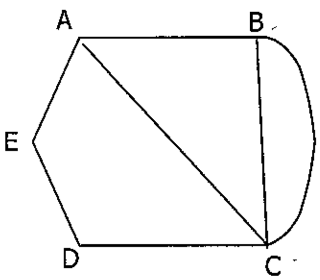
15.3 Notes: Hamilton Paths and Circuits

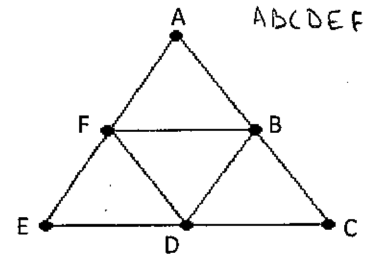
A path that passes through each VERTEX of a graph exactly once is called a Hamilton Path. If a Hamilton path begins and ends at the same vertex and passes through all other vertices exactly once, it is called a Hamilton Circuit. A Hamilton path or circuit does not need to pass through every edge!

1) Find a Hamilton path and a Hamilton circuit for each of the given graphs:

AEDCB   AEDCBA



H. Path ABCDEF   Circuit ABCDEFA

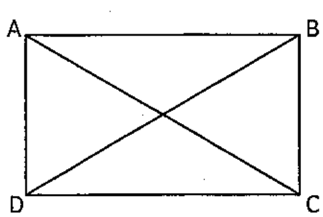


Not Complete

A **complete graph** is a graph in which all the vertices are connected to each other. (edge between each pair vertices) **A complete graph with three or more vertices will** always **contain a Hamilton circuit.** (Please note: some graphs that are not complete graphs, might contain a Hamilton circuit.)

Complete

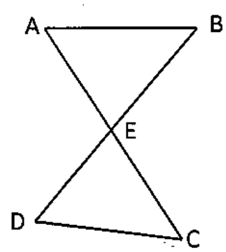
2) Refer to the graphs below. a) Is the graph a complete graph? b) Does the graph contain a Hamilton circuit? If so, name 2 of them that start at A.



Complete

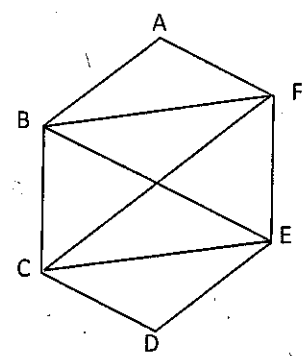
A B C D A   A C D B A

A D C B A



Not complete

No Hamilton circuit. Would have to pass E twice



Not Complete

A B C D E F A

A F E D C B A

A F C D E B A

A **complete graph** with *n* vertices will have **(n–1)!** different Hamilton circuits.

$n = 3$ $2! = 2$

$n = 4$ $3! = 3 \cdot 2 \cdot 1 = 6$

3) How many Hamilton circuits are there in a complete graph with 6 vertices?

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

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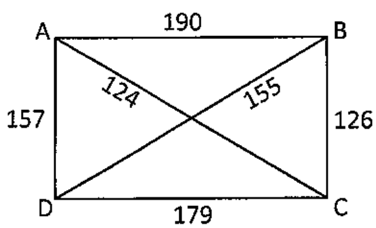
A graph whose edges have numbers attached to them is called a **weighted graph**. (See graph below.)

Sales directors for large companies are often required to visit regional offices in a number of different cities. How can these visits be scheduled in the cheapest possible way? Suppose a sales director lives in city A and is required to fly to cities B, C, and D. The weighted graph below shows the cost of airfare between each city. What is the cost for a trip described by the Hamilton circuit A, B, D, C, A?



190
155
179
124
648

The traveling salesperson problem is the problem of finding a Hamilton circuit in a complete weighted graph for which the sum of the edges is a **minimum**. Such a Hamilton circuit is called the **optimal Hamilton circuit** or the **optimal solution.**

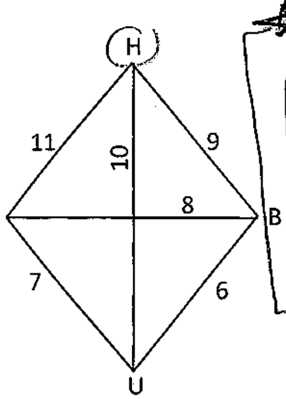
One way to find an optimal Hamilton circuit is using the **Brute Force Method**:

1) Model the problem with a complete, weighted graph.
2) Make a list of all possible Hamilton circuits. Start from A   A B C D A = B C D A B
3) Find the sum of the weights of each of the Hamilton circuits.
4) The Hamilton circuit with the smallest sum is the optimal solution.

4) Find the optimal solution for the problem above: 6 possible H circuits

A B C D A	190 + 126 + 179 + 157	652
A B D C A	190 + 155 + 179 + 124	648
★ A C B D A	124 + 126 + 155 + 157	(562)
A C D B A	124 + 179 + 155 + 190	648
★ A D B C A	157 + 155 + 126 + 124	(562)
A D C B A	157 + 179 + 126 + 190	652

5) You drive a Hummer and only get 5mpg. You have 3 errands to do, and want to travel the least distance in order to save on the cost of gas. You need to go to the bank to deposit a check, go to Paradise Bakery to pick up some cookies for your math teacher, and go to UPS to pick up a package. What's the optimal solution to this problem? The graph below shows the number of miles between each location. Assume you start and end at home.



★ (H B U P H)	9	6	7	11	(33)
H B P U H	9	8	7	10	34
H U B P H	10	6	8	11	35
H U P B H	10	7	8	9	34
H P B U H	11	8	6	10	35
★ (H P U B H)	11	7	6	9	(33)