Review 4.5 Solving Log / Exp Equations

(1) $5^{3x+5} = 2$ or $5^{3x+5} = 2^{2x}$

$\log 5^{3x+5} = \log 2$ (I) Take Log both sides

$(3x+5)\log 5 = \log 2$

$3x \log 5 + \cancel{5 \log 5} = \log 2$

$\quad\quad\quad - \cancel{5 \log 5} \quad - 5\log 5$

$$\frac{3x\cancel{\log 5}}{\cancel{3\log 5}} = \frac{\log 2 - 5\log 5}{3\log 5}$$

$$x = \frac{\log 2 - 5\log 5}{3\log 5}$$

EXACT

Calculator for Approximate answer

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(2) $\log_2 (x+3) = 4$ (I) Convert to Exponential

$2^4 = x+3$

$16 = x+3$

$-3 \quad\quad -3$

$\boxed{13 = x}$

CHECK:

$13+3 = 16$ ✓ OK ✓

(3) $\cancel{\log_3}(2x-4) = \cancel{\log_3}(x)$ (I) Chop Logs

$$2x - \cancel{4} = x$$
$$\quad\ \cancel{+4} \qquad +4$$

$$2x = \cancel{x} + 4$$
$$-x \quad \cancel{-x}$$

$$\boxed{x = 4}$$

Check:

$2(4) - 4 = 4$ + ✓

$x = 4$ + ✓

4.6 Exponential Growth or Decay → radioactive decay

bacteria, compound interest, population

$$y = y_0 e^{kt}$$

k is a Constant

y_0 = starting value

$k > 0$ growth

$k < 0$ decay

ex: Continuously compounded

$$A = Pe^{rt}$$

4.6 $\boxed{\text{Ex 1}}$ Day 1 (3)

	Year	Carbon Dioxide (ppm)
$x=0$	1990	353
$x=10$	2000	375
	2075	590
	2175	1090
$x=285$	2275	2000

Exponential Growth

$y = y_0 e^{kt}$

$y_0 = 353$

$y = 353 e^{kx}$

We know in 2275, the CO_2 level is 2000 ppm.

In 2275 $x = 285$ since $2275 - 1990 = 285$

$$\frac{2000}{353} = \frac{\cancel{353}}{\cancel{353}} e^{k285}$$

$$\ln\left(\frac{2000}{353}\right) = \cancel{\ln e}^{k285}$$

$$\frac{\ln\left(\frac{2000}{353}\right)}{285} = \frac{k\,\cancel{285}}{\cancel{285}}$$

$$\boxed{.00609} = k$$

$\ln e^{k285} = z$

$\log_e e^{k285} = z$

$e^z = e^{k285}$

$z = k285$

$\ln e^{k285} = k285$

$\boxed{\text{Ex 2}}$ Find the doubling time for Money \$

Interest rate, r, = 3%
Compounded Continuously $A = Pe^{rt}$
How long for the \$ to double?

$$A = Pe^{rt}$$

$$\frac{2P}{P} = \frac{Pe^{rt}}{P}$$

$$2 = e^{.03t}$$

$$\ln 2 = \cancel{\ln e}^{.03t}$$

$$\frac{\ln 2}{.03} = \frac{.0\cancel{3}t}{\cancel{.03}}$$

$$23.10 = t$$

Takes about 23 years to double.

$\Rightarrow \ln e^{.03t}$

$$\log_e e^{.03t} = Z$$

$$e^Z = e^{.03t}$$

$$Z = .03t$$

$$\ln e^{.03t} = .03t$$

$\boxed{\text{Ex 3}}$ World population $f(t) = 6.947e^{.00745t}$

Based on the model, what will be population in 2025? $t = 0$ in 2010

We will use $t = 15$ for 2025.

$$f(t) = 6.947e^{.00745(15)} \approx 7.768 \text{ Billion}$$

4.6 $\boxed{\text{Ex4}}$ Radioactive Decay

600g of a radioactive substance.
3 years later only 300g remain.

(a) Find exponential function that models decay.

$$y = y_0 e^{kt} \qquad y_0 = 600$$

$$y = 600e^{kt}$$

$$\frac{300}{600} = \frac{600e^{k3}}{600}$$

$$.5 = e^{k3}$$

$$\ln .5 = \ln e^{k3}$$

$$\frac{\ln .5}{3} = \frac{k\cancel{3}}{\cancel{3}}$$

$$\boxed{-0.231 \approx k}$$

(b) Equation is $y = 600e^{-0.231t}$

How much remains after 6 years?

$$y = 600e^{-0.231(6)}$$

$$\boxed{y \approx 150 \text{ grams}}$$