3.3 Numbers of Zeros Theorem

A polynomial of degree n, has at most n distinct zeros.

Ex: $f(x) = x^3 + 3x^2 + 3x + 1$ ← degree 3

Have at most 3 distinct zeros.

$f(x) = (x+1)^3$ Actually has 1 distinct zero at $x = -1$. Multiplicity of 3.

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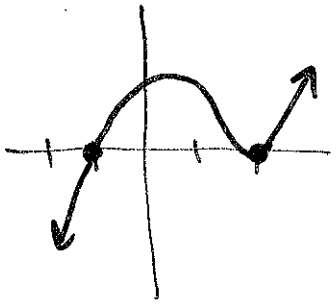
ex. $f(x) = (x+1)(x-2)^2$

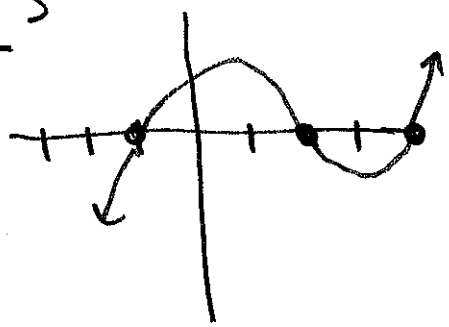
Zeros: $x = -1$ $x = +2$

Multiplicity: 1 2



Ex 4 Find a function of degree 3 with real coefficients that satisfies the given conditions.

(a) Zeros of $-1, 2$ and 4; $f(1) = 3$



$f(x) = a(x+1)(x-2)(x-4)$

$f(1) = 3 = a(1+1)(1-2)(1-4)$

$3 = a(2)(-1)(-3)$

$\frac{3}{6} = \frac{a\,6}{6}$

$\frac{1}{2} = a$

$f(x) = \frac{1}{2}(x+1)(x-2)(x-4)$

3.3 Ex 4 (b)

-2 is a zero of multiplicity 3; $f(-1) = 4$

$$f(x) = a(x+2)^3$$

$$f(-1) = 4 = a(-1+2)^3$$

$$4 = a(1)^3$$

$$4 = a$$

$$\boxed{f(x) = 4(x+2)^3}$$

$$f(x) = 4x^3 + 24x^2 + 48x + 32$$