

Graphs, Paths, Circuits Review Sheet

Graph

- Vertex – points of reference in a graph
- Edge – lines connecting vertices
- Loop – edge that connects to the same vertex

Path – sequence of adjacent vertices and edges connecting them

Circuit - a path that begins and ends at the same vertex

Equivalent Graphs - graphs with the same number of vertices connected to each other in the same way

Connected Graphs – at least one path connecting two vertices

Disconnected Graphs – graphs made of pieces that are by themselves connected

Bridge – edge that if removed would leave graph disconnected.

Graph Theory Vocabulary

- Degree of a vertex – number of edges at vertex
- Even vertex – vertex with even number of edges
- Odd Vertex – vertex with odd number of edges
- Adjacent Vertices – two vertices with at least one edge connecting them

Euler VS. Hamilton

Euler	Hamilton						
<p>Euler Path:</p> <ul style="list-style-type: none"> Travels through every edge of a graph once and only once <p>Euler Circuit:</p> <ul style="list-style-type: none"> Travels through every edge of a graph once and only once but begins and ends at the same vertex <p>Euler's Theorem:</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px;">2 Odd vertices</td> <td style="width: 50%; padding: 5px;">At least one Euler Path</td> </tr> <tr> <td style="width: 50%; padding: 5px;">No odd vertices</td> <td style="width: 50%; padding: 5px;">At least one Euler Circuit</td> </tr> <tr> <td style="width: 50%; padding: 5px;">More than 2 Odd vertices</td> <td style="width: 50%; padding: 5px;">No Euler Path or Circuit</td> </tr> </table> <p>Fleury's Theorem:</p> <ol style="list-style-type: none"> If graph has two odd vertices (Euler Path), start at one of the two odd vertices. If graph has no odd vertices (Euler Circuit), start at any vertex 	2 Odd vertices	At least one Euler Path	No odd vertices	At least one Euler Circuit	More than 2 Odd vertices	No Euler Path or Circuit	<p>Hamilton Path:</p> <ul style="list-style-type: none"> Travels through every vertex of a graph once and only once <p>Hamilton Circuit:</p> <ul style="list-style-type: none"> Travels through every vertex of a graph once but begins and ends at same vertex <p>Complete Graph – All vertices are connected to each other</p> <ul style="list-style-type: none"> Complete graphs always contain Hamilton circuits Complete graph with n vertices has (n-1)! Different Hamilton circuits <p>Weighted Graph – graph whose edges have numbers attached to them</p> <p>Finding Optimal Hamilton Circuit using Brute Force Method:</p> <ol style="list-style-type: none"> Model problem with complete, weighted graph Make list of all possible Hamilton Circuits Find the sum of the weights of each Circuit Circuit with smallest sum is the optimal solution
2 Odd vertices	At least one Euler Path						
No odd vertices	At least one Euler Circuit						
More than 2 Odd vertices	No Euler Path or Circuit						