Geometry: 2.4-2.6 Notes

NAME______________________

<u>2.4 Use and understand properties of equality</u>__________________________Date:____________________

Define Vocabulary:

equation

solve an equation

formula

Core Concepts

Algebraic Properties of Equality

Let *a*, *b*, and *c* be real numbers.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \bullet c = b \bullet c$, $c \neq 0$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.
Substitution Property of Equality	If $a = b$, then a can be substituted for *b* (or *b* for *a*) in any equation or expression.

Examples: Justifying steps. Solve the equations and justify each step.

<u>WE DO</u>

$\mathbf{2x - 5 = 13}$

<u>YOU DO</u>

$\mathbf{-2x - 9 = 10x - 17}$

Distributive Property

Let *a*, *b*, and *c* be real numbers.

Sum $a(b + c) = ab + ac$ **Difference** $a(b - c) = ab - ac$

Examples: Using the distributive property. Solve the equations and justify each step.

<u>WE DO</u>

$2(x + 1) = -4$

<u>YOU DO</u>

$3(3x + 14) = -3$

Examples: Solve the equation for the given variable.

<u>WE DO</u>

$9x + 2y = 5;\ y$

<u>YOU DO</u>

$\frac{1}{15}s - \frac{2}{3}t = -2;\ s$

Examples: Solve the real-life problem.

<u>WE DO</u>

The formula for the surface area *S* of a cone is $S = \pi r^2 + \pi rs,$ where *r* is the radius and *s* is the slant height. Solve the formula for *s*. Justify each step. Then find the slant height of the cone when the surface area is 220 square feet and the radius is 7 feet. Approximate to the nearest tenth.

Reflexive, Symmetric, and Transitive Properties of Equality

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Examples: Using properties of equality. Name the property of equality that the statement illustrates.

WE DO

1) If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.

2) $34° = 34°$

YOU DO

3) $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 5$. So, $m\angle 1 = m\angle 5$.

Assignment	

2.5 Write two-column proofs.______________________________________ Date:____________________

Define Vocabulary:

Proof

Two-column proof

Theorem

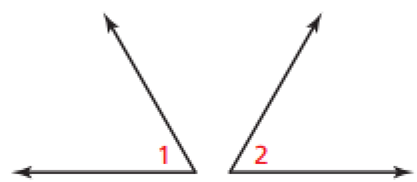
Writing Two-Column Proofs

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is the explanation for the corresponding statement.

Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

statements based on facts that you know or on conclusions from deductive reasoning

definitions, postulates, or proven theorems that allow you to state the corresponding statement

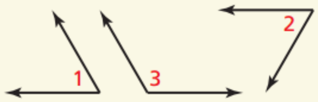
The number of statements will vary.

Remember to give a reason for the last statement.

Examples: Writing a Two-Column Proof

<u>WE DO</u>

Given $\angle 1$ is supplementary to $\angle 3$.
$\angle 2$ is supplementary to $\angle 3$.
Prove $\angle 1 \cong \angle 2$



<u>YOU DO</u>

Given T is the midpoint of $\overline{SU}$.

Prove $x = 5$

STATEMENTS	REASONS
1. T is the midpoint of $\overline{SU}$.	**1.** ______________________
2. $\overline{ST} \cong \overline{TU}$	**2.** Definition of midpoint
3. $ST = TU$	**3.** Definition of congruent segments
4. $7x = 3x + 20$	**4.** ______________________
5. ________________	**5.** Subtraction Property of Equality
6. $x = 5$	**6.** ______________________

Theorem 2.1 Properties of Segment Congruence

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Theorem 2.2 Properties of Angle Congruence

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Examples: Name the property that the statement illustrates.

<u>WE DO</u>

a. If $\angle RST \cong \angle TSU$ and $\angle TSU \cong \angle VWX$, then $\angle RST \cong \angle VWX$.

b. If $\overline{GH} \cong \overline{JK}$, then $\overline{JK} \cong \overline{GH}$.

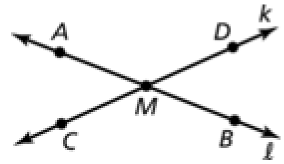
<u>YOU DO</u>

a. $\angle A \cong \angle A$

b. If $\overline{PQ} \cong \overline{JG}$ and $\overline{JG} \cong \overline{XY}$, then $\overline{PQ} \cong \overline{XY}$.

Examples:
WE DO

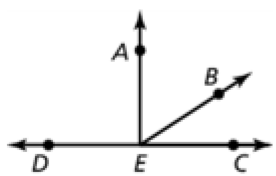
Given $\overline{AB}$ and $\overline{CD}$ bisect each other at point M and $\overline{BM} \cong \overline{CM}$.
Prove $AB = AM + DM$



STATEMENTS	REASONS
1. $\overline{BM} \cong \overline{CM}$	**1.** Given
2. $\overline{CM} \cong \overline{DM}$	**2.** ______________________
3. $\overline{BM} \cong \overline{DM}$	**3.** ______________________
4. $BM = DM$	**4.** ______________________
5. ______________________	**5.** Segment Addition Postulate (Post. 1.2)
6. $AB = AM + DM$	**6.** ______________________

YOU DO

Given $\angle AEB$ is a complement of $\angle BEC$.
Prove $m\angle AED = 90°$



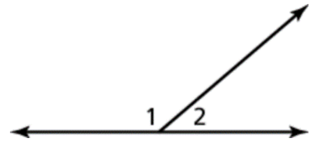
STATEMENTS	REASONS
1. $\angle AEB$ is a complement of $\angle BEC$.	**1.** Given
2. ______________________	**2.** Definition of complementary angles
3. $m\angle AEC = m\angle AEB + m\angle BEC$	**3.** ______________________
4. $m\angle AEC = 90°$	**4.** ______________________
5. $m\angle AED + m\angle AEC = 180°$	**5.** Definition of supplementary angles
6. ______________________	**6.** Substitution Property of Equality
7. $m\angle AED = 90°$	**7.** ______________________

Assignment	

Postulate 2.8 Linear Pair Postulate

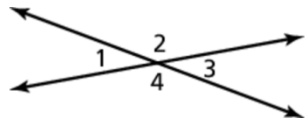
If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$.



Theorem 2.6 Vertical Angles Congruence Theorem

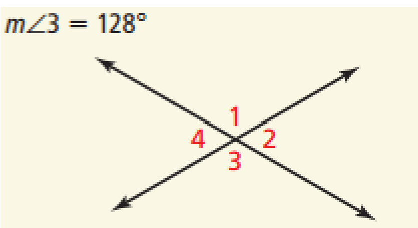
Vertical angles are congruent.



$\angle \mathbf{1} \cong \angle \mathbf{3}, \angle \mathbf{2} \cong \angle \mathbf{4}$

Examples: Use the diagram and the given angle measure to find the other three angle measures.

<u>WE DO</u>

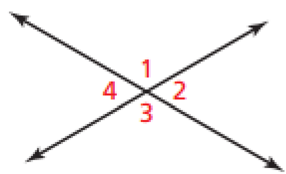


<u>YOU DO</u>

4. $m\angle 1 = 117°$

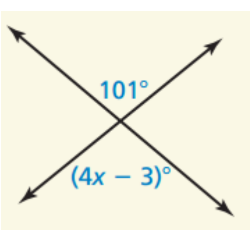
5. $m\angle 2 = 59°$

6. $m\angle 4 = 88°$

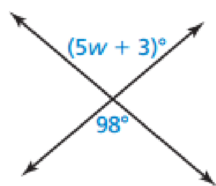


<u>Examples:</u> Find the value of the variable.

<u>WE DO</u>



<u>YOU DO</u>



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