

Notes 6.3
Applications of Linear Equations

STRATEGY FOR SOLVING WORD PROBLEMS

- Step 1 Read the problem carefully several times until you can state in your own words what is given and what the problem is looking for. Let x (or any variable) represent one of the unknown quantities in the problem.
- Step 2 If necessary, write expressions for any other unknown quantities in the problem in terms of x .
- Step 3 Write an equation in x that models the verbal conditions of the problem.
(Review Table 6.2 p. 332 Algebraic Translations of English Phrases)
- Step 4 Solve the equation and answer the problem's question.
- Step 5 **Check** the solution in the **original wording of the problem**, not in the equation obtained from the words.

Example 1

One number exceeds another by 18. The sum of the numbers is 84. What are the two numbers?

$x + 18$ ↑ x

x $x - 18$
 $x + x - 18 = 84$

$2x = 102$
 $x = 51$

$x + x + 18 = 84$
 $2x + 18 = 84$
 $\quad -18 \quad -18$
 $\hline 2x = 66$
 $\frac{2x}{2} = \frac{66}{2}$
 $x = 33$

$y = 33 + 18$
 $y = 51$

CHECK
51 exceeds 33 by 18 ✓

Example 2

When a number is decreased by 65% of itself, the result is 28. What is the number?

x $x - .65x$

$x - .65x = 28$
 $x(1 - .65) = 28$
 $.35x = 28$
 $\frac{.35x}{.35} = \frac{28}{.35}$
 $x = 80$

80 is decreased by $.65(80)$
CHECK by $-.65(80)$
 $80 - .65(80) \stackrel{?}{=} 28$
 $80 - 52 =$
 $28 = 28$ ✓

For examples 3 and 4, write the English phrase as an algebraic expression. Then simplify the expression. Let x represent the number.

Example 3

A number decreased by the difference between twelve and the number

x $x - (12 - x)$
 $x - 12 + x$
 $2x - 12$

Example 4

Five decreased by four times the sum of a number and ten

$$5 - 4(x + 10)$$

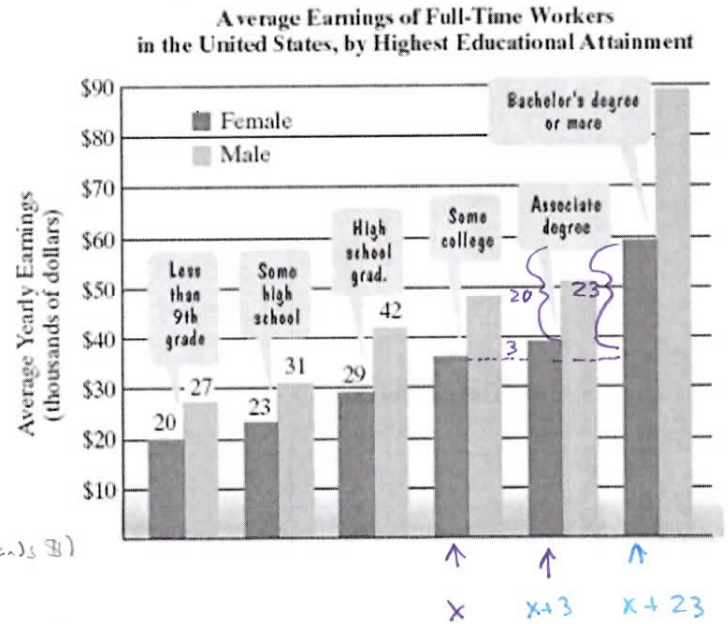
$$5 - 4x - 40$$

$$-35 - 4x$$

$-4x - 35$

Example 5 Similar to Ex 1 pg 333

The average yearly salary of a woman with an associate degree exceeds that of a woman with some college by \$ 3 thousand. The average yearly salary of a woman with a bachelor's degree or more exceeds that of a woman with some college by \$ 23 thousand. Combined, three women with each of these educational attainments earn \$ 134 thousand. Find the average yearly salary of women with each of these levels of education.



- ① $x =$ salary woman some college (thousands \$)
- ② $x + 3 =$ associate
- $x + 23 =$ bach

③ $x + (x + 3) + (x + 23) = 134$

④
$$\begin{array}{r} 3x + 26 = 134 \\ -26 \quad -26 \\ \hline 3x = 108 \\ \hline x = 36 \end{array}$$

$x = 36$ thousand \$

Some college

Associate

$= 39$ thousand \$

$= 59$ thousand \$

CHECK Makes sense on graph

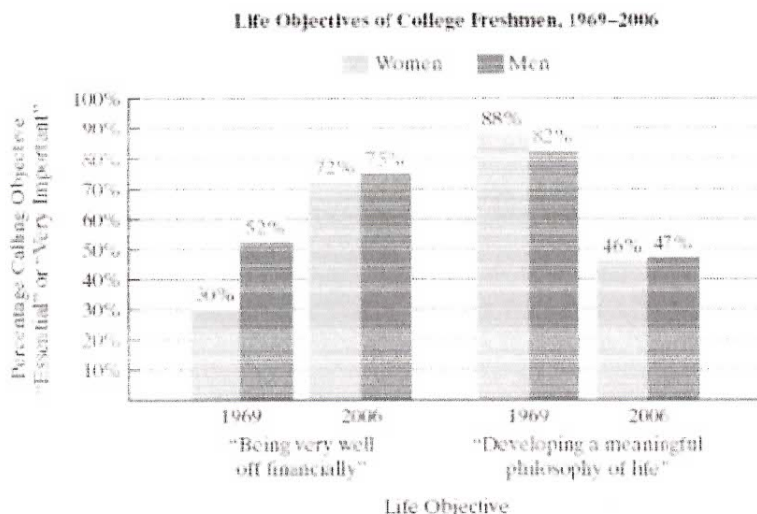
$36 + 39 + 59 = 134$ ✓

STOP

See Ex 2 ps 334

Example 6

The bar graph shows that the freshmen class of 2006 was less interested in developing a philosophy of life than the freshmen of 1969 had been. In 1969, 88% of the women considered this objective essential or very important. Since then, this percentage has decreased by approximately 1.1 each year. If this trend continues, by which year will only 33% of female freshmen consider "developing a meaningful philosophy of life" essential or very important?



- ① $x = \#$ years after 1969 when 33% women meaningful phil of life important
- ② No other unknown quantities
- ③ Write equation Constant Rate of Δ Linear
 $y = mx + b$
 $33 = -1.1x + 88$

- ④ Solve eqn

$$\frac{-55}{-1.1} = \frac{-1.1x}{-1.1}$$

$$50 = x$$
 Answer: $1969 + 50 = \boxed{2019}$
- ⑤ Check
 $88 - 1.1(50) = 33$
 $88 - 55 = 33$

Example 7 See Ex 3 ps 335

You are choosing between two long-distance telephone plans. Plan A has a monthly fee of \$ 15 with a charge of \$ 0.08 per minute for all long-distance calls. Plan B has a monthly fee of \$ 3 with a charge of \$ 0.12 per minute for all long-distance calls. For how many minutes of long-distance calls will the costs for the two plans be the same?

- ① $x = \#$ minutes of long-distance calls when both plans cost same
- ② No other unknown quantities
- ③
$$15 + .08x = 3 + .12x$$

$$\begin{array}{r} 15 + .08x = 3 + .12x \\ -3 \quad -.08 \quad -3 \quad -.08 \\ \hline 12 = .04x \\ \frac{12}{.04} = \frac{.04x}{.04} \\ 300 = x \end{array}$$
- ④

	<u>A</u>	<u>B</u>
monthly	\$15	\$3
/minute	\$.08/min	\$.12/min

$$\boxed{x = 300 \text{ minutes}}$$

- ⑤ Check Cost A $15 + .08(300) = \$39$ Cost B $3 + .12(300) = \$39$ ✓

Like Ex 4 p. 336

Example 8 $x - .30x$

After a 30% price reduction, you purchase a new computer for \$ 840. What was the computer's price before the reduction?

①
②

x

③ $1x - .30x = 840$

④ $x(1-.3) = 840$

$$\frac{.7x}{.7} = \frac{840}{.7}$$

$$x = \$1200$$

⑤ CHECK

$$1200 - .3(1200)$$

$$1200 - 360 = 840 \checkmark$$

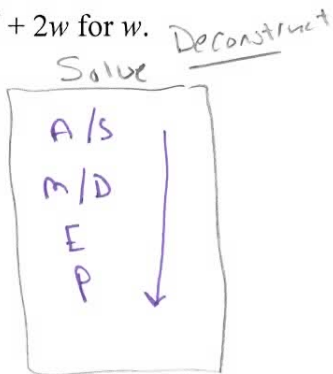
Example 9 (Ex 5)

Solve the formula $P = 2l + 2w$ for w . Deconstruct

$$\begin{array}{r} P = 2l + 2w \\ -2l \quad -2l \\ \hline \end{array}$$

$$\frac{P-2l}{2} = \frac{2w}{2}$$

$$\frac{P-2l}{2} = w$$



Example 10

Solve the formula $S = B + \frac{1}{2}Pl$ for P .

$$\begin{array}{r} S = B + \frac{1}{2}Pl \\ -B \quad -B \\ \hline \end{array}$$

$$2(S-B) = \left(\frac{1}{2}Pl\right)2$$

$$\frac{2(S-B)}{L} = \frac{P}{L}$$

$$\frac{2(S-B)}{L} = P$$

CHECK:
Solve for S

Example 11

Solve the formula $A = P(1+rt)$ for r

$$\frac{A}{P} = \frac{P(1+rt)}{P}$$

$$\frac{A}{P} = 1 + rt$$

$$\frac{A}{P} - 1 = \frac{rt}{t}$$

$$\frac{A}{P} - 1 = r$$

Check: Solve for A
 $\frac{A}{P} - 1 = rt$
 $\frac{A}{P} = rt + 1$
 $A = P(rt+1) \checkmark$

Example 12

Solve the formula $X = Y + AY$ for A

$$\begin{array}{r} X = Y + AY \\ -Y \quad -Y \\ \hline \end{array}$$

$$\frac{X-Y}{Y} = \frac{AY}{Y}$$

$$\frac{X-Y}{Y} = A$$

$$\frac{X}{Y} - 1$$

CHECK:
Solve for X
 $X - Y = AY$
 $X = AY + Y \checkmark$