples:** • Write an equation for a circle with a radius of 2 units and center at (1, 3). • Write an equation for a circle given that the endpoints of the diameter are (-2, 7) and (4, -8). • Find the center and radius of the circle $4x^2 + 4y^2 - 4x + 2y – 1 = 0$. **HS.G-GPE.4.** Students may use geometric simulation software to model figures and prove simple geometric theorems. **HS.G-GMD.1.** Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. **HS.G-MG.1.** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

B. Use coordinates to prove simple geometric theorems algebraically 4. Use coordinates to prove simple geometric theorems algebraically. *For example, ~~prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle~~; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **Geometric measurement and dimension—G-GMD** A. Explain volume formulas and use them to solve problems 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, ~~volume of a cylinder, pyramid, and cone~~. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **Modeling with Geometry—G-MG** A. Apply geometric concepts in modeling situations 1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	**Common Core State Standards for Mathematical Practice** 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. Circle problems provide an opportunity for complex problem-solving situations that consolidate many different geometric relationships (**MP.2** and **MP.4**). In developing the relationships, students will look for patterns, make conjectures, and then construct logical arguments to justify their conjectures. Again, allow opportunities for students to share their own reasoning and critique the reasoning of others in these problem situations (**MP.5**).	

Unit 9: Geometric modeling in two dimensions Suggested number of days: 10		
In prior units of this course, students learned about many geometric relationships and developed a mathematical system. This unit provides the opportunity to bring together all of the relationships students have learned in this course and apply them to real-world situations. The unit should present in-depth problems that require students to draw on their understanding of geometric figures and strategically use the tools they have been developing throughout the course.		
Common Core State Standards for Mathematical Content	**Comments**	**Explanations and Examples**
Modeling with Geometry—G-MG A. Apply geometric concepts in modeling situations 1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). 2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	In this unit, focus the modeling examples on two-dimensional objects only. Unit 10 will bring in three-dimensional objects. **Common Core State Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. The problems presented in this unit should require students to struggle and collaborate, thus building their mathematical persistence (**MP.1**). Students should see ways to use the geometric relationships they have been learning throughout the course to model real-world situations (**MP.4**). As students model situations geometrically, they will often have to decontextualize the problem and apply geometric properties (**MP.2**).	**HS.G-MG.1, -MG.2, and –MG.3.** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

Unit 10: Understanding and modeling with three-dimensional figures   Suggested number of days: 20

This unit explores three-dimensional geometry including representations of real-world situations with three-dimensional objects and calculating volume. Students make connections between two-dimensional and three-dimensional representations of objects. Students culminate the course with modeling problems involving three-dimensional objects, allowing them again to integrate their knowledge and apply complex geometric reasoning.

Common Core State Standards for Mathematical Content	Comments	Explanations and Examples
Geometric measurement and dimension—G-GMD A. Explain volume formulas and use them to solve problems 1. Give an informal argument for the formulas for ~~the circumference of a circle, area of a circle~~, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	In **G-GMD.A.1**, focus on three dimensional objects. Students have worked with the volume formulas in middle school, so the focus here should be on developing arguments for the formulas and applying them to more complex situations. **Common Core State Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively 4. Model with mathematics.	**HS.G-GMD.1.** Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. **HS.G-GMD.3.** Missing measures can include but are not limited to slant height, altitude, height, diagonal of a prism, edge length, and radius. **HS.G-GMD.4.** Students may use geometric simulation software to model figures and create cross sectional views. **Example:** • Identify the shape of the vertical, horizontal, and other cross sections of a cylinder.
B. Visualize relationships between two-dimensional and three dimensional objects 4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **Modeling with Geometry—G-MG** A. Apply geometric concepts in modeling situations 1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	The problems presented in this unit should require students to struggle and collaborate, thus building their mathematical persistence (**MP.1**). Students should see ways to use the geometric relationships they have been learning throughout the course to model real-world situations (**MP.4**). As students model situations geometrically, they will often have to decontextualize the problem and apply geometric properties (**MP.2**).	**HS.G-MG.1, -MG.2, and –MG.3.** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

2. Apply concepts of density based on ~~area and~~ volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).		