Key

Notes: Section 12.5 Problem Solving with Normal Distribution

Recall the z-score formula $\frac{data\ item - mean}{standard\ deviation}$

A **percentile** is a measurement that tells you what percent of the data falls below that measurement. If you scored in the 89th percentile on the ASVABs, then 89% of the people taking the test scored below your score.

1) A student scored in the 75th percentile on the SAT. What does this mean?

   75% scored below the student

The 68-95-99.7 rule works when data values are exactly 1, 2, or 3 standard deviations from the mean. What if a data value is 1.7 standard deviations from the mean? How do you find what percent of the data falls above or below that data value??? Table 12.14 to the rescue! This table gives numerous z-scores and their percentiles. The table shows the percent of data that falls **BELOW** that z-score.

2) In a normal distribution, a data value is 1.5 standard deviations below the mean.

   a) What's the z-score for this data value? -1.5

   b) What's the percentile for this data value? 6.68 % 6.68 P

   c) What percent of the data falls below this value? 6.68%

   d) What percent of the data falls above this value? 100 - 6.68 = 93.32 %

3) In a normal distribution, a data value is 2.7 standard deviations above the mean.

   a) What's the z-score for this data value? 2.7

   b) What's the percentile for this data value? 99.65 % 99.65P

   c) What percent of the data falls below this value? 99.65

   d) What percent of the data falls above this value? 100 - 99.65 = .35%

4) The distribution of monthly charges for cellphone plans in the US is approximately normal with a mean of \$62 and a standard deviation of \$18. What percentage of plans have charges that are less than \$83.60?

62 80 98

Compute the z-score, Use the table to get the percentile, answer the question

$Z_{83.60} = \frac{83.60-62}{18} = 1.2 \rightarrow$ 88.49% 88.49 percentile

5) Female adult heights in North America are approximately normally distributed with a mean of 65 inches and a standard deviation of 3.5 inches. What percentage of North American women have heights that exceed 69.9 inches?

Compute the z-score, Use the table to get the percentile, answer the question

$Z = \frac{69.9-65}{3.5} = 1.4 \rightarrow$ 91.92%

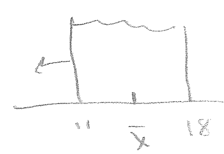
$100 - 91.92 = \boxed{8.08\%}$

6) The distribution for the life of refrigerators is approximately normal with a mean of 14 years and a standard deviation of 2.5 years. What percentage of refrigerators have lives between 11 years and 18 years?

Compute both z-scores, Use the table to get the percentile, answer the question

$Z: \frac{11-14}{2.5} = -1.2 \rightarrow 11.51\%$ $\frac{18-14}{2.5} = 1.6 \rightarrow 94.52\%$

$94.52 - 11.51 = \boxed{83.01\%}$



7) The weights for 24-month-old toddler boys are normally distributed with a mean of 28 pounds and a standard deviation of 2 pounds. Find the percentage of 24-month-old toddler boys who weigh

28
26 30
24 32
22

a) More than 32 pounds

2.5%

$Z = \frac{32-28}{2} = 2 \rightarrow$

$100 - 97.72 = \boxed{2.28\%}$

b) Less than 22 pounds

.15%

$Z: \frac{22-28}{2} = -3 \rightarrow \boxed{0.13\%}$

(#6 continued) The weights for 24-month-old toddler boys are normally distributed with a mean of 28 pounds and a standard deviation of 2 pounds. Find the percentage of 24-month-old toddler boys who weigh

c) Between 23.8 and 27.5 pounds

$z: \frac{23.8-28}{2} = -2.1 \rightarrow 1.79$

$\frac{27.5-28}{2} = -.25 \rightarrow 40.13$

$40.13 - 1.79 = \boxed{38.34\%}$

d) Between 26 and 30.4 pounds

$z: \frac{26-28}{2} = -1 \rightarrow 15.87\%$

$\frac{30.4-28}{2} = 1.2 \rightarrow 88.49$

$88.49 - 15.87 = \boxed{72.62\%}$

8) The amount of daily cell phone usage time for teens in the U.S. is normally distributed with a mean of 4.46 hours and a standard deviation of 1.44 hours.

$\bar{X} = 4.46 \quad s = 1.44$

a) What percentage of these young Americans talk more than 3.74 hours daily?

$z = \frac{3.74 - 4.46}{1.44} = -0.5 \rightarrow 30.85\%$ below

$100 - 30.85 = \boxed{69.15\%}$

b) What percentage talk less than 8.06 hours daily?

$z = \frac{8.06 - 4.46}{1.44} = 2.5 \rightarrow \boxed{99.38\%}$ below

c) Give the percentage that talk between 3.74 and 8.06 hours per day?

$99.38 - 30.85 = \boxed{68.53\%}$

8) Cholesterol levels in men 18 to 24 years of age are normally distributed with a mean of 178 and a standard deviation of 40.

$\bar{X} = 178$

$S = 40$

a) In what percentile is a man with a cholesterol level of 190?

$z = \frac{190-178}{40} = 0.3 \rightarrow \boxed{61.79\ \%}$

b) What percent of men have a cholesterol level less than 130?

$z = \frac{130-178}{40} = -1.2 \rightarrow \boxed{11.51\ \%}$

c) What percent of men have a cholesterol level greater than 222?

$z = \frac{222-178}{40} = 1.1 \rightarrow 86.43$

$100 - 86.43 = \boxed{13.57\ \%}$

d) What cholesterol level corresponds to the 90th percentile, the level at which treatment may be necessary?

$90\% \rightarrow 1.3 = \frac{X-178}{40} = \boxed{230}$

9)The heights for American women aged 18 to 24 are normally distributed with a mean of 65 inches and a standard deviation of 2.5 inches. In order to serve in the U.S. Army, women must be between 58 inches and 72.5 inches tall. What percentage of women are ineligible to serve based on their heights?

4' 10"

6' ½"

$\bar{X} = 65$

$s = 2.5$

$z = \frac{58-65}{2.5} = -2.8$

Below

$\rightarrow 0.26\%$

$z = \frac{72.5-65}{2.5} = 3$

$\rightarrow 99.87\%$

Above

$100 - 99.87 = 0.13$

$0.26 + .13 = \boxed{.39\%}$