NOTES 15.2 Euler Paths and Euler Circuits

Euler's Path:

A path that travels through every EDGE of a graph once and only once.

Trace w/o lifting pencil

Euler's Circuit:

A circuit that travels through every EDGE of a graph once and only once.

An Euler circuit MUST begin and end at the same vertex.

Every Euler circuit is an Euler path. However, not every Euler path is an Euler circuit.

EULER'S THEOREM

If the graph has: ***You will find:***

Number of ODD Vertices	Number of EVEN vertices	EULER PATH	EULER CIRCUIT
EXACTLY 2		AT LEAST ONE Starts at one odd vertex and ends at the other odd vertex	NONE
NONE	ALL	At least one Euler path	AT LEAST ONE Starts and ends at any vertex
MORE THAN 2		NONE	NONE

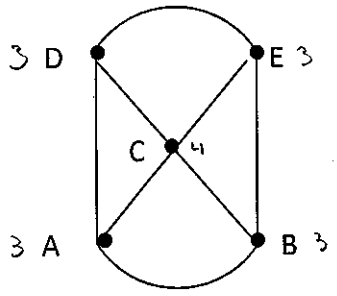
odd
2 None
=2 path
0 circuit | path
>2 3 or more

2 odd

(a – c) A graph or a connected graph is described. Determine whether the graph has an Euler path (but not an Euler circuit), an Euler circuit, or neither an Euler path nor an Euler circuit. Explain your answer.

No odd
3 or more odd

a)



Neither. 4 odd vertices

b) The graph has 16 even vertices and two odd vertices.

At least one path, no circuit

c) The graph has 12 even vertices and no odd vertices.

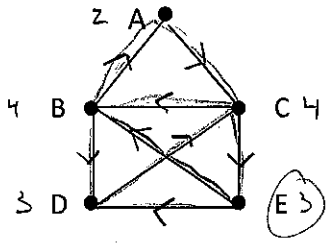
At least one path and circuit

Ex 1) REVISIT KONINGSBERG BRIDGE

a) Explain why the graph has at least one Euler path

2 odd vertices

b) Use trial and error to find an Euler path that starts at E (odd) and ends at D (odd)



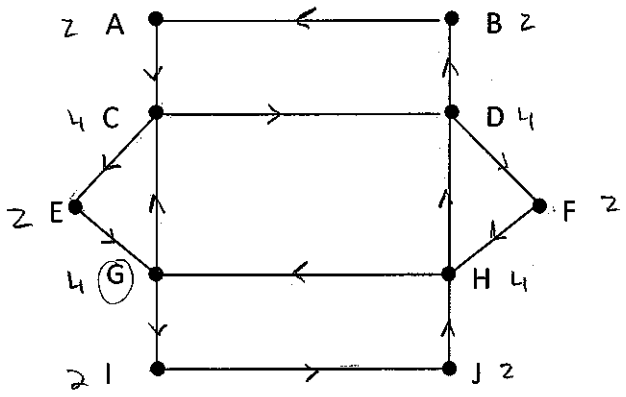
E B A C B D C E D

Ex 2)

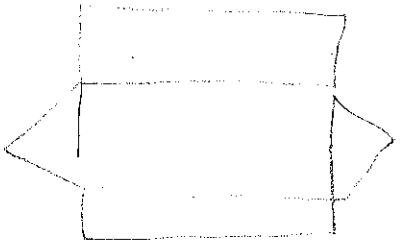
a) Explain why the graph has at least one Euler circuit

Zero odd vertices

b) Use trial and error to find an Euler circuit that starts and ends at G



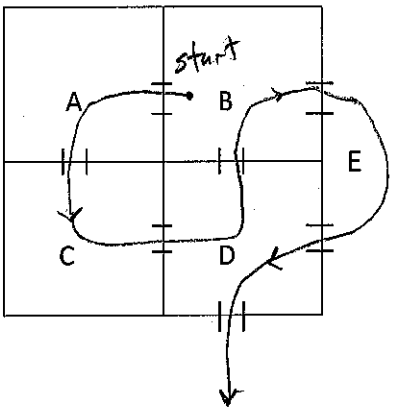
G C D B A C E G I J
H D F H G

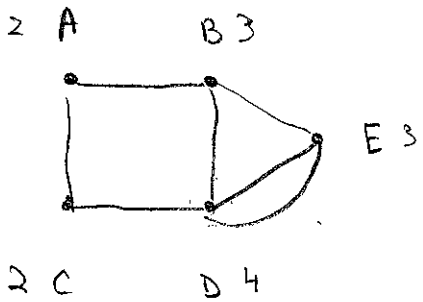


APPLICATIONS OF EULER'S THEOREM

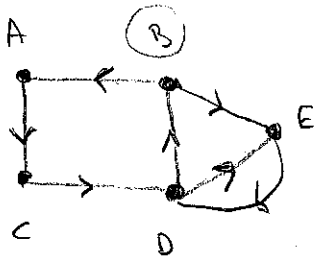
Ex 3) For the following floor plan:

a) Draw a graph that models the connecting relationships in each floor plan.
b) Use your graph to determine if it is possible to find a path that uses each door only once.
c) If such a path is possible, show it in your graph in part (a). Then trace this route in the floor plan in a manner that is clear to a person strolling through the house.





2 odd ∃1 path Yes



B A C D B E D E

FLEURY'S ALGORITHM

Used to find Possible Euler Paths and Euler Circuits without using Trial and Error

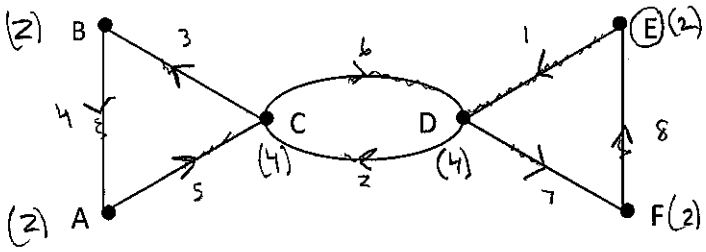
If Euler's Theorem indicates the existence of an Euler path or Euler circuit, one can be found using the following procedure:

1. If the graph has exactly two odd vertices (and therefore an Euler path), choose one of the two odd vertices as the starting point.
   If the graph has no odd vertices (and therefore and Euler circuit), choose any vertex as the starting point.

2. Number edges as you trace through the graph according to the following rules:
   - After you have traveled over an edge, erase it and draw a dashed line to identify edges that you have already used (this is because you must travel each edge exactly once).
   - To choose the next edge, choose one that is **not a bridge**. Travel over an edge that is a bridge only if there is no alternative.

Ex 4) Use the given graph to determine:

a) Whether the graph has an Euler path and Euler circuit, or neither. Both
b) If the graph has an Euler path or circuit, use Fleury's Algorithm to find it.



Circuit

E D C B A C D F E