



Arizona Mathematics Standards

Third Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
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Arizona Mathematics Standards 3rd Grade

Third Grade Overview

Operations and Algebraic Thinking (OA)

Note: Grade 3 expectations in this domain are limited to multiplication through 10×10 and division with both quotients and divisors less than or equal to 10.

- Represent and solve problems involving whole number multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten (NBT)

Note: A range of algorithms may be used.

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions (NF)

Note: Grade 3 expectations are limited to fractions with denominators: 2, 3, 4, 6, 8

- Understand fractions as numbers.

Measurement and Data (MD)

- Solve problems involving measurement.
- Represent and interpret data.
- Geometric measurement: Understand concepts of area and perimeter.

Geometry (G)

- Reason with shapes and their attributes.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Third Grade: Critical Areas

In third grade, instructional time should focus on three critical areas:

- 1. Develop competency in multiplication and division and strategies for multiplication and division within 100.**
- 2. Develop understanding of fractions as numbers, especially unit fractions.**
- 3. Develop understanding of the structure of rectangular arrays and of area.**

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models as described in *Table 2*. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. By the end of 3rd grade, students are fluent in multiplication and division within 100.

(2) Students develop an understanding of fractions as numbers, beginning with unit fractions. Students understand that the size of a fractional part is relative to the size of the whole. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on recognizing equal numerators or denominators.

(3) Students develop an understanding of area as an attribute of two-dimensional regions by making connections among standards in the domains OA, NBT and G. They measure the area of a shape by finding the total number of same-size units required to cover the shape without gaps or overlaps. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By working with arrays, students connect area to multiplication and justify using multiplication to determine the area.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Arizona Mathematics Standards 3rd Grade

Operations and Algebraic Thinking (OA)

Note: Grade 3 expectations in this domain are limited to whole number multiplication through 10×10 and whole number division with both quotients and divisors less than or equal to 10.

3.OA.A Represent and solve problems involving whole number multiplication and division.	3.OA.A.1	Interpret products of whole numbers as the total number of objects in equal groups (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each).
	3.OA.A.2	Interpret whole number quotients of whole numbers (e.g., interpret $56 \div 8$ as the number of objects in each group when 56 objects are partitioned equally into 8 groups, or as a number of groups when 56 objects are partitioned into equal groups of 8 objects each). <i>See Table 2.</i>
	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. <i>See Table 2.</i>
	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times \square = 48$, $5 = \square \div 3$, $6 \times 6 = \square$.</i> <i>See Table 2.</i>
3.OA.B Understand properties of multiplication and the relationship between multiplication and division.	3.OA.B.5	Apply properties of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.)
	3.OA.B.6	Understand division as an unknown-factor problem (e.g., find $32 \div 8$ by finding the number that makes 32 when multiplied by 8).
3.OA.C Multiply and divide within 100.	3.OA.C.7	Fluently multiply and divide within 100. By the end of Grade 3, know from memory all multiplication products through 10×10 and division quotients when both the quotient and divisor are less than or equal to 10.
3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Utilize understanding of the Order of Operations when there are no parentheses.
	3.OA.D.9	Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends).
	3.OA.D.10	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Arizona Mathematics Standards 3rd Grade

Number and Operations in Base Ten (NBT)

Note: A range of algorithms may be used.

3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.	3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
	3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
	3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 using strategies based on place value and the properties of operations (e.g., 9×80 , 5×60).

Number and Operations – Fractions (NF)

Note: Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8.

3.NF.A Understand fractions as numbers.	3.NF.A.1	Understand a fraction ($1/b$) as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
	3.NF.A.2	<p>Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Understand that each part has size $1/b$ and that the end point of the part based at 0 locates the number $1/b$ on the number line.</p> <p>b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Understand that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line including values greater than 1.</p> <p>c. Understand a fraction $1/b$ as a special type of fraction that can be referred to as a unit fraction (e.g. $1/2$, $1/4$).</p>
	3.NF.A.3	<p>Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent if they have the same relative size compared to 1 whole.</p> <p>b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.</p> <p>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.</p> <p>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols $>$, $=$, or $<$, and justify conclusions.</p>

Arizona Mathematics Standards 3rd Grade

Measurement and Data (MD)

3.MD.A Solve problems involving measurement.	3.MD.A.1a	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., representing the problem on a number line diagram).
	3.MD.A.1b	Solve word problems involving money through \$20.00, using symbols \$, ".", ¢.
	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using metric units. (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much"). <i>See Table 2.</i>
3.MD.B Represent and interpret data.	3.MD.B.3	Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. <i>See Table 1.</i>
	3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch to the nearest quarter-inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
3.MD.C Geometric measurement: Understand concepts of area and perimeter.	3.MD.C.5	Understand area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
	3.MD.C.6	Measure areas by counting unit squares (e.g., square cm, square m, square in, square ft, and improvised units).
	3.MD.C.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure. Apply this technique to solve problems in real-world contexts.

Arizona Mathematics Standards 3rd Grade

3.MD.C (cont.)	3.MD.C.8	Solve real-world and mathematical problems involving perimeters of plane figures and areas of rectangles, including finding the perimeter given the side lengths, finding an unknown side length. Represent rectangles with the same perimeter and different areas or with the same area and different perimeters.
Geometry (G)		
3.G.A Reason with shapes and their attributes.	3.G.A.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples quadrilaterals that do not belong to any of these subcategories.
	3.G.A.2	Partition shapes into b parts with equal areas. Express the area of each part as a unit fraction $1/b$ of the whole. (Grade 3 expectations are limited to fractions with denominators $b = 2,3,4,6,8$.)

Arizona Mathematics Standards 3rd Grade

Standards for Mathematical Practice

3.MP.1	<p>Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?” to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.</p>
3.MP.2	<p>Reason abstractly and quantitatively.</p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.</p>
3.MP.3	<p>Construct viable arguments and critique the reasoning of others.</p> <p>Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.</p>
3.MP.4	<p>Model with mathematics.</p> <p>Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>

Arizona Mathematics Standards 3rd Grade

3.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.
3.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.
3.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.
3.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.

Arizona Mathematics Standards 3rd Grade

Table 1. Common Addition and Subtraction Problem Types/Situations.¹

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown ²
Put Together / Take Apart³	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

¹Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

²These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes or results* in but always does mean *is the same quantity as*.

³Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

Arizona Mathematics Standards 3rd Grade

Table 2. Common Multiplication and Division Problem Types/Situations.¹

	Unknown Product	Group Size Unknown (“How many in each group?” Division)	Number of Groups Unknown (“How many groups?” Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p>Measurement example.</p> <p>You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p>Measurement example.</p> <p>You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p>Measurement example.</p> <p>You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays,² Area³	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p>Area example.</p> <p>What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p>Area example.</p> <p>A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p>Area example.</p> <p>A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Compare	<p>A straw hat costs \$6. A baseball hat costs 3 times as much as the straw hat. How much does the baseball hat cost?</p> <p>Measurement example.</p> <p>A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A baseball hat costs \$18 and that is 3 times as much as a straw hat costs. How much does a blue straw cost?</p> <p>Measurement example.</p> <p>A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A baseball hat costs \$18 and a straw hat costs \$6. How many times as much does the baseball hat cost as the straw hat?</p> <p>Measurement example.</p> <p>A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

¹The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

²The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

³Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.