Mini-Lecture 11.2

Permutations

$_nP_r = \frac{n!}{(n-r)!}$ ex: $_5P_3 = \frac{5!}{2!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1} = 5\cdot4\cdot3$

Order Matters

Examples:

1. Seven seats are positioned in a row at a movie theatre. Alice, Betty, Craig, Dan, Evelyn, Frank, and Gavin want to sit together in this row.
   a. How many different ways can they be arranged?

   $7! = 7\ 6\ 5\ 4\ 3\ 2\ 1 = {}_7P_7 =$ **5040**

   b. How many different ways can they be arranged if Betty sits in the second seat?

   $6\ \ B\ 5\ 4\ 3\ 2\ 1$ $6! =$ **720** $_6P_6$

   c. How many different ways can they be arranged if Craig and Gavin want to sit in the aisle seats?

   $2\ 5\ 4\ 3\ 2\ 1\ 1 =$ **240**

2. You need to arrange 12 of your favorite photographs on the mantel above a fireplace. How many ways can you arrange the photographs?

   $12! = {}_{12}P_{12} =$ **479,001,600**

3. Evaluate the factorial expressions:

   a. $\frac{8!}{3!}$ **6720** b. $\frac{1045!}{1044!}$ **1045** c. $(8-3)!$ **120** d. $9!-7!$ **357,840** e. $\left(\frac{20}{5}\right)!$ **24** f. $\frac{15!}{(15-3)!}$ $\frac{15!}{12!} = 15\cdot14\cdot13$ **2730**

Pg 611 4. Use the formula for $_nP_r$ to evaluate each expression. $= \frac{n!}{(n-r)!}$

   a. $_{10}P_4$ $\frac{10!}{6!} = 10\cdot9\cdot8\cdot7$ **5040**

   b. $_3P_3$ $\frac{3!}{0!}$ **6**

   c. $_3P_0$ $\frac{3!}{(3-0)!}$ **1**

   d. $_{145}P_2$ $\frac{145!}{143!}$ **20,880**

   e. $_{25}P_1$ $\frac{25!}{24!}$ **25**

5. A club with fifteen members is to choose four officers – president, vice-president, treasure, and secretary. If each office is to be held by one person and no person can hold more than one office, in how many ways can those offices be filled?

   $15\ 14\ 13\ 12 =$ **32,760** $_{15}P_4$ # of Permutations for 15 objects taken 4 at a time $\frac{15!}{(15-4)!} = \frac{15\cdot14\cdot13\cdot12\cdot11!}{11!}$

Pg 613 6. In how many distinct ways can the letters of the word SASSAFRAS be arranged?

   $\frac{n!}{p!\,q!\,r!\ldots}$ $\frac{9!}{4!\,3!} = \frac{9\cdot8\cdot7\cdot6\cdot5\cdot4!}{3\cdot2\cdot1\ \ 4!} =$ **2520** S-4 A-3 F,R

7. There are fifteen balloons – 3 blue, 4 red, 2 green, 3 yellow, and 3 orange. In how many distinct ways can the balloons be arranged?

   $\frac{15!}{3!\,4!\,2!\,3!\,3!}$ $\frac{15\cdot14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4!}{3\cdot2\cdot1\cdot3\cdot2\cdot1\cdot3\cdot2\cdot1\ \ 4!}$ **126,126,000**

   $\frac{15!}{4!\,3^3\,2^4}$