Arizona Mathematics Standards

Introduction

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
December, 2016

Introduction

The Arizona Mathematics Standards are a connected body of mathematical understandings and competencies that provide a foundation for all students. These standards are coherent, focused on important mathematical concepts, rigorous, and well-articulated across the grades. Concepts and skills that are critical to the understanding of important processes and relationships are emphasized.

The need to understand and use a variety of mathematical strategies in multiple contextual situations has never been greater. Utilization of mathematics continues to increase in all aspects of everyday life, as a part of cultural heritage, in the workplace, and in scientific and technical communities. Today's changing world will offer enhanced opportunities and options for those who thoroughly understand mathematics.

Mathematics education should enable students to fulfill personal ambitions and career goals in an information age. The National Council of Teachers of Mathematics (NCTM) document, *Principles and Standards for School Mathematics,* asks us to, "*Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need it*".[1] The Arizona Mathematics Standards are intended to facilitate this vision.

What the Arizona Mathematics Standards Are

The Arizona Mathematics Standards define the knowledge, understanding, and skills that need to be taught and learned so all students are ready to succeed in credit-bearing, college-entry courses and/or in the workplace. The Arizona Mathematics Standards are the foundation to guide the construction and evaluation of mathematics programs in Arizona K-12 schools and the broader Arizona community.

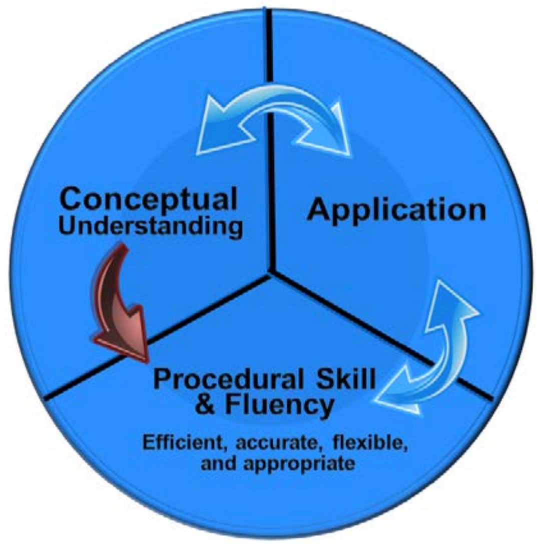
The Arizona Mathematics Standards are:

- Focused in coherent progressions across grades K-12.
- Aligned with college and workforce expectations.
- Inclusive of rigorous content and applications of knowledge through higher-order thinking.
- Research and evidence based.

[1] National Council of Teachers of Mathematics, (2000) *Principles and Standards for School Mathematics*, NCTM Publications, Reston, VA.

The Arizona Mathematics Standards are well articulated across grades K-8 and high school. The Arizona Mathematics Standards are the result of a process designed to identify, review, revise or refine, and create high-quality, rigorous mathematics standards. The Arizona Mathematics Standards are coherent, focus on deep mathematical content knowledge, and address a balance of rigor, which includes conceptual understanding, application, and procedural skills and fluency.

Balanced approach to rigor found in the Arizona Mathematics Standards



What the Arizona Mathematics Standards Are NOT

The standards are not the curriculum.
While the Arizona Mathematics Standards may be used as the basis for curriculum, the Arizona Mathematics Standards are not a curriculum. Therefore, identifying the sequence of instruction at each grade – what will be taught and for how long – requires concerted effort and attention at the district and school levels. The standards do not dictate any particular curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local governing board. The Arizona Department of Education defines standards, curriculum, and instruction as:

> **Standards** – **What** a student needs to know, understand, and be able to do by the end of each grade/course. Standards build across grade levels in a progression of increasing understanding and through a range of cognitive-demand levels.
>
> **Curriculum** –The resources used for teaching and learning the standards. Curricula are adopted at the local level by districts and schools. Curriculum refers to the **how** in teaching and learning the standards.
>
> **Instruction** – The methods used by teachers to teach their students. Instructional techniques are employed by individual teachers in response to the needs of all the students in their classes to help them progress through the curriculum in order to master the standards. Instruction refers to the **how** in teaching and learning the standards.

The standards are not instructional practices.
While the Arizona Mathematics Standards define the knowledge, understanding, and skills that need to be effectively taught and learned for *each and every* student to be college and workplace ready, the standards are not instructional practices. The educators and subject matter experts who worked on the Mathematics Standards Subcommittee and Workgroups ensured that the Arizona Mathematics Standards are free from embedded pedagogy and instructional practices. The Arizona Mathematics Standards do not define how teachers should teach and must be complimented by well-developed, aligned, and appropriate curriculum materials, as well as effective instructional practices.

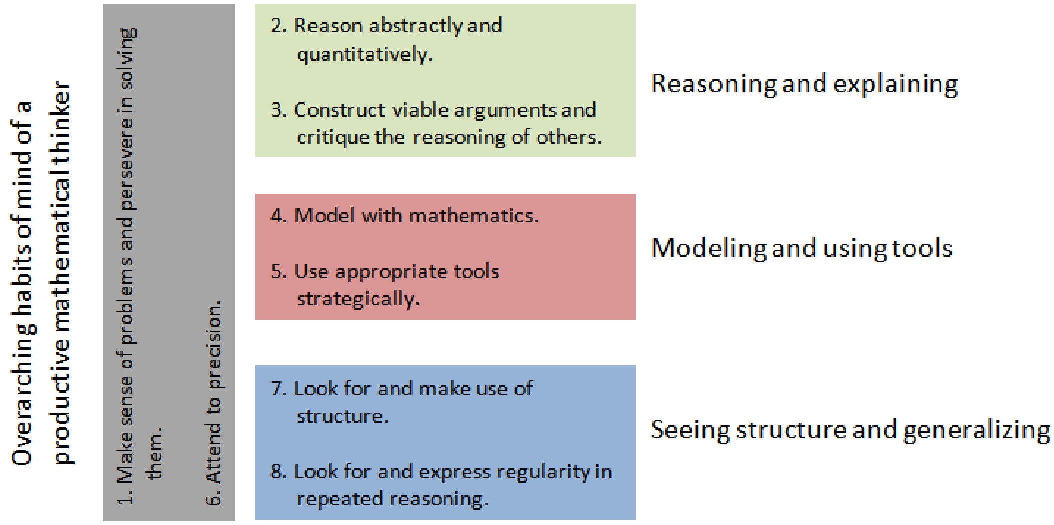
The standards do not necessarily address students who are far below or far above the grade level.
No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. The Arizona Mathematics Standards do not define the intervention methods or materials necessary to support students who are well below or well above grade level expectations. It is up to the teacher, school, and district to determine the most effective instructional methods and curricular resources and materials to meet all students' needs.

Organization of the Standards

Two Types of Standards – Mathematical Content and Practice Standards

The Arizona Mathematics Standards include two types of standards: Standards for Mathematical Practice (identical for each grade level or course) and mathematical content standards (different at each grade level or course). Together these standards address both "habits of mind" that students should develop to foster mathematical understanding, and what students need to know, understand, and be able to do regarding mathematics content.

Standards for Mathematical Practice [2]



Educators at all levels should seek to develop expertise in their students through the **Standards for Mathematical Practice (MP)**. Although students exhibit these habits of mind at every grade level, the demonstration of these practices will build in complexity throughout the child's educational experience. These practices rest on the two sets of important "processes and proficiencies"; the NCTM Process Standards (1989,2000) and the Strands of Mathematical proficiency specified in the National Research Council's report, *Adding It Up: Helping Children Learn Mathematics* (2001), each of which has longstanding importance in mathematics education.

[2] McCallum, William. (2011). *http://www.ascd.org/ascd-express/vol8/805-parker.aspx* .

The following narratives describe the eight Standards for Mathematical Practice. **(MP)**

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.
2. **Reason abstractly and quantitatively.**
   Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.
3. **Construct viable arguments and critique the reasoning of others.**
   Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
4. **Model with mathematics.**
   Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**
   Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
6. **Attend to precision.**
   Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
7. **Look for and make use of structure.**
   Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
8. **Look for and express regularity in repeated reasoning.**
   Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

The eight Standards for Mathematical Practice describe ways in which students are expected to engage within the mathematical content standards. The Arizona Standards for Mathematical Practice reflect the interaction of skills necessary for success in math coursework as well as the ability to apply math knowledge and processes within real-world contexts. The Standards for Mathematical Practice highlight the applied nature of math within the workforce and clarify the expectations held for the use of mathematics in and outside of the classroom. The Standards for Mathematical Practice complement the math content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Standards for Mathematical Content

Grade Level or High School Course											
K	1	2	3	4	5	6	7	8	Algebra 1	Geometry	Algebra 2
Standards for Mathematical Practice											
Counting & Cardinality						Ratios & Proportional Relationships		Functions	Functions		Functions
Operations & Algebraic Thinking						Expressions & Equations			Algebra		Algebra
Number & Operations in Base Ten						The Number System			Number & Quantity	Number & Quantity	Number & Quantity
			Fractions								
Geometry						Geometry				Geometry	
Measurement & Data						Statistics & Probability			Statistics & Probability		Statistics & Probability
Domains									Conceptual Categories		

The mathematical content standards represent a cohesive progression of mathematical ideas across grades kindergarten through high school course Algebra 2. The mathematics content standards are rigorous and represent relevant mathematical content knowledge essential to success in the workplace and career and credit bearing college courses.

The Arizona Mathematics Standards call for the mathematical practices and mathematical content to be inextricably linked. One cannot solve problems without understanding and using mathematical content.[3] The Standards for Mathematical Practice complement the math content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. These connections are essential to support the development of students' broader mathematical understanding.

[3] California State Standards-Mathematics. (2013).

Reading the Mathematical Content Standards in Kindergarten – Grade 8

In kindergarten through grade 8, the Arizona Mathematics Standards are organized by grade level and then by domains (clusters of standards that address "big ideas" and support connections of topics across the grades), clusters (groups of related standards inside domains), and the standards (what students should know, understand, and be able to do). ***The standards do not dictate curriculum or pedagogy.*** The numbering of standards within a grade level does not imply an instructional sequence (3.NBT.A.**2** is not required to be taught before 3.NBT.A.**3**), nor does the numerical coding imply vertical alignment from one grade to the next.

There are eleven domains within the K-8 Standards. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades. Critical areas are included before each grade to support the implementation of the mathematical content standards.

1. CC: Counting and Cardinality
2. OA: Operations and Algebraic Thinking
3. NBT: Number and Operations in Base Ten
4. MD: Measurement and Data
5. NF: Number and Operations – Fractions
6. G: Geometry
7. RP: Ratios and Proportional Relationships
8. NS: The Number System
9. EE: Expressions and Equations
10. F: Functions
11. SP: Statistics and Probability

Domains are intended to convey coherent groupings of content. All domains are **bold** and **centered**.
Clusters are groups of related standards. Cluster headings are **bolded**.
Standards define what students should know, understand and be able to do. Standards are numbered.

Domain

Number and Operations – Fractions (NF) *Note: Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8.*		
3.NF.A Understand fractions as numbers.	3.NF.A.1	Understand a unit fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts 1/*b*.

Cluster **Standard**

The code for each standard begins with the grade level, followed by the domain code, the cluster letter, and the number of the standard. For example, 3.NF.A. 1 would be the first standard (1), in the first cluster (A), in the domain of Number and Operations-Fractions (NF) of the standards for grade 3.

Grade ⟶ **3.NF.A.1**

Domain **Cluster** **Standard**

<u>Reading the Mathematical Content Standards in High School</u>

The Arizona Mathematics high school standards are organized into three main courses and also include additional and extended or plus (**P**) standards that could be included in a 4th credit mathematics course or to extend instruction in Algebra 1, Geometry or Algebra 2. The Arizona Mathematics high school courses are organized into Algebra 1, Algebra 2, and Geometry courses. Districts can use a variety of pathways including traditional or integrated pathways in which students can master the high school mathematics standards over the path of four courses.

In high school, the Arizona Mathematics Standards are organized by course and then by conceptual category and domains (clusters of standards that address "big ideas" and support connections of topics across the grades), clusters (groups of related standards inside domains), and the standards (what students should know, understand. and be able to do). ***The standards do not dictate curriculum or pedagogy.*** The numbering of standards within a grade level does not imply an instructional sequence (A1.F-LE.A.**2** is not required to be taught before A1.F-LE.A.**3**).

The high school mathematics standards are organized by Conceptual Category, Domain, Cluster and Standard. There are six conceptual categories for high school.

- N: Number and Quantity
- A: Algebra
- F: Functions
- G: Geometry
- S: Statistics and Probability
- CM: Contemporary Mathematics

Conceptual Categories portray a coherent view of higher mathematics. All conceptual categories are in **bold, *italicized*, centered** and **larger** font. **Domains** are intended to convey coherent groupings of content. All domains are **bold and centered.**

Clusters are groups of related standards. Cluster headings are **bolded**.
Standards define what students should know, understand and be able to do. Standards are numbered.

Conceptual Category → *Number and Quantity - N*

Domain → The Real Number System (N – RN)

		Number and Quantity - N
		The Real Number System (N – RN)
A1.N-RN.B Use properties of rational and irrational numbers.	A1.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Cluster **Standard**

The conceptual categories portray a coherent view of higher mathematics based on the realization that students' work on a broad topic, such as functions, crosses a number of traditional course boundaries. As local school districts develop a full range of courses and curriculum in higher mathematics, the organization of standards by conceptual categories offers a starting point for discussing course content.

The code for each high school mathematics standard begins with the identifier for the course or course level (A1, G, A2, P) the conceptual category code, followed by the domain code, cluster letter, and the number of the standard. For example, A1.F-LE.A.1 would be the first standard (1), in the first cluster (A), in the domain of Linear, Quadratic, and Exponential Models (LE) in the conceptual category of Functions (F) in an Algebra 1 course (A1).

Course → **A1.F-LE.A.1**

Conceptual Category **Domain** **Cluster** **Standard**

The high school standards specify the mathematics that all students should study in order to be college, career and workplace ready. While certain standards progress across all mathematics courses, the most notable is the Quantities domain (N-Q). These standards (N-Q.A.1, N-Q.A.2, N-Q.A.3) are essential to problem solving and reinforce the standards of mathematical practice. Mathematical modeling applications are also embedded throughout all of the courses and are best interpreted in relation to other standards and not as a collection of isolated topics.

Additional High School Standards - Plus

The Arizona Mathematics Standards contain an additional set of standards that are found outside the limits of a high school Algebra 1, Geometry, or Algebra 2 minimum course of study as outlined by the Arizona Mathematics Standards. The high school conceptual category Contemporary Mathematics (CM) includes the domain Discrete Mathematics (DM). This additional conceptual category and standards could be included in a fourth credit math course. These additional high school standards are represented with the code **P**.

The plus (**P**) standards are standards that are found outside the limits of a high school Algebra 1, Algebra 2, or Geometry minimum course of study as outlined by the standards. The Plus standards are represented with the code **P**. The plus standards are intended to be included in honors, accelerated, advanced courses, fourth credit courses, as well as extensions of the regular courses (Algebra 1, Algebra 2, and Geometry).

Key Considerations for Standards Implementation

Addition/Subtraction and Multiplication/Division Problem Types or Situations

There are important distinctions among different types of addition/subtraction and multiplication/division problems that are reflected in the ways that children think about and solve them.[4] Table 1 and Table 2 describe different problem types that provide a structure for selecting problems for instruction and interpreting how children solve them. This is a critical consideration for standards implementation. When planning instruction, educators must provide all students with the opportunity to learn and experience all different problem types associated with a given standard. Without the opportunity to learn and experience different problem types, students cannot truly master and apply the grade level standards in future mathematical tasks and experiences.

Table 1 and 2 can be found in the Introduction, Glossary and at the end of grade level standards that would utilize Table 1 and/or Table 2.

Table 1: Addition and Subtraction Situations

Table 1 provides support to clarify the varied problem structures necessary to build student conceptual understanding of addition and subtraction, focusing on developing student flexibility. In order to fully implement the standards, students must solve problems from all problem subcategories relevant to the grade level. All problem types should not be mastered at all grades in Kindergarten through fifth grade. Guidance on what problem types could be mastered at each grade level is available in the Progressions for Operations and Algebraic Thinking document.[5]

Table 2: Multiplication and Division Situations

Table 2 provides support to clarify the varied problem structures necessary to build student conceptual understanding of multiplication and division, focusing on developing student flexibility. In order to fully implement the standards, students must solve problems from all problem subcategories relevant to the grade level.

---

[4] Carpenter, T., Fennema, E., Franke, M., Levi, L., Empson, S. (1999). *Children's Mathematics Cognitively Guided Instruction*.

[5] University of Arizona Institute for Mathematics and Education. (2011). *Progression on Counting and Cardinality and Operations and Algebraic Thinking.*

Table 1. Common Addition and Subtraction Problem Types/Situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together / Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean ***makes*** or ***results*** in but always does mean ***is the same quantity as***.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

Table 2. Common Multiplication and Division Problem Types/Situations.[1]

	Unknown Product 3 x 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 x ? = 18 and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? x 6 = 18 and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example*****.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example*****.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example*****.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? ***Area example*****.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? ***Area example*****.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? ***Area example*****.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A straw hat costs $6. A baseball hat costs 3 times as much as the straw hat. How much does the baseball hat cost? ***Measurement example*****.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A baseball hat costs $18 and that is 3 times as much as a straw hat costs. How much does a straw hat cost? ***Measurement example*****.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A baseball hat costs $18 and a straw hat costs $6. How many times as much does the baseball hat cost as the straw hat? ***Measurement example*****.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	*a x b = ?*	*a x ? = p,* and *p ÷ a = ?*	*? x b = p,* and *p ÷ b = ?*

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Fluency in Mathematics

Wherever the word *fluently* appears in a content standard, the word includes *efficiently, accurately, flexibly, and appropriately*. Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.[6]

- **Efficiency**—carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem.
- **Accuracy**—reliably produces the correct answer.
- **Flexibility**—knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to double-check.
- **Appropriately**—knows when to apply a particular procedure.

Please see standards 2.OA.B.2 and 3.OA.C.7 for standards related to addition and subtraction of within 20 and multiplication within 100. Both of these standards show mastery involves "from memory" as an outcome. By the end of 2nd and 3rd grade, these procedural fluency standards should be automatic recall by students.

Fluency Expectations, K-8

Specific mathematics standards in K-6 state fluency as the intended end of grade level outcome. Some standards in grades 7-8 do not explicitly state ***fluently*** within the standard but based on the definition of fluency, we want students to *efficiently, accurately, flexibly, and appropriately* problem solve.

Fluency Expectations, High School

The high school standards do not always set explicit expectations for fluency but fluency is important in high school mathematics. For example, fluency in algebra can help students get past the need to manage computational details so that they can observe structure and patterns in problems. Therefore, this section makes recommendations about fluencies that can serve students well as they learn and apply mathematics.

Table 3, Fluency Expectations across All Grade Levels can also be found in the Glossary.

---

[6] National Council of Teachers of Mathematics, Inc. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA. Diane Briars (2016) NCTM. djbmath@comcast.net. Russell, S. J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics, 7*(3), 154–158.

Table 3. Fluency Expectations Across All Grade Levels.

Grade	Coding	Fluency Expectations
K	**K.OA.A.5**	Fluently add and subtract within 5.
1	**1.OA.C.5**	Fluently add and subtract within 10.
2	**2.OA.B.2** **2.NBT.B.5**	Fluently add and subtract within 20. By the end of 2nd grade, know from memory all sums of two one-digit numbers. Fluently add and subtract within 100.
3	**3.NBT.A.2** **3.OA.C.7**	Fluently add and subtract within 1000. Fluently multiply and divide within 100. By the end of 3rd grade, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.
4	**4.NBT.B.4**	Fluently add and subtract multi-digit whole numbers using a standard algorithm.
5	**5.NBT.B.5**	Fluently multiply multi-digit whole numbers using a standard algorithm.
6	**6.NS.B.2** **6.NS.B.3** **6.EE.A.2**	Fluently divide multi-digit numbers using a standard algorithm. Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. Write, read, and evaluate algebraic expressions.
7	**7.NS.A.1.d** **7.NS.A.2.c** **7.EE.B.4.a**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers. Fluently solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$
8	**8.EE.C.7**	Fluently solve linear equations and inequalities in one variable.
Algebra 1	**A1.F-IF.C.7** **A1.A-SSE.A.2**	Graph functions expressed symbolically and show key features of the graph. Use structure to identify ways to rewrite numerical and polynomial expressions.
Geometry	**G.G-SRT.B.5** **G.G-GPE.B** **G.SRT.C.8**	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing a real-world context. Use coordinates to prove geometric theorems algebraically. Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles utilizing real-world context.
Algebra 2	**A2.A-SSE.A.2** **A2.F-BF.B** **A2.A-REI.B.4**	Use the structure of an expression to identify ways to rewrite it. Build new functions from existing functions. Fluently solve quadratic equations in one variable.

Disciplinary Literacy in Mathematics

The English Language Arts (ELA) Standards provide an integrated approach to literacy to help guide instruction throughout all disciplines. Therefore, the ELA standards in reading, writing, speaking and listening, should be integrated throughout K-12 mathematics teaching and learning.

By incorporating ELA Standards, and critical thinking in instruction, educators provide students with opportunities to develop literacy in mathematics instruction. The Standards for Mathematical Practice (MP) naturally link to the ELA Standards. By engaging in a multitude of critical thinking experiences linked to the Standards for Mathematical Practice, students will:

- Construct viable arguments through proof and reasoning.
- Critique the reasoning of others.
- Process and apply reasoning from others.
- Synthesize ideas and make connections to adjust the original argument.

The goal of using literacy skills in mathematics is to foster a deeper conceptual understanding of the mathematics and provide students with the opportunity to read, write, speak, and listen within a mathematics discourse community.

Technology Integration in Mathematics

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. Electronic technologies – calculators and computers – are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately. They can support student investigations in every area of mathematics, including geometry, statistics, algebra, measurement, and number. When technology tools are available, students can focus on decision making, reflection, reasoning, and problem solving. Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions.[7]

It is the goal in teaching, learning, and assessing mathematical understanding that technology tools are used appropriately and strategically. (MP 5) Students should choose a tool, including a calculator when it is relevant and useful to the problem at hand. It is suggested that calculators in elementary grades serve as aids in advancing student understanding without replacing other calculation methods. Calculator use can promote the higher-order thinking and reasoning needed for problem solving in our information and technology based society. Their use can also assist teachers and students in increasing student understanding of and fluency with arithmetic operations, algorithms, and numerical relationships and enhancing student motivation. Strategic calculator use can aid students in recognizing and extending numeric, algebraic, and geometric

[7] National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA.

patterns and relationships.[8] It is suggested that teaching, learning, and assessment of mathematics include calculator use in grades 5 through high school on assessments or parts of assessments.

Understand and Use Formulas

When planning instruction around the Arizona Mathematics Standards, it is important to point out that conceptual understanding is developed prior to mastery of procedural skill and fluency. This also applies to certain concepts or big ideas related to the use of formulas. Understanding an individual formula includes knowledge of each part of the equation. The formula should be developed from a foundation of conceptual understanding, and formula mastery should include this understanding as well as use of the formula in specific applied problems.

Mathematical Modeling

In the course of a student's mathematics education, the word "model" is used in many ways. Several of these, such as manipulatives, demonstration, role modeling, and conceptual models of mathematics, are valuable tools for teaching and learning. However, these examples are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer questions utilizing real-world context.

Within the standards document, the mathematical modeling process should be used with standards that include the phrase "real-world context."

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena. This process includes:

- Using the Language of mathematics to quantify real-world phenomena and analyze behaviors.
- Using math to explore and develop our understanding of real world problems.
- An iterative problem solving process in which mathematics is used to investigate and develop deeper understanding.

Students have to do the same mathematics to answer a mathematical modeling question, but they are forced to reconcile their answer with reality, making the mathematics more relevant and interesting. Making judgments about what matters and assessing the quality of a solution are components of mathematical modeling. Within the mathematical modeling process, students' opinions matter and influence the answer to the question.

[8] National Council of Teachers of Mathematics. (2015). *Calculator Use in Elementary Grades-NCTM position statement.*

The Modeling Process

The Arizona Mathematics Standards state that Mathematical Modeling is a process made up of the following components:

IDENTIFY THE PROBLEM

We identify something in the real world we want to know, do, or understand. The result is a question in the real world.

MAKE ASSUMPTIONS AND IDENTIFY VARIABLES

We select information that seems important in the question and identify relations between them. We decide what information and relationships are relevant, resulting in an idealized version of the original question.

DO THE MATH

We translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This formulation is the model. We do the math to see what insights and results we get.

ANALYZE AND ASSESS THE SOLUTION

We consider: Does it address the problem? Does it make sense when translated back into the real world? Are the results practical, the answers reasonable, and the consequences acceptable?

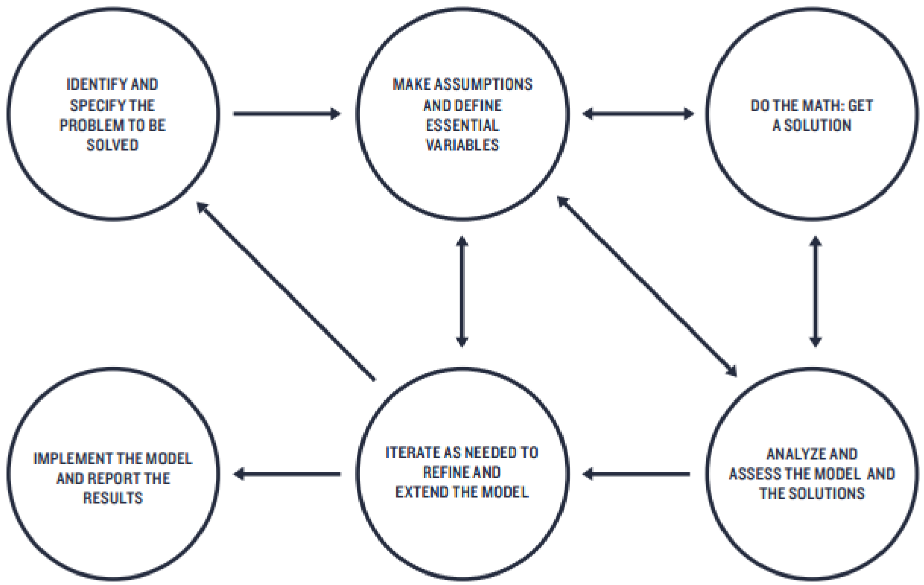
ITERATE

We iterate the process as needed to refine and extend our model.

IMPLEMENT THE MODEL

For real-world, practical applications, we report our results to others and implement the solution.

Mathematical modeling is often pictured as a cycle, since we frequently need to come back to the beginning and make new assumptions to get closer to a usable result. Mathematical modeling is an iterative problem-solving process and therefore is not referenced by individual steps. The following representation reflects that a modeler often bounces back and forth through the various stages.



The practice of mathematical modeling emphasizes the integration of the standards for mathematics content and the standards for mathematical practice. Mathematical modeling also integrates mathematical literacy and the ELA standards. Similarly, the P21 twenty first century skills of creativity and innovation, critical thinking and problem solving, communication, and collaboration are all accessible via modeling.[9]

[9] Framework for 21st century Learning. http://www.p21.org/our-work/p21-framework