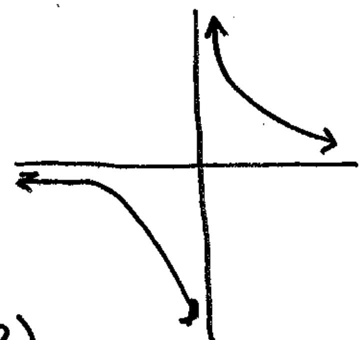
3.5 Rational Functions

$$f(x) = \frac{p(x)}{q(x)}$$

Ex: $f(x) = \frac{1}{x}$, $f(x) = \frac{x+1}{2x^2+5x-3}$

Reciprocal Function

$$f(x) = \frac{1}{x}$$



Domain: $(-\infty, 0) \cup [0, \infty)$

$\{x \mid x \neq 0\}$

Range: $(-\infty, 0) \cup (0, \infty)$

Discontinuous at $x = 0$

Odd Function, symmetric with respect to origin

ie. point $(1,1)$ also point $(-1,-1)$

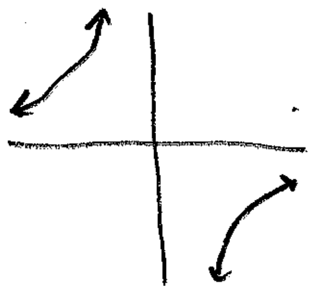
Decreasing on both sides of vertical asymptote

Horizontal asymptote: x-axis

Vertical asymptote: y-axis

3.5 $\boxed{\text{Ex 1}}$ Day 1 2

Graph $y = -\dfrac{2}{x} = -2\left(\dfrac{1}{x}\right)$



reflect over x- or y-axis
Stretch by 2 vertically

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Increasing on $(-\infty, 0)$ and $(0, \infty)$

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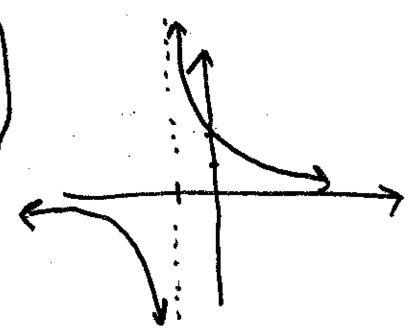
$\boxed{\text{Ex 2}}$ $f(x) = \dfrac{2}{(x+1)} = 2\left(\dfrac{1}{x+1}\right)$



Compare to $f(x) = \dfrac{1}{x}$

x has been replaced by $x+1$
so graph moves Left 1.

Domain: $(-\infty, -1) \cup (-1, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Decrease: $(-\infty, -1)$ and $(-1, \infty)$

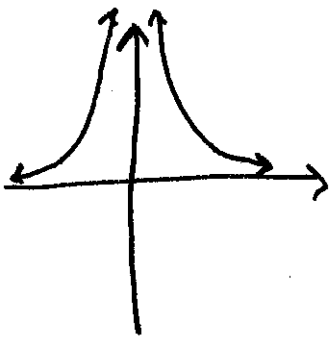
Increase: None

Vertical Asymptote $x = -1$

Horizontal Asymptote $y = 0$

3.5 continued

Rational Function $f(x) = \frac{1}{x^2}$



Domain $(-\infty, 0) \cup (0, \infty)$

Range $(0, \infty)$

Increase $(-\infty, 0)$ **Decrease** $(0, \infty)$

Discontinuous at $x = 0$

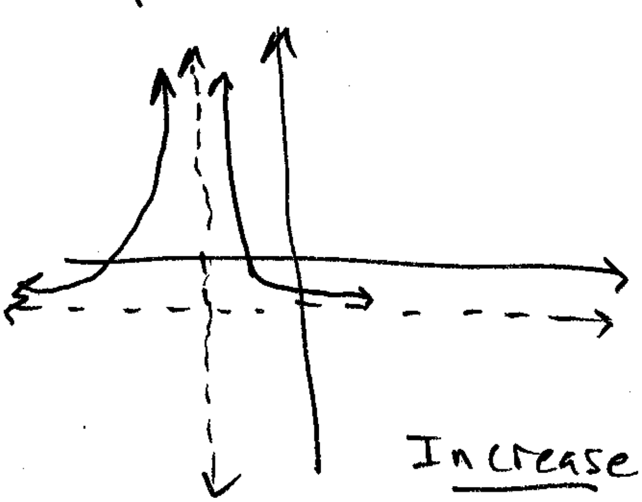
Vertical Asymptote y-axis

Horizontal Asymptote x-axis

Even function symmetric wrt y-axis. with respect to

$\boxed{\text{Ex 3}}$ $g(x) = \frac{1}{(x+2)^2} - 1$

Compared to $f(x) = \frac{1}{x^2}$, $g(x)$ moves down 1
Left 2



Domain: $(-\infty, -2) \cup (-2, \infty)$

Range: $(-1, \infty)$

Hor. Asym. $y = -1$

Vert. Asym. $x = -2$

Increase: $(-\infty, -2)$ **Decrease**: $(-2, \infty)$

Determining Asymptotes

(1) VERTICAL ASYMPTOTE
Find by setting denominator $= 0$ and solve for x.

$f(x) = \dfrac{1}{x-a}$

$$\begin{aligned} x - a &= 0 \\ +a\ &\ +a \\ \hline x &= a \end{aligned}$$ ← Vert. Asym.

(2) OTHER ASYMPTOTES

(a) Numerator has a lesser degree than Denominator, then Horizontal Asymptote is $y = 0$ (x-axis)

Ex: $f(x) = \dfrac{x}{x^2}$ degree 1 / degree 2

(b) Numerator and Denominator have same degree, Horizontal Asymptote is ratio of leading coefficients.

Ex: $f(x) = \dfrac{2x^2 + 3x + 2}{5x^2 + 4}$

Horizontal Asymptote → $y = \dfrac{2}{5}$