

Unit 7 7.3 Geometric Sequences

①

$n = 1, 2$
 a_1, a_2
 $1, 2, 4, 8, \dots$
 $\downarrow \quad \downarrow \quad \downarrow$
 $\times 2 \quad \times 2 \quad \times 2$

Common ratio $\frac{4}{2} = 2$ "r" = $\frac{a_{n+1}}{a_n}$

Ex: $2, 8, 32, 128, \dots$ $r = \frac{8}{2} = 4$
 $\downarrow \quad \downarrow$
 $\times 4 \quad \times 4$

$a_1 = 2$

$a_2 = 2 \cdot 4 = \underline{2 \cdot 4^1}$

$a_3 = 2 \cdot 4 \cdot 4 = \underline{2 \cdot 4^2}$

$a_4 = 2 \cdot 4 \cdot 4 \cdot 4 = \underline{2 \cdot 4^3}$

nth Term of a Geometric Sequence

$a_n = a_1 r^{(n-1)}$

Ex 1 How much will be earned on day 20 if daily wages follow the sequence 1, 2, 4, 8, 16 cents?

$a_n = a_1 r^{n-1} = a_n = 1 (2^{n-1})$

$a_{20} = 1 (2^{20-1})$

$= 2^{19} = 524,288 \text{¢}$

$= \boxed{\$5242.88}$

7.3 Ex 2 Find a_5 and a_n for the
geometric sequence $4, 12, 36, 108, \dots$

(2)

$$r = \frac{a_{n+1}}{a_n} = \frac{a_{1+1}}{a_1} = \frac{a_2}{a_1} = \frac{12}{4} = 3$$

$$r = 3 \rightarrow \frac{12}{4} = \textcircled{3 = r}$$

$$\textcircled{a_1 = 4}$$

$$a_n = a_1 r^{(n-1)}$$

$$\boxed{a_n = 4 \cdot 3^{n-1}}$$

↗

a_n

$$\begin{aligned} a_5 &= 4 (3^{5-1}) \\ &= 4 (3^4) \end{aligned}$$

$$\boxed{a_5 = 324}$$

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7.3 Ex 3 Determine r and a_1 from the 3
geometric sequence with $a_3 = 20$ and $a_6 = 160$.

$$\begin{array}{ccccccc} \overline{a_1} & \overline{a_2} & \overline{20} & \overline{a_4} & \overline{a_5} & \overline{160} & \\ & & a_3 & & & a_6 & \\ & & \vee & \vee & \vee & & \\ & & \times r & \times r & \times r & & \end{array}$$

$$a_6 = a_3 r^3$$

$$\frac{160}{20} = \frac{20}{20} r^3 \quad \rightarrow \quad (8)^{\frac{1}{3}} = (r^3)^{\frac{1}{3}}$$

$$8^{\frac{1}{3}} = r$$

$2 = r$

Find a_1

$$a_n = a_1 r^{n-1}$$

$$20 = a_1 2^{3-1}$$

$$20 = a_1 2^2$$

$$\frac{20}{4} = a_1 \frac{4}{4}$$

$5 = a_1$

7.3

Ex 4Modeling a population of fruit flies (4)

Population of fruit flies grows such that each generation is 1.5 times as large as the last generation. [$r = 1.5$]

There are 100 insects in the first generation.

[$a_1 = 100$] How many flies would be in the fourth generation? Round to nearest whole number. [0, 1, 2, 3, ...]

$$a_n = a_1 r^{n-1}$$

$$a_4 = 100 (1.5^{4-1})$$

$$a_4 = 100 (1.5^3) = 100 (3.375) =$$

$$= 337.5 \approx$$

338 fruit flies in 4th generation.