

Unit 9 7.3 Geometric Sequences + Series

①

Geometric Sequence,

$$2, 4, 6, 8$$

$\begin{array}{c} \downarrow \\ +2 \end{array}$
 $\begin{array}{c} \downarrow \\ +2 \end{array}$
 $\begin{array}{c} \downarrow \\ +2 \end{array}$

Common difference, $d = 2$ Arithmetic

$$2, 4, 8, 16$$

$\begin{array}{c} \downarrow \\ \times 2 \end{array}$
 $\begin{array}{c} \downarrow \\ \times 2 \end{array}$
 $\begin{array}{c} \downarrow \\ \times 2 \end{array}$

Common ratio, $r = 2$ Geometric

Geometric Series Sum of the terms of a geometric sequence

nth term of geometric sequence (explicit formula)

$$a_n = a_1 r^{n-1}$$

Ex 1 Finding nth term of a Geometric Sequence

$$1, 2, 4, 8, 16, \dots$$

Find 20th term.

$$a_n = a_1 r^{n-1}$$

$$a_1 = 1$$

$$r = 2$$

$$a_n = (1) 2^{n-1}$$

$$= a_n = 2^{n-1}$$

$$a_{20} = 2^{20-1}$$

$$= 2^{19}$$

$$= 524,288 = a_{20}$$

Ex 2 Determine a_5 and a_n for

$$4, 12, 36, 108$$

$\begin{array}{c} \downarrow \\ \times 3 \end{array}$
 $\begin{array}{c} \downarrow \\ \times 3 \end{array}$
 $\begin{array}{c} \downarrow \\ \times 3 \end{array}$

$$a_1 = 4 \quad r = 3$$

$$a_n = a_1 r^{n-1} = 4(3^{n-1}) = a_n$$

$$a_5 = 4(3^{5-1}) = 4(3^4) = 324$$

Sum of the first n terms of
a Geometric Sequence

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad r \neq 1$$

Ex 5 Geometric Sequence 1, 2, 4, 8, ...

Find sum of 1st 20 terms

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad a_1 = 1 \quad r = 2$$

$$S_{20} = \frac{1(1-2^{20})}{(1-2)} = 10\,485\,75$$

Ex 6 Evaluate $\sum_{i=1}^6 2 \cdot 3^i$

$$a_1 = 2 \cdot 3^1 = 6 \quad > \times 3$$

$$a_2 = 2 \cdot 3^2 = 18 \quad > \times 3$$

$$a_3 = 2 \cdot 3^3 = 54$$

$$r = 3$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_6 = \frac{6(1-3^6)}{(1-3)} = \boxed{2184}$$