(Compound) Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- A: Future Value; Value after interest added
- P: principal; present value; Amount deposit for savings; Loan Amount
- r: interest rate (decimal)
- n: # times compounded each year
- t: time in years

Ex 7 Suppose $1000 is deposited in an account paying 4% interest per year compounded quarterly (4 times per year).

(a) Amount after 10 years.

$A = ?$ $P = 1000$ $r = .04$ $n = 4$ $t = 10$

$$A = 1000\left(1 + \frac{.04}{4}\right)^{4(10)} = \boxed{\$1488.86}$$

(b) Interest earned? $1488.86 - 1000 = \boxed{\$488.86}$

Compounded:	n
Annually	1
Semi-annual	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Continuous → new equation

Ex 8 Finding Present Value

Becky must pay a lump sum of $6000 in 5 years.

(a) Amount to deposit today at 3.1% compounded annually?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$6000 = P\left(1 + \frac{.031}{1}\right)^{(1 \times 5)}$$

$$\frac{6000}{1.031^5} = \frac{P(1.031)^5}{1.031^5}$$

$$\$5150.60 = P$$

(b) Becky only has $5000 now. What interest rate is necessary to reach $6000 in 5 years?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$6000 = 5000\left(1 + \frac{r}{1}\right)^{1 \times 5}$$

$$\frac{6000}{5000} = \frac{5000}{5000}(1+r)^5$$

$$\left(\frac{6}{5}\right)^{\frac{1}{5}} = (1+r)^{5\,\frac{1}{5}}$$

$$\left(\frac{6}{5}\right)^{\frac{1}{5}} = 1 + r$$

$$-1 \qquad -1$$

$$\left(\frac{6}{5}\right)^{\frac{1}{5}} - 1 = r \simeq .0371 \rightarrow \boxed{3.71\% \text{ interest rate}}$$

Ex 9 **Continuous Compounding**

e is an irrational number

$e \approx 2.718....$

$A = Pe^{rt}$ We are continuously PERT!

$5000 is deposited at 3% interest rate compounded continuously for 5 years. Future value at the end of 5 years?

$$A = Pe^{rt} = 5000\, e^{(.03 \times 5)}$$

$$= \boxed{\$\,5809.17}$$