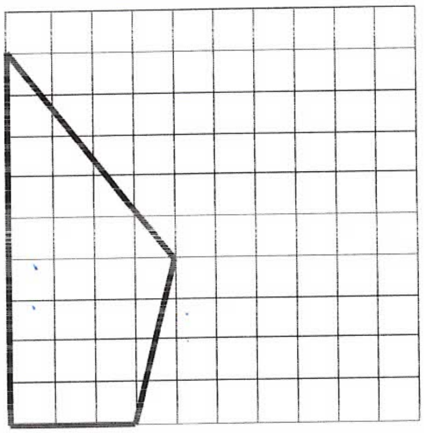
Notes 7.5 LINEAR PROGRAMMING

Ex 1) Given the objective function, find its value at each corner of the graphed region. What is the maximum and minimum of the objective function?

Objective function: $z = 30x + 45y$



(x,y)	z = 30x + 45y	=
(0,0)	0 + 0	= 0
(3,0)	30(3) + 45(0) =	90
(4,4)	30(4) + 45(4) = 120 + 180 =	300
(0,9)	30(0) + 45(9) = 0 + 405	= 405

$\boxed{\text{Max: } 405 \quad \text{min: } 0}$

For problems 2 – 4:

a) Graph the system of inequalities (representing the constraints)

b) Find the value of the objective function at each corner of the graphed region. If lines cross in the region, use elimination or substitution to find the point of intersection.

c) Use the coordinates from (b) to find the values at which the maximum occurs.

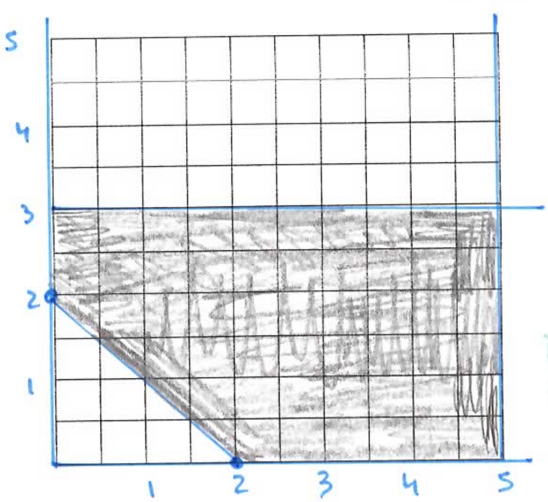
Ex 2) objective function: $z = 5x - 2y$

Constraints: $0 \le x \le 5$

$0 \le y \le 3$

$X + y \ge 2$

x	y
0	2
2	0



	5x-2y	
(2,0)	5(2) - 2(0) = 10 - 0	= 10
(5,0)	5(5) - 2(0) = 25 - 0	= 25
(5,3)	5(5) - 2(3) = 25 - 6 =	19
(0,3)	5(0) - 2(3) = 0 - 6	= -6
(0,2)	5(0) - 2(2) = 0 - 4 =	-4

$\boxed{\text{max: } 25 \qquad \text{min: } -6}$

Ex 3) Objective Function $Z = 2x + 4y$

Constraints: $x \ge 0 \quad y \ge 0$

- $X + 3y \ge 6$
- $X + y \ge 3$
- $X + y \le 9$

x	y
0	2
6	0

x	y
0	3
3	0

x	y
0	9
9	0

$-1(x + 3y \ge 6)$

$x + y \ge 3$

$-x - 3y \ge -6$

$\dfrac{-2y}{-2} = \dfrac{-3}{-2}$

$y = \frac{3}{2}$

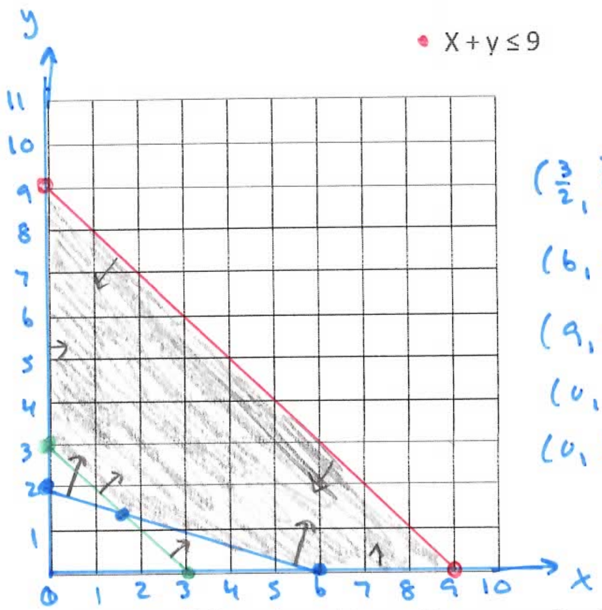
$x + y = 3$

$x + \frac{3}{2} = 3$

$x = 3 - \frac{3}{2}$

$x = \frac{3}{2}$

$(\frac{3}{2}, \frac{3}{2})$



$Z = 2x + 4y$

$(\frac{3}{2}, \frac{3}{2})$ $\quad 2(\frac{3}{2}) + 4(\frac{3}{2}) = 9$

$(6, 0)$ $\quad 2(6) + 4(0) = 12$

$(9, 0)$ $\quad 2(9) + 4(0) = 18$

$(0, 9)$ $\quad 2(0) + 4(9) = 36$

$(0, 3)$ $\quad 2(0) + 4(3) = 12$

Max: 36

Min: 9

Steps to follow for a Linear Programming Problem:

1	Identify variables x = y =	What is it that you want to make, produce, sell deliver, ...?
2	Write the objective function Z =	What is it that you want to maximize or minimize?
3	Write your constraints in form of inequalities	What are the limitations?
4	Graph each inequality. Use x and y intercepts	Use the right scale!
5	Shade the feasible region. List all corners that form the region.	Use elimination or substitution to find the point of every intersecting lines that are not on the axis.
6	Plug in all coordinates into objective function to find the max. or min.	
7	Answer the question of the problem	

Checkpoints 1-4:

A company manufactures bookshelves and desks for computers. Let *x* represent the number of bookshelves manufactured daily and *y* represent the number of desks manufactured daily. The company's profits are $25 per bookshelf and $55 per desk. Write the objective function that describes the company's daily profit, *z*, from *x* bookshelves and *y* desks.

x = # book shelves

y = # desks

$z = 25x + 55y$

To maintain high quality, the company should not manufacture more than a total of 80 bookshelves and desks per day. Write an inequality that describes this constraint.

$x + y \leq 80$

To meet customer demand, the company must manufacture between 30 and 80 bookshelves per day, inclusive. Furthermore, the company must manufacture at least 10 and no more than 30 desks per day. Write an inequality that describes each of these sentences. Then summarize what you have described about this company by writing the objective function for its profits and the three constraints.

$30 \leq x \leq 80$

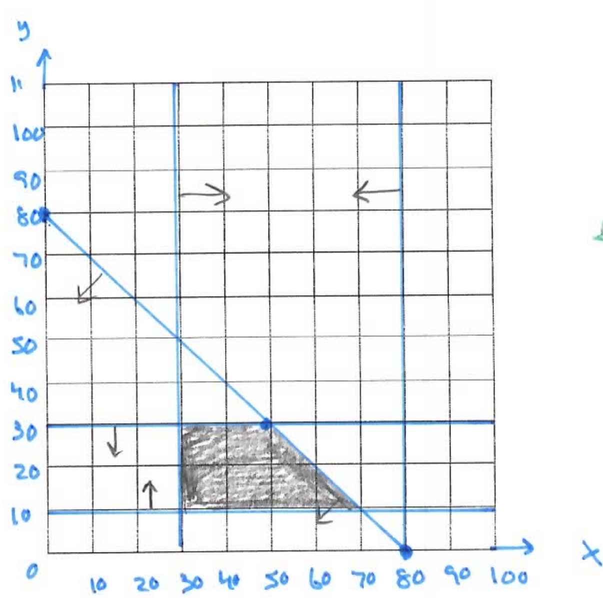
$10 \leq y \leq 30$

x	y
0	80
80	0

- $x + y \leq 80$

$z = 25x + 55y$

How many bookshelves and how many desks should be manufactured per day to obtain a maximum profit? What is the maximum daily profit?



$z = 25x + 55y$

$(30, 10)$ $25(30) + 55(10) = 1300$

$(70, 10)$ $25(70) + 55(10) = 2300$

$(50, 30)$ $25(50) + 55(30) = \boxed{2900}$

$(30, 30)$ $25(30) + 55(30) = 2400$

$y = 30$

$x + y = 80$

$x + 30 = 80$

$x = 50$

50 bookshelves and 30 desks to maximize profit at $2900.