Unit 4 Review

Kinematics

Kinematics

- When air resistance is not taken into consideration, released objects will experience acceleration due to gravity, also known as freefall.

- Projectile motion can be predicted and controlled using kinematics

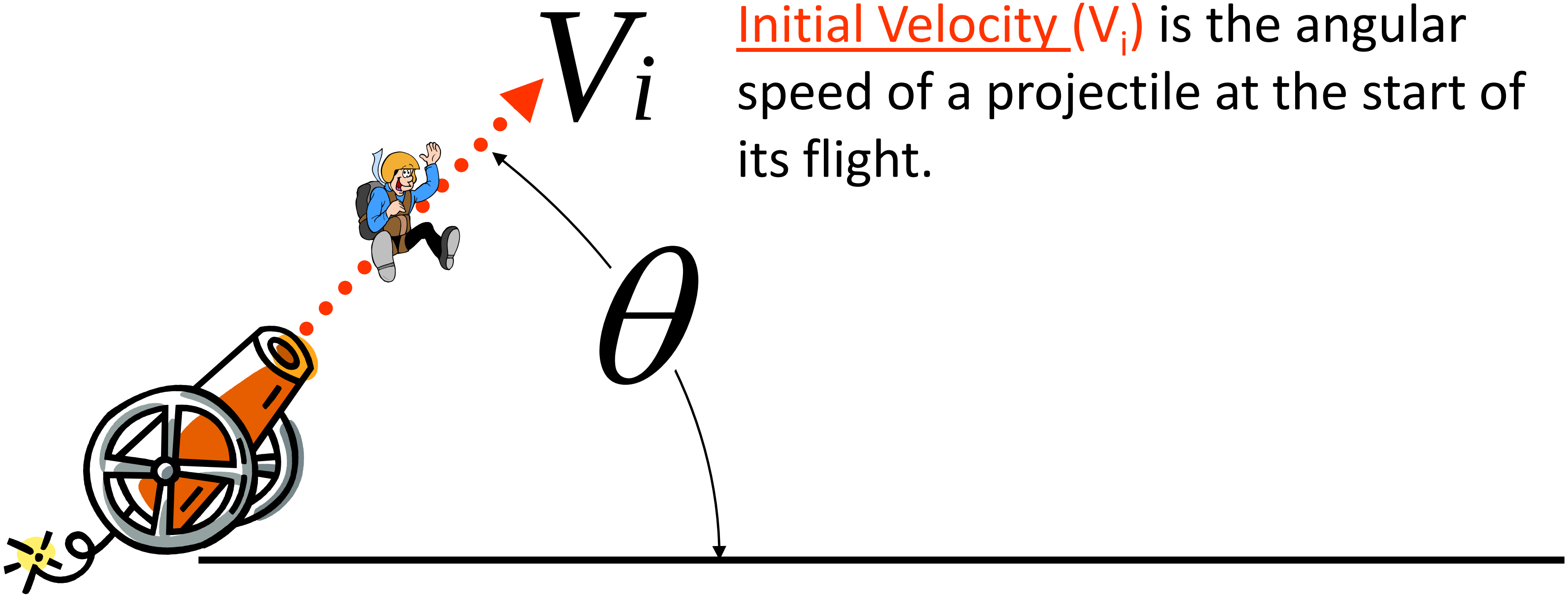
Human Cannonball Training

Speaking the Lingo

$\theta = \text{Theta}$

Firing Angle (θ) is measured in degrees. It is the angle at which the projectile left the cannon.

Initial Velocity (V_i) is the angular speed of a projectile at the start of its flight.



Human Cannonball Training

Calculating Initial Velocity

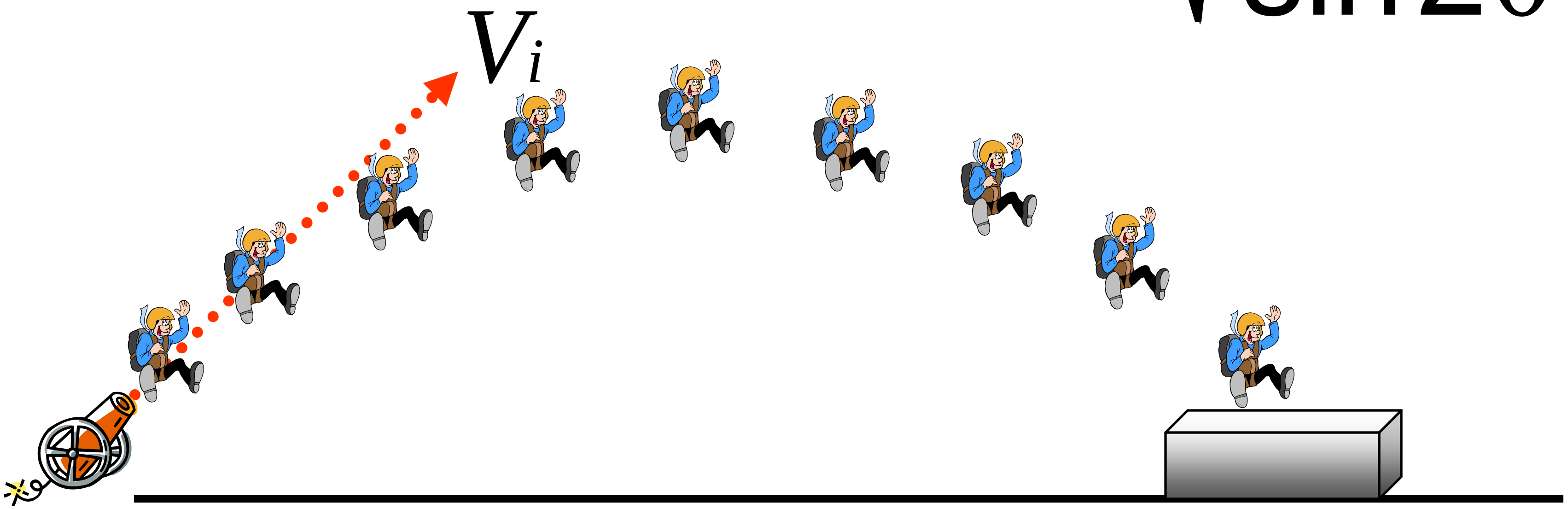
$V_i =$ Initial Velocity

$g =$ Gravitational Acceleration

$x =$ Horizontal Distance Traveled

$\theta =$ Firing Angle

$$V_i = \sqrt{\frac{-gx}{\sin 2\theta}}$$



Human Cannonball Training

Calculating Horizontal Distance

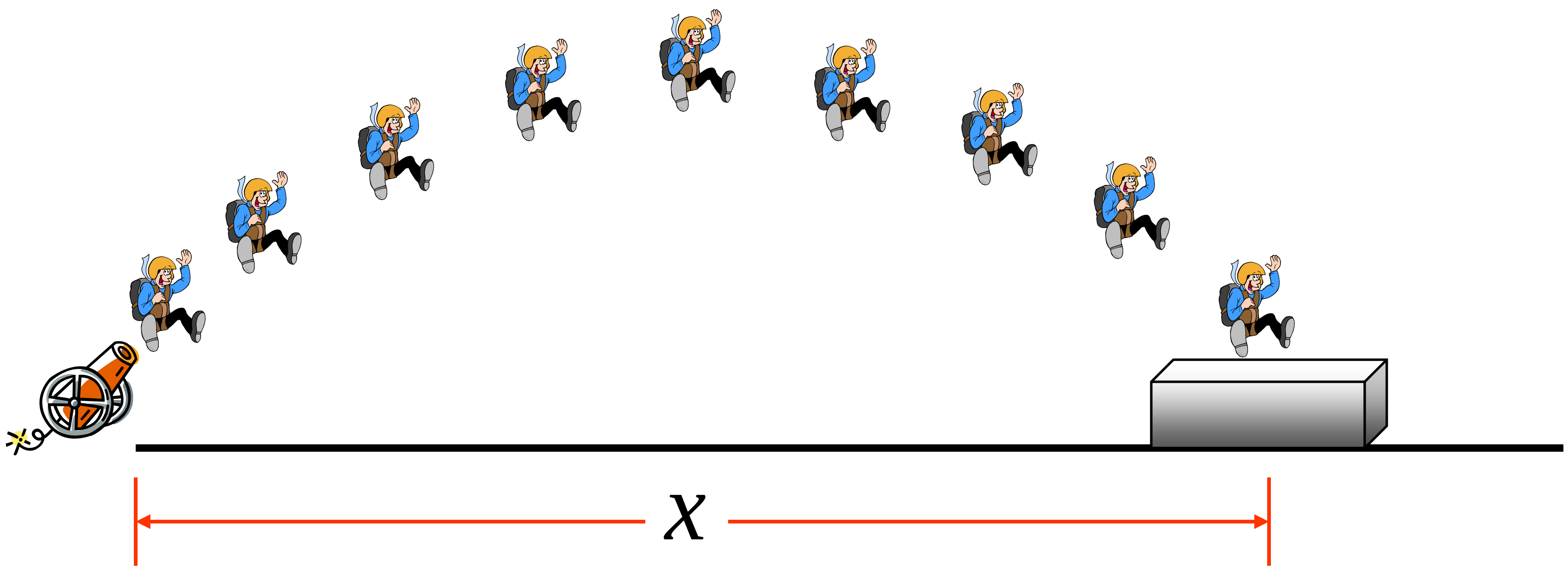
$V_i =$ Initial Velocity

$g =$ Gravitational Acceleration

$x =$ Horizontal Distance Traveled

$\theta =$ Firing Angle

$$x = \frac{V_i^2 \sin 2\theta}{-g}$$



Human Cannonball Training

Calculating Firing Angle

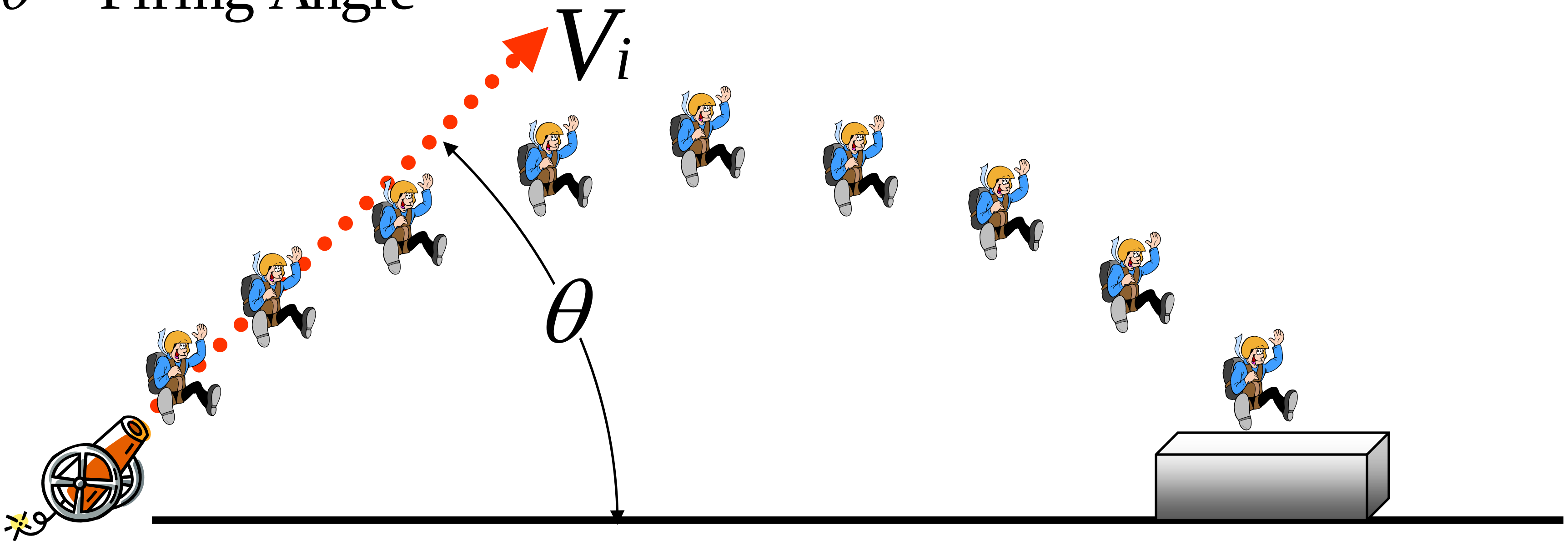
V_i = Initial Velocity

g = Gravitational Acceleration

x = Horizontal Distance Traveled

θ = Firing Angle

$$2\theta = \sin^{-1}\left(\frac{-gx}{V_i^2}\right)$$



Kinematics important info

Horizontal Motion:

- Velocity is constant!!!
- $v_x = v_{ix}$ Vix = $V_i \cos \Theta$

Kinematics important info

Vertical Motion:

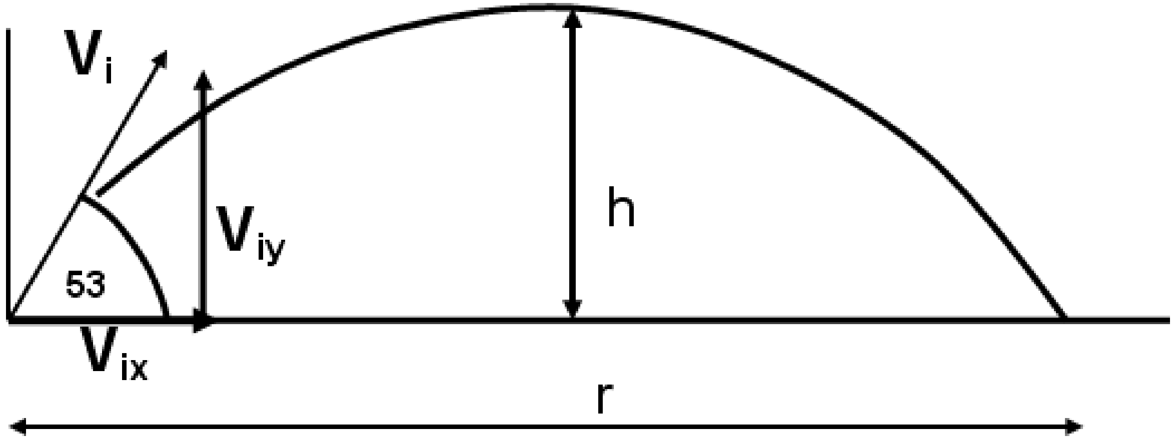
-Velocity changes with time due to gravity

-Viy = Vi sin theta

-**Velocity is zero in the y direction at peak**

Projectile Motion Problem

A ball is fired from a device, at a rate of 160 ft/sec, with an angle of 53 degrees to the ground.



Projectile Motion Problem

- Find the x and y components of V_i.
- What is the initial vertical velocity?
- What is the ball's range (the distance traveled horizontally)?

Projectile Motion Problem

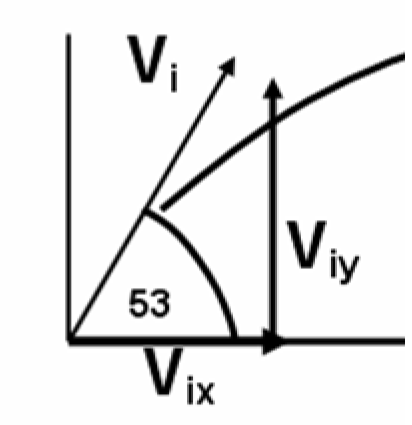
Find the x and y components of V_i.

$V_{ix} = V_i \cos \Theta$

$V_{ix} = (\cos 53)(160 \text{ ft/sec})$

$V_{ix} = (.6018)(160 \text{ ft/sec})$

$V_{ix} = 96 \text{ ft/sec}$



Projectile Motion Problem

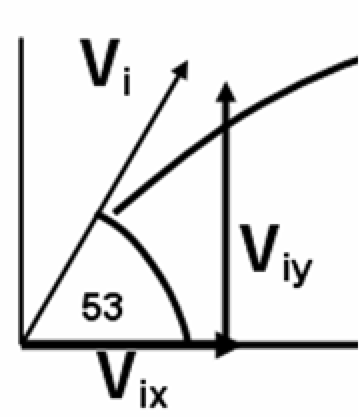
Find the initial vertical velocity.

$$V_{iy} = V_i \sin \Theta$$

$$V_{iy} = (\sin 53)(160 \text{ ft/sec})$$

$$V_{iy} = (.7986)(160 \text{ ft/sec})$$

$$V_{iy} = 128 \text{ ft/sec}$$



Projectile Motion Problem

What is the ball's range (the distance traveled horizontally)?

$$x = \frac{V_i^2 \sin 2\theta}{-g}$$

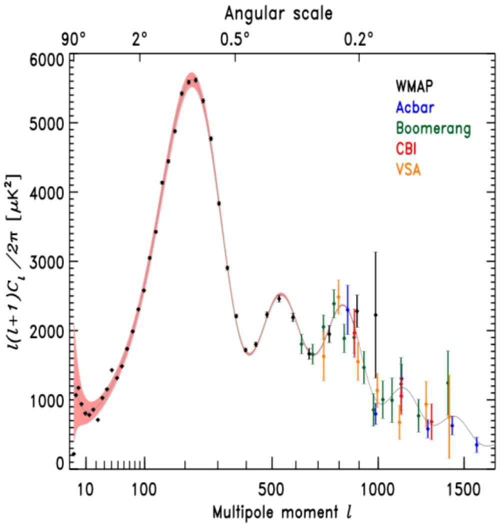
Vi = 160 ft/sec
Theta = 53 degrees
G = -32 ft/sec/sec

$$x_{\max} = 768\, ft$$

Statistics

Statistics

The collection, evaluation, and interpretation of data



Engineers use statistics to make informed decisions based on established principles.

Statistics

Statistics

Descriptive Statistics

Describe collected data

Inferential Statistics

Generalize and evaluate a population based on sample data

Statistics is based on both theoretical and experimental data analysis

Methods of Determining Probability

- Empirical
- Experimental observation

  Example – Process control
- Theoretical

  Uses known elements
- Example – Coin toss, die rolling
- Subjective

  Assumptions

  Example – I think that . . .

Probability

The number of times an event will occur divided by the number of opportunities

$$P_x = \frac{F_x}{F_a}$$

P_x = Probability of outcome x

F_x = Frequency of outcome x

F_a = Absolute frequency of all events

Expressed as a number between 0 and 1

fraction, percent, decimal, odds

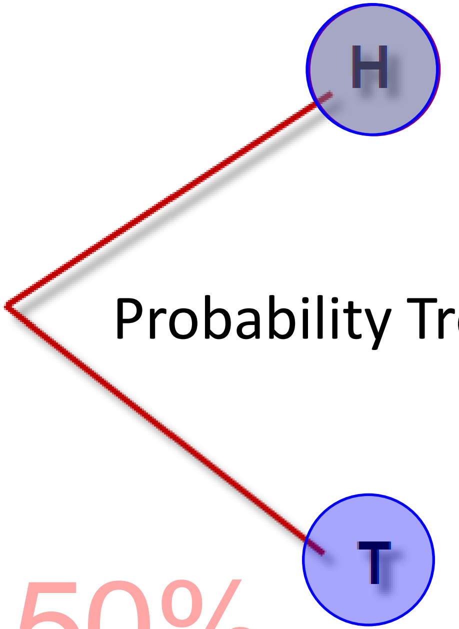
Total probability of all possible events totals 1

Probability

What is the probability of a tossed coin landing heads up?

How many desirable outcomes? 1

How many possible outcomes? 2



Probability Tree

$$P_x = \frac{F_x}{F_a} \quad P = \frac{1}{2} = .5 = 50\%$$

What is the probability of the coin landing tails up?

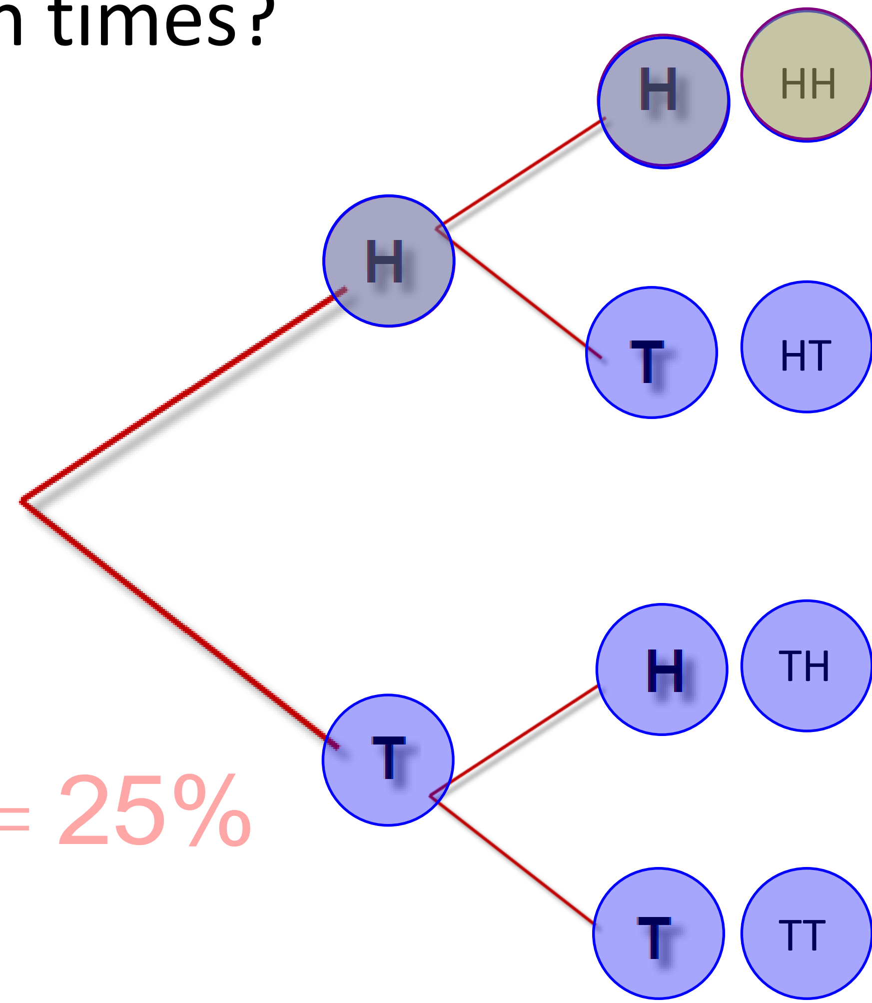
Probability

What is the probability of tossing a coin twice and it landing heads up both times?

How many desirable outcomes? 1

How many possible outcomes? 4

$$P_x = \frac{F_x}{F_a} \quad P = \frac{1}{4} = .25 = 25\%$$



Binomial Process

Each trial has only two possible outcomes

yes-no, on-off, right-wrong

Trial outcomes are independent

Tossing a coin does not affect future tosses

$$P_x = \frac{n!\left(p^x\right)\left(q^{n-x}\right)}{x!\left(n-r\right)!}$$

Bernoulli Process

$$P_x = \frac{n!\left(p^x\right)\left(q^{n-x}\right)}{x!\left(n-x\right)!}$$

P = Probability

x = Number of times an outcome occurs within n trials

n = Number of trials

p = Probability of success on a single trial

q = Probability of failure on a single trial

Probability Distribution

What is the probability of tossing a coin three times and it landing heads up two times?

$$P_x = \frac{n!\left(p^x\right)\left(q^{n-x}\right)}{x!\left(n-x\right)!}$$

$$P = \frac{3 \times 2 \times 1 \times \left(0.5^2\right)\left(0.5^2\right)}{\left(2 \times 1\right)\left(1 \times 1\right)}$$

$$P = .375 = 37.50\%$$

Law of Large Numbers

The more trials that are conducted, the closer the results become to the theoretical probability

Trial 1: Toss a single coin 5 times

H,T,H,H,T

P = .600 = 60%

Trial 2: Toss a single coin 500 times

H,H,H,T,T,H,T,T,……T

P = .502 = 50.2%

Theoretical Probability = .5 = 50%

Probability

AND (Multiplication)

Independent events occurring simultaneously

Product of individual probabilities

If events A and B are independent, then the probability of A and B occurring is:

$$P = P(A) \times P(B)$$

Probability

AND (Multiplication)

What is the probability of rolling a 4 on a single die?

How many desirable outcomes? 1

How many possible outcomes? 6

$$P_4 = \frac{1}{6}$$

What is the probability of rolling a 1 on a single die?

How many desirable outcomes? 1

How many possible outcomes? 6

$$P_1 = \frac{1}{6}$$

What is the probability of rolling a 4 and then a 1 using two dice?

$$P = (P_4)(P_1) = \frac{1}{6} \bullet \frac{1}{6} = \frac{1}{36} = .0278 = 2.78\%$$

Probability

OR (Addition)

Independent events occurring individually

Sum of individual probabilities

If events A and B are mutually exclusive, then the probability of A or B occurring is:

$$P = P(A) + P(B)$$

Probability

OR (Addition)

What is the probability of rolling a 4 on a single die?

How many desirable outcomes? 1

How many possible outcomes? 6

$$P_4 = \frac{1}{6}$$

What is the probability of rolling a 1 on a single die?

How many desirable outcomes? 1

How many possible outcomes? 6

$$P_1 = \frac{1}{6}$$

What is the probability of rolling a 4 or a 1 on a single die?

$$P = (P_4) + (P_1) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = .3333 = 33.33\%$$