

### 3.3 Rational Zeros Theorem

①

$$f(x) = \underset{q}{\textcircled{6}}x^4 + 7x^3 - 12x^2 - 3x + \underset{p}{\textcircled{2}}$$

① List all possible rational zeros

p can be  $\pm 1$   $\pm 2$  ← Factors

q can be  $\pm 1$   $\pm 2$   $\pm 3$   $\pm 6$  ← Factors

Possible rational zeros

$$\frac{p}{q}$$

$$\frac{\pm 1}{1} \quad \frac{\pm 2}{1}$$

$$\frac{\pm 1}{2} \quad \frac{\pm 2}{2} \quad , \quad \frac{\pm 1}{3} \quad \frac{\pm 2}{3} \quad , \quad \frac{\pm 1}{6} \quad \frac{\pm 2}{6}$$

Final list:  $\boxed{\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}}$

Possible Rational Zeros

**Ex 3**  $f(x) = 6x^4 + 7x^3 - 12x^2 - 3x + 2$

(2)

$\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$  Possible Rational Zeros

$$\begin{array}{r|rrrrr}
 1 & 6 & 7 & -12 & -3 & 2 \\
 & & 6 & 13 & 1 & -2 \\
 \hline
 & 6 & 13 & 1 & -2 & 0
 \end{array}$$

$f(x) = (x-1)(6x^3 + 13x^2 + x - 2)$

$$\begin{array}{r|rrrr}
 -2 & 6 & 13 & 1 & -2 \\
 & & -12 & -2 & 2 \\
 \hline
 & 6 & 1 & -1 & 0
 \end{array}$$

$f(x) = (x-1)(6x^2 + x - 1)(x+2)$

$$\begin{array}{r|rrr}
 \frac{1}{3} & 6 & 1 & -1 \\
 & & 2 & 1 \\
 \hline
 & 6 & 3 & 0
 \end{array}$$

$\frac{6x}{3} + \frac{3}{3}$   
 $3(2x+1)$

$6x^2 + x - 1$   
 $\frac{6x^2}{3x} + \frac{3x}{3x} \Big| \frac{-2x-1}{-1-1}$

$\frac{-6}{3(-2)} \Big| \frac{1}{3-2} = 1$

$3x(2x+1) - 1(2x+1)$   
 $(3x-1)(2x+1)$

$f(x) = 3(x-1)(x+2)(2x+1)(x-\frac{1}{3})$   


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 $f(x) = (x-1)(x+2)(3x-1)(2x+1)$

$3x-1=0 \Rightarrow x=\frac{1}{3}$       $2x+1=0 \Rightarrow x=-\frac{1}{2}$

Linear Factors

Actual Rational Zeros

$x-1=0$   
 $\frac{+1}{+1}$   
 $x=1$

$x+2=0$   
 $\frac{-2}{-2}$   
 $x=-2$

$3x-1=0$   
 $\frac{+1}{+1}$   
 $3x=1$   
 $x=1/3$

$2x+1=0$   
 $\frac{-1}{-1}$   
 $2x=-1$   
 $x=-1/2$

#39  $f(x) = 1x^3 - 2x^2 - 13x - 10$

List all possible rational zeros

$\frac{p}{q} = \frac{-10}{1} \rightarrow \boxed{\pm 1, \pm 2, \pm 5, \pm 10}$

1 | 1 -2 -13 -10  
    1 -1

-1 | 1 -2 -13 -10  
     -1 3 10  
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     1 -3 -10 0

$f(x) = (x+1)(x^2 - 3x - 10)$

$\begin{array}{r} -10 \mid -3 \\ -5(2) \end{array} \begin{array}{l} -3 \\ -5+2 = -3 \end{array}$

$x^2 - 3x - 10$   
 $(x-5)(x+2)$

$f(x) = (x+1)(x-5)(x+2)$  Linear Factors

Actual rational zeros:

$x = -1$   
 $x = 5$   
 $x = -2$

$x + y = 0$   
 $\frac{-1}{-1}$   

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 $x = -1$

$x + y = 0$   
 $\frac{-2}{-2}$   

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 $x = -2$

$x - y = 0$   
 $\frac{+5}{+5}$   

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 $x = 5$