

Unit 9 7.2 Arithmetic Sequences + Series

①

Arithmetic Sequence Common difference, d,
between terms.

$$5, 9, 13, 17, \dots$$

$$\begin{array}{cccc} \vee & \vee & \vee & \\ +4 & +4 & +4 & \end{array}$$

Common difference
 $d = 4$

$$d = a_{n+1} - a_n = 13 - 9 = 4$$

Ex 1 Finding the common difference

$$-9, -7, -5, -3, -1, \dots$$

$$\begin{array}{cccc} \vee & \vee & & \\ +2 & +2 & & \end{array}$$

$$d = -7 - (-9) = -7 + 9 = \boxed{2}$$

Ex 2 Find the first 5 terms of the arithmetic sequence

① The first term is 7, the common difference is -3

$$a_1 = 7 \quad d = -3$$

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & \\ 7 & 4 & 1 & -2 & -5 & \\ \vee & \vee & \vee & \vee & & \\ -3 & -3 & -3 & -3 & & \end{array}$$

n th term of an arithmetic sequence

$$a_n = a_1 + (n-1)d$$

Ex 3 Determine a_{13} and a_n for the arithmetic sequence $-3, 1, 5, 9, \dots$

$$a_1 = -3 \quad d = 4$$

$$\begin{aligned} a_n &= -3 + (n-1)4 \\ &= -3 + 4n - 4 \end{aligned}$$

$$a_n = -7 + 4n = 4n - 7$$

$$a_{13} = -7 + 4(13) = 45$$

Arithmetic Series is the sum of the terms of an arithmetic sequence.

$$S_n = \frac{n}{2} (a_1 + a_n) \quad \text{or} \quad S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

Use when first and last terms known

Use when only 1st term is known

Ex 7. Arithmetic Sequence $-9, -5, -1, 3, 7, \dots$

(a) Evaluate S_{12}

$$a_1 = -9 \quad d = 4$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d] \quad \text{2nd formula}$$

$$S_{12} = \frac{12}{2} [2(-9) + (12-1)4]$$

$$= 6 [-18 + 44] = \boxed{156}$$

(b) Evaluate the sum of the first 60 positive integers. $1, 2, 3, \dots, 60$

Know 1st and last terms so use 1st formula.

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_{60} = \frac{60}{2} (1 + 60)$$

$$= 30 (61) = \boxed{1830}$$

Ex 9 Evaluate the sum

(a) $\sum_{i=1}^{10} (4i+8) = 12+16+\dots = 300$

$i=1$

i	$4i+8$
1	$4(1)+8 = 12$
2	$4(2)+8 = 16$
\vdots	

(4)

Ex 9 Continuedfind a_1 and a_{10}

$$a_1 = (4(1) + 8) = 12$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$a_{10} = 4(10) + 8 = 48$$

$$S_{10} = \frac{10}{2} (12 + 48)$$

$$= 5(60)$$

$$S_{10} = 300$$

(b) Evaluate the sum

$$\sum_{k=3}^9 (4 - 3k) = -98$$