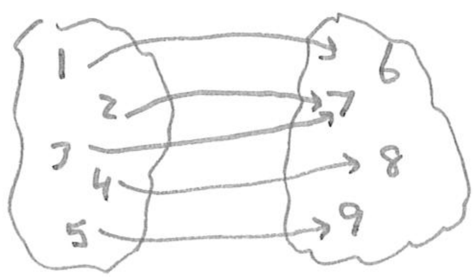
4.1 Inverse Functions

One-to-one function

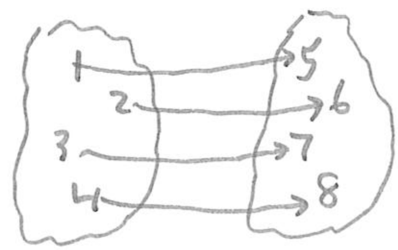
each x-value corresponds to only one y-value.

$\therefore$ $f(a) = f(b)$ implies $a = b$

Meaning if two range values are equal, their corresponding domain values are equal.



Not one-to-one



One-to-one

$\boxed{\text{Ex 1}}$ Deciding whether functions are one-to-one

(a) $f(x) = -4x + 12$

If can show $f(a) = f(b)$ leads to $a = b$, then the function is one-to-one.

$$f(a) = f(b)$$

$$-4a + \cancel{12} = -4b + \cancel{12}$$

$$\underline{\quad -\cancel{12} \qquad\qquad -\cancel{12}}$$

$$\frac{-4a}{-4} = \frac{-4b}{-4}$$

$a = b$ Therefore, $f(x) = -4x + 12$ is one-to-one.

$\boxed{\text{Ex 1}}$ (b) $f(x) = \sqrt{25 - x^2}$

Can show NOT one-to-one if different domain values (x) correspond to same range value (y).

$f(3) = \sqrt{25 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$f(-3) = \sqrt{25 - (-3)^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$f(3) = 4$ AND $f(-3) = 4$ NOT One-to-One.

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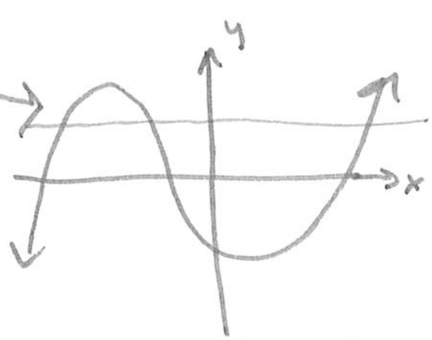
$\boxed{\text{Ex 2}}$ Horizontal Line Test

Reminder: Vertical Line Test

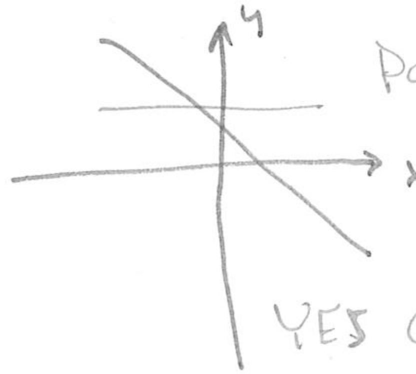
Use to determine whether a relation is a function.

Function → NO Function → YES



Fails Hor. Line Test

NOT One-to-One



Passes Horizontal Line Test

YES One-to-one

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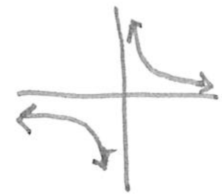
4.1 Day 1

Note: In general, a function that is increasing or decreasing on the entire domain, must be one-to-one.

Ex: $f(x) = -x$, $g(x) = x^3$ $h(x) = \frac{1}{x}$



Summary: Tests to Determine whether a function is one-to-one.

1. Show that $f(a) = f(b)$ implies $a = b$
2. If two x-values produce the same y-value NOT one-to-one.
3. Sketch graph and use horizontal line test.
4. If function increases or decreases over entire domain, it is one-to-one.

Review: Composition:

$f(x) = 2x$ $g(x) = x+1$

$(f \circ g)(x) = f(g(x)) = 2(x+1) = 2x+2$

$(g \circ f)(x) = g(f(x)) = (2x)+1 = 2x+1$

Inverse Functions

Certain one-to-one functions "undo" each other. These are inverse functions.

$$10 \to \boxed{} \to 85 \to \boxed{} \to 10$$

$g(x) = 8x+5$ $\qquad$ $f(x) = \frac{1}{8}x - \frac{5}{8}$

Inverse function

If functions are inverse,

$f(g(2)) = 2$ $\qquad$ $g(f(2)) = 2$

$f(g(x)) = x$ $\qquad$ $g(f(x)) = x$

$$\underbrace{(f \circ g)(x) = x \qquad (g \circ f)(x) = x}_{\text{proves inverse functions}}$$

$\boxed{\text{Ex 3}}$ $f(x) = x^3 - 1$ and $g(x) = \sqrt[3]{x+1}$

Test if inverse functions:

Does $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$

$(f \circ g)(x) = f(\underline{g(x)})$

$= (\sqrt[3]{x+1})^3 - 1 = x+1-1 = x$

$(g \circ f)(x) = g(f(x))$

$= \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3-1+1}$

$= \sqrt[3]{x^3} = x$

$\boxed{\therefore f(x) \text{ and } g(x) \text{ are inverses}}$

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$f(x) = x^3 - 1$ has inverse $f^{-1}(x) = \sqrt[3]{x+1}$

f^{-1} f-inverse

Note: For inverse functions, the domain of f is the range of f^{-1}.

and the range of f is the domain of f^{-1}.

4.1 $\boxed{\text{Ex 4}}$ Finding Inverses of One-to-One Functions.

Find the inverse of each function that is one-to-one.

(a) $F = \{(-2,1)(-1,0)(0,1)(1,2)(2,2)\}$

Is it one-to-one? $\boxed{\text{NO}}$

(b) $G = \{(3,1)\ (0\ 2)\ (2\ 3)\ (4\ 0)\}$

Is this one-to-one? $\boxed{\text{YES}}$

$G^{-1} = \{(1,3)\ (2,0)\ (3,2)\ (0,4)\}$

(c)

Year	Number Hurricanes
2009	3
2010	12
2011	7
2012	10
2013	2

One-to-One? $\boxed{\text{YES}}$

$f^{-1}(x) = \{(3,2009)\ (12,2010)\ (7,2011)\ (10,2012)\ (2,2013)\}$

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53