Section 8.3 Notes- Compound Interest

The Difference between Simple Interest and Compound Interest:

Simple Interest	Interest is paid on your **investment or principal** and **NOT** on any **interest added**
Compound Interest	Interest paid on **BOTH** on the **principal** and on **all interest that has been added**

SIMPLE Interest	COMPOUND Interest
Imagine you deposit $1000 in Honest John's Money Holding Service, which promises to pay 5% interest each year.	Suppose you place $1000 in a bank account that pays the same 5% interest per year. But instead of paying you the interest directly, the bank adds the interest to your account.
After one year:	After one year:
After the second year:	After the second year:
After the third year:	After the third year:

Example: Savings Bond

While banks almost always pay compound interest, bonds usually pay simple interest. Suppose you invest $1000 in a savings bond that pays simple interest of 10% per year. How much total interest will you receive in 5 years? If the bond paid compound interest, would you receive more or less total interest? Explain.

End of	Simple Interest		Compound Interest	
Year	Interest	New Balance	Interest	New Balance

Calculating the Amount in an account for Compound Interest Paid _n_ times a year
If you deposit _P_ dollars at a rate _r_ (in decimal form) subject to compound interest paid _n_ times a year, then the amount, _A_, of money in the account after _t_ years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{(nt)}$$

Annually n= ___ Semi-Annually n=___ Quarterly n=___ Monthly n=___Weekly n=___

Examples:

1) You deposit $1000 in a savings account at a bank that has a rate of 4%.

   a) Find the amount, A, of money in the account after 5 years subject to interest compounded once a year. Round to the nearest cent.

   b) Find the interest.

2) You deposit $4200 in a savings account that has a rate of 4%. The interest is compounded quarterly.

    a) How much money will you have after ten years? Round to the nearest cent.

    b) Find the interest after ten years.

Calculating the Amount in an account when Compounding Interest Continuously
If you deposit P dollars at a rate r (in decimal form) and the interest is compounded continuously, then the amount, A, of money in the account after t years is given by

$$A = Pe^{(rt)}$$ where $e \approx 2.71828$ (use the e^x button on your calculator)

Examples:

3) A sum of $10,000 is invested at an annual rate of 8%. Find the balance in the account after 5 years subject to

    a) quarterly compounding

    b) compounding continuously

4) Calculating Present Value.
How much money should be deposited today in an account that earns 7% compounded weekly so that it will accumulate to $10,000 in eight years?

More Examples:

1) Find the accumulated value of an investment of $5,000 for 10 years at an interest rate of 3.75% if the money is

   a) compounded quarterly

   b) compounded continuously

2) Suppose that you have $10,000 to invest. Which investment yields the greater return over 5 years: 7.55% compounded quarterly or 7.6% compounded semi-annually?

3) How much money should be deposited today in an account that earns 5% compounded quarterly so that it will accumulate to $15,000 in ten years?

4) You deposit $5000 in a bank account that pays an APR of 3% and compounds interest monthly. How much money will you have after 5 years? Compare this amount to the amount you'd have if interest were paid only once each year.

5) You deposit $100 in an account with an APR of 8% and continuous compounding. How much will you have after 10 years?

6) College Fund at 3%

   Suppose you put money in an investment with an interest rate of APR = 3%, compounded annually, and leave it there for the next 18 years. How much would you have to deposit now to realize $100,000 after 18 years?

7) Repeat Example 6, but with an interest rate of 5% and monthly compounding. Compare the results.

Section 8.3 Day 2

Effective Annual Yield- (or the effective rate) is the simple interest rate that produces the same amount of money in an account at the end of one year as when the account is subjected to compound interest at a stated rate.

The effective annual yield is often included in the information about investments or loans. Because it's the true interest rate you are earning or paying, it is the number you should pay attention to. **If you are selecting the best investment from two or more investments, the best choice is the account with the greatest effective annual yield.**

When borrowing money, the effective rate or effective annual yield is called the **annual percentage rate. If all other factors are equal and you are borrowing money, select the option with the least annual percentage rate.**

The stated interest rate is called the **nominal rate**.

Example:
1) You deposit $6000 in an account that pays 10% interest compounded monthly.

   a) Find the future value after one year.

   b) Determine the effective annual yield. (This is a simple interest rate.)

   We use the future value formula for simple interest to determine the simple interest rate that produces the future value found in part a for the deposit of $6000 after one year

2) You deposit $7,500 in an account that pays 4.75% interest compounded monthly.

   a) Find the future value after one year.

   b) Determine the effective annual yield.

Calculating Effective Annual Yield

Suppose that an investment has a nominal interest rate, *r*, in decimal form, pays compound interest *n* times per year. The investment's effective annual yield, Y, in decimal form is given by

$$Y = \left(1 + \frac{r}{n}\right)^{n} - 1$$ the decimal form of Y should then be converted to a percent.

Example:

3) What is the effective annual yield of an account paying 8% compounded quarterly?

4) Determine the effective annual yield, to the nearest tenth of a percent, for each investment. Then select the better investment.

    a) 6% compounded semi-annually; 5.85% compounded daily

    b) 8% compounded monthly; 8.25% compounded quarterly

Additional Examples:

5) At the time of her grandson's birth, a grandmother deposits $5,000 in an account that pays 6% compounded monthly. What will be the value of the account at the child's 21st birthday, assuming no other deposits or withdrawals are made during this period?

6) Parents wish to have $120,000 available for a child's education. If the child is now 1 year old, how much money must be set aside at 8% compounded monthly to meet their financial goal when the child is 18?