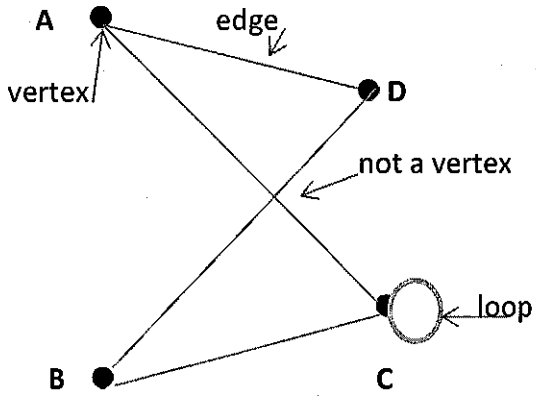
Intro: Subway Map, 6 degrees of separation*, mail routes
Salesmen, garbage collection, baseball schedules, UPS deliver
NYC sing 1970s

Notes 15.1 Graphs, Paths, and Circuits

Definition of a Graph:

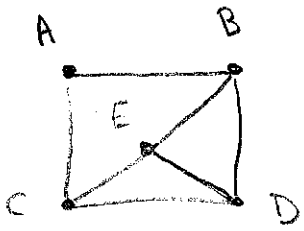
A **graph** consists of a finite set of points, called vertices (singular is vertex), and line segments or curves, called edges, that start and end at vertices. An edge that starts and ends at the same vertex is called a loop.

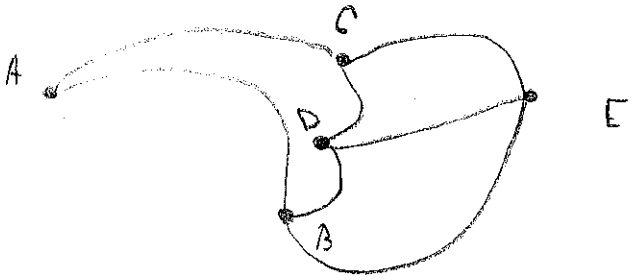


Two graphs are **equivalent** if they have the same number of vertices connected to each other in the same way.

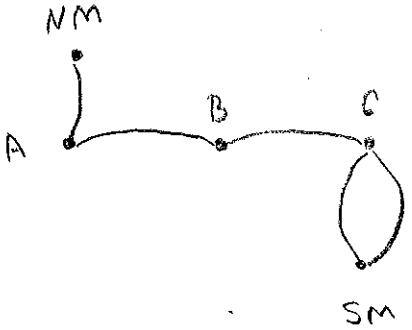
Ex 1) Draw two equivalent graphs for the following description:

The vertices are A, B, C, D, and E, and the edges are $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, $\overline{BE}$, $\overline{CE}$, $\overline{CD}$, and $\overline{DE}$

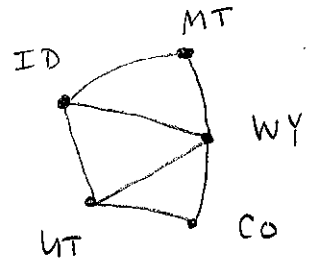




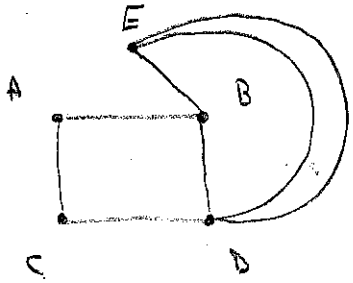
Ex 2) **Read example #2** on page 821 and do below Check point #2:



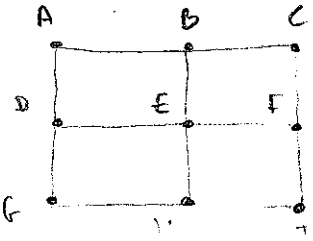
Ex 3) **Read example #3** on page 822 and do below check point #3:



Ex 4) **Read example #4** on page 823 and do below check point #4:



Ex 5) **Read example #5** on page 824 and do below check point #5:

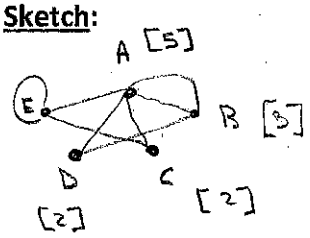


STOP

pg 825 Fig 5.16

Graph Theory Vocabulary:

Sketch:



Degree of a vertex: # edges to vertex Ex:

Even Vertex: Even # edges Ex: C, D, E

Odd Vertex: Odd # edges Ex: A, B

Adjacent Vertices: At least one edge connecting Ex: A and B, B and D, A and C, C and E, A and D, E and E ★, A and E

A **path** in a graph is a sequence of adjacent vertices and the edges connecting them.

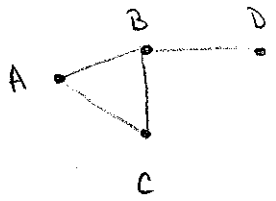
***Although a *vertex can appear on the path more than once*, **an edge can be part of a path only** once ***

A **circuit** is a path that begins and ends a the same *vertex.*

path, circuit

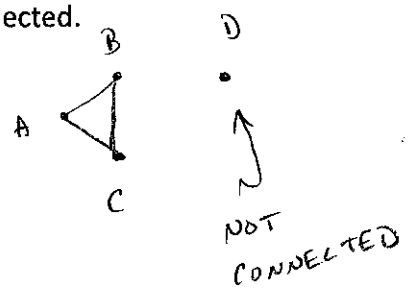
Connected Graphs: For any two vertices, there is alt least one edge connecting them.

Ex:



Disconnected Graphs: Graphs made up of components (pieces) that are by themselves connected.

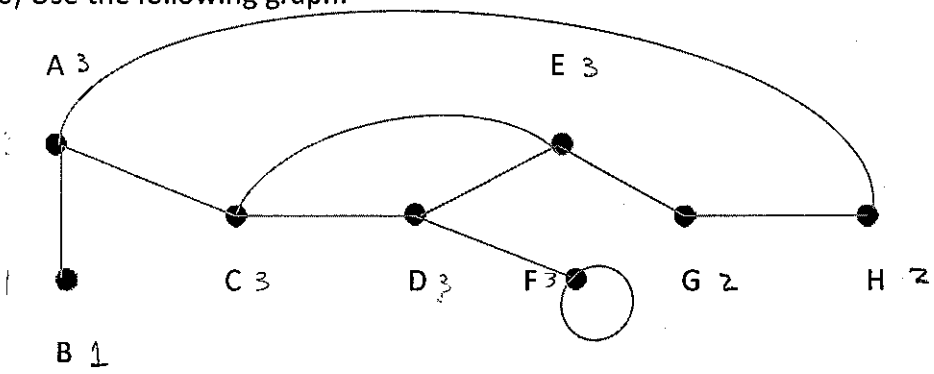
Ex:



A **bridge** is an edge (path) that if *removed* from a connected graph would leave behind a *disconnected graph.*

Ex: edge BD edge BC is NOT a bridge

Ex 6) Use the following graph:



a) Find the degree at each vertex in the graph:

b) Identify the even vertices and identify the odd vertices:

G, H A, B, C, D, E, F

c) Which vertices are adjacent to vertex C?

A, D, E

d) Which vertices are adjacent to vertex G?

E, H

e) Use vertices to describe two paths that start at vertex A and end at vertex H. Are there any others?

A, H A, C, E, G, H A, C, D, E, G, H Yes

f) Is there an edge that is considered a bridge? Why or why not?

A B
D F Removing it would disconnect B from the rest

g) Which edge could be removed to create a bridge in the graph? Which edge would be that bridge?

AH GH