

R.5 Rational Expressions

The quotient (division / fraction) of two polynomials.

$$\frac{x+6}{x+2}, \quad \frac{(x+6)(x+4)}{(x+2)(x+4)}, \quad \frac{2p^2+7p-4}{5p^2+20p}$$

Domain (x-values) all real numbers, except numbers that make the denominator (bottom) zero.

ex: $\frac{x+6}{x+2} \leftarrow \begin{array}{r} 0 \neq x+2 \\ -2 \quad -2 \\ \hline -2 \neq x \end{array}$

-2 cannot be in domain, because denominator will equal ϕ , which makes the expression undefined.

Domain $\{x \mid x \neq -2\}$

Ex 1 Find the domain

$$\frac{(x+6)(x+4)}{(x+2)(x+4)}$$

Domain $\{x \mid x \neq -2, -4\}$

$$(x+2)(x+4) \neq 0$$

$$\begin{array}{r} x+2 \neq 0 \\ -2 \quad -2 \\ \hline x \neq -2 \end{array}$$

$$\begin{array}{r} x+4 \neq 0 \\ -4 \quad -4 \\ \hline x \neq -4 \end{array}$$

Lowest Terms of a Rational Expression

↳ when the greatest common factor is 1

Ex 2

Write rational expression in lowest terms

(a)

$$\frac{2x^2 + 7x - 4}{5x^2 + 20x}$$

$\swarrow a=2$ $\swarrow b=7$ $\swarrow c=-4$
 \rightarrow

$$\begin{array}{r|l} -8 & 7 \\ \hline 8(-1) & 8-1=7 \end{array}$$

↙

$$5x(x+4)$$

$$\frac{2x^2 + 8x}{2x} \left| \frac{-1x - 4}{-1} \right.$$

$$2x(x+4) - 1(x+4)$$

$$(x+4)(2x-1)$$

$$\frac{\cancel{(x+4)}(2x-1)}{5x \cancel{(x+4)}} = \boxed{\frac{2x-1}{5x}}$$

$$\text{Domain: } \{x \mid x \neq 0, -4\}$$

(b)

$$\frac{6-3x}{x^2-4} = \frac{3(2-x)}{(x-2)(x+2)} = \frac{3(-1)(-2+x)}{(x-2)(x+2)} = \frac{-3(x-2)}{\cancel{(x-2)}(x+2)}$$

diff. of squares

$$= \boxed{\frac{-3}{x+2}}$$

$$\text{Domain: } \{x \mid x \neq -2, +2\}$$

$$\frac{2x+4}{6} = \frac{2(x+2)}{6} = \boxed{\frac{x+2}{3}}$$