Skiers seek soft, freshly fallen snow because it gives a smooth “floating” ride. Of course, the ride up the mountain isn’t nearly as much fun—especially if the ski lifts are on the fritz!
# Chapter 1 Overview

This chapter introduces students to the concept of functions. Lessons provide opportunities for students to explore functions, including linear, exponential, quadratic, linear absolute value functions, and linear piecewise functions through problem situations, graphs, and equations. Students will classify each function family using graphs, equations, and graphing calculators. Each function family is then defined and students will create graphic organizers that represent the graphical behavior and examples of each.

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<td>1.1 Understanding Quantities and Their Relationships</td>
<td>N.Q.2 F.LE.1.b</td>
<td>1</td>
<td>This lesson provides opportunities for students to explore quantities and their relationships with each other through eight different problem situations. Questions ask students to identify the independent and dependent quantity for each and match a numberless graph to each scenario. Questions then focus students to compare and contrast the different graphs.</td>
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<td>1.2 Analyzing and Sorting Graphs</td>
<td>F.IF.1 F.IF.5</td>
<td>1</td>
<td>This lesson provides twenty-two different graphs (18 functions and 4 non-functions) for students to analyze and compare. Questions ask students to sort the graphs into different groups based on their own rationale, and then students will identify the grouping rationale of others. Finally, they distinguish between graphs of functions and non-functions.</td>
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<td>F.IF.5, F.IF.9, A.REI.10, F.IF.1, F.IF.2, F.IF.7.a</td>
<td>2</td>
<td>This lesson provides opportunities for students to explore graphical behavior and the form of the equation for different functions. Student will sort the eighteen graphs of functions identified in the previous lesson according to specific graphical behaviors and use a graphing calculator to match an equation to each graph. Questions then ask students to sort the graphs based on the form of the equations. This leads students to identifying one of five different functions: linear, exponential, quadratic, linear absolute value, and linear piecewise. Finally, students paste each graph with its corresponding equation into the appropriate graphic organizer and describe the graphical behavior of each function.</td>
</tr>
<tr>
<td>1.4</td>
<td>F.IF.1, F.IF.4, F.IF.7.a, F.IF.9, F.LE.1.b, F.LE.2, A.CED.2</td>
<td>1</td>
<td>This lesson revisits the eight scenarios presented in the first lesson of this chapter. Questions ask students to complete a table by identifying the function family represented by the scenario and the attributes of the function with respect to graphical behavior.</td>
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# Skills Practice Correlation for Chapter 1

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<td>1.3 Recognizing Algebraic and Graphical Representations of Functions</td>
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<td>1.4 Recognizing Functions by Characteristics</td>
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A Picture Is Worth a Thousand Words
Understanding Quantities and Their Relationships

LEARNING GOALS
In this lesson, you will:
• Understand quantities and their relationships with each other.
• Identify the independent and dependent quantities for a problem situation.
• Match a graph with an appropriate problem situation.
• Label the independent and dependent quantities on a graph.
• Review and analyze graphs.
• Describe similarities and differences among graphs.

ESSENTIAL IDEAS
• There are two quantities that change in problem situations.
• When one quantity depends on another, it is said to be the dependent quantity.
• The quantity that the dependent quantity depends upon is called the independent quantity.
• The independent quantity is used to label the x-axis.
• The dependent quantity is used to label the y-axis.
• Graphs can be used to model problem situations.

KEY TERMS
• dependent quantity
• independent quantity

MATHEMATICS COMMON CORE STANDARDS
N-Q Quantities
Reason quantitatively and use units to solve problems.

2. Define appropriate quantities for the purpose of descriptive modeling.

F-LE Linear, Quadratic, and Exponential Models
Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
Overview

Several problem situations are presented for students to identify the independent and dependent quantities. They are then given graphs that model each scenario and will match each graph to the appropriate scenario and label each axis using the independent and dependent quantities. Several questions are posed which focus the students on various characteristics of each graph, their similarities and differences. Some graphical behaviors are compared and discussed, such as increasing, decreasing, curved, linear, smooth (continuous), and maximum and minimum values.
Use the graph shown to answer each question.

Emma bought a new video game. The graph describes the number of hours Emma spent playing the game over a period of 7 days.

1. How would you label the x-axis?
   The x-axis would be labeled Time (days).

2. How would you label the y-axis?
   The y-axis would be labeled Time (number of hours playing game).

3. Explain your reasoning for choosing each label.
   The scenario stated Emma played the game over a period of 7 days. I chose the x-axis to represent number of days because each point lies on a different day.

4. What does the highest point on the graph represent with respect to the scenario?
   The highest point describes the number of hours Emma played the video game on the 3rd day, four hours.

5. What does the lowest point on the graph represent with respect to the scenario?
   The lowest point describes the number of hours Emma played the video game on the 7th day, zero hours.
1.1 Understanding Quantities and Their Relationships

LEARNING GOALS

In this lesson, you will:
• Understand quantities and their relationships with each other.
• Identify the independent and dependent quantities for a problem situation.
• Match a graph with an appropriate problem situation.
• Label the independent and dependent quantities on a graph.
• Review and analyze graphs.
• Describe similarities and differences among graphs.

KEY TERMS

• dependent quantity
• independent quantity

How interesting would a website be without pictures or illustrations? Does an inviting image on a magazine cover make you more likely to buy it? Pictures and images aren’t just for drawing your attention, though. They also bring life to text and stories.

There is an old proverb that states that a picture is worth a thousand words. There is a lot of truth in this saying—and images have been used by humans for a long time to communicate. Just think: would you rather post a story of your adventure on a social media site, or post one picture to tell your thousand-word story in a glance?
Problem 1
In the first activity, students will identify the independent and dependent quantity in five different statements and describe their identification process. Next, students identify the independent and dependent quantities in eight different scenarios.

Grouping
- Ask a student to read the introduction before Question 1. Discuss as a class.
- Have students complete Questions 1 and 2 independently. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2
Which quantity forces the other quantity to change?

PROBLEM 1 What’s the Dependency?

Have you ever planned for a party? You may have purchased ice, gone grocery shopping, selected music, made food, or even cleaned in preparation. Many times, these tasks depend on another task being done first. For instance, you wouldn’t make food before grocery shopping, now would you?

Let’s consider the relationship between:

- the number of hours worked and the money earned.
- your grade on a test and the number of hours you studied.
- the number of people working on a particular job and the time it takes to complete a job.
- the number of games played and the number of points scored.
- the speed of a car and how far the driver pushes down on the gas pedal.

There are two quantities that are changing in each situation. When one quantity depends on another in a problem situation, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity.

1. Circle the independent quantity and underline the dependent quantity in each statement.

2. Describe how you can determine which quantity is the independent quantity and which quantity is the dependent quantity in any problem situation.

   The independent quantity is the quantity that stands alone and is not changed by the other quantities.
   The dependent quantity depends on the independent quantity. The independent quantity causes a change in the dependent quantity.
Grouping
Have students complete Question 3 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Question 3

Something’s Fishy
- Does the amount of time change or determine the number of gallons of water emptied, or does the gallons of water emptied change or determine the amount of time?
- How is time measured in this scenario?
- How is water measured in this scenario?

Smart Phone, but Is It a Smart Deal?
- Does the amount of time the money was borrowed change or determine the amount of interest paid, or does the amount of interest paid change or determine the amount of time the money was borrowed?
- How is time measured in this scenario?
- How is interest measured in this scenario?

3. Read each scenario and then determine the independent and dependent quantities. Be sure to include the appropriate units of measure for each quantity.

Something’s Fishy
Candice is a building manager for the Crowley Enterprise office building. One of her responsibilities is cleaning the office building’s 200-gallon aquarium. For cleaning, she must remove the fish from the aquarium and drain the water. The water drains at a constant rate of 10 gallons per minute.
- independent quantity: time (minutes)
- dependent quantity: water (gallons)

Smart Phone, but Is It a Smart Deal?
You have had your eye on an upgraded smart phone. However, you currently do not have the money to purchase it. Your cousin will provide the funding, as long as you pay him interest. He tells you that you only need to pay $1 in interest initially, and then the interest will double each week after that. You consider his offer and wonder: is this really a good deal?
- independent quantity: time (weeks)
- dependent quantity: interest (dollars)
Can’t Wait to Hit the Slopes!

- Does the amount of time he was on the ski lift change or determine the distance the lift traveled, or does the distance the lift traveled change or determine the amount of time he was on the ski lift?
- How is time measured in this scenario?
- How is distance measured in this scenario?

It’s Magic

- Does the number of cuts in the rope change or determine the length of each piece of rope, or does the length of each piece of rope change or determine the number of cuts in the rope?
- How is the length of the pieces of rope measured in this scenario?

Can’t Wait to Hit the Slopes!

Andrew loves skiing—he just hates the ski lift ride back up to the top of the hill. For some reason the ski lift has been acting up today. His last trip started fine. The ski lift traveled up the mountain at a steady rate of about 83 feet per minute. Then all of a sudden it stopped and Andrew sat there waiting for 10 minutes! Finally, the ski lift began to ascend up the mountain to the top.

- independent quantity: 
  time (minutes)
- dependent quantity: 
  distance (feet)

It’s Magic

The Amazing Aloysius is practicing one of his tricks. As part of this trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 20-foot rope and then cuts it in half. He then takes one of the halves and cuts that piece in half. He repeats this process until he is left with a piece so small he can no longer cut it. He wants to know how many total cuts he can make and the length of each remaining piece of rope after the total number of cuts.

- independent quantity: 
  number of cuts
- dependent quantity: 
  length of each piece of rope (feet)
**Baton Twirling**

- If Jill wants to twirl around more times, what impact will it have on the maximum height of the baton?
- How is time measured in this scenario?
- How is the height of the baton measured in this scenario?

**Music Club**

- If Jermaine wants to purchase more songs, what impact will it have on the cost?
- How is cost measured in this scenario?

---

**Baton Twirling**

Jill is a drum major for the Altadena High School marching band. She has been practicing for the band’s halftime performance. For the finale, Jill tosses her baton in the air so that it reaches a maximum height of 22 feet. This gives her 2 seconds to twirl around twice and catch the baton when it comes back down.

- independent quantity: time (seconds)
- dependent quantity: height of baton (feet)

---

**Music Club**

Jermaine loves music. He can lip sync almost any song at a moment’s notice. He joined *Songs When I Want Them*, an online music store. By becoming a member, Jermaine can purchase just about any song he wants. Jermaine pays $1 per song.

- independent quantity: number of songs
- dependent quantity: cost (dollars)
A Trip to School

- Can Myra change the distance she has to walk to school, or can she change the time it takes to walk to school?
- How is time measured in this scenario?
- How is distance measured in this scenario?

Jelly Bean Challenge

- Does Mr. Wright determine the number of jelly beans guessed, or the number of jelly beans they are off by?

A Trip to School

On Monday morning, Myra began her 1.3-mile walk to school. After a few minutes of walking, she walked right into a spider's web—and Myra hates spiders! She began running until she ran into her friend Tanisha. She stopped and told Tanisha of her adventurous morning and the icky spider's web! Then they walked the rest of the way to school.

- independent quantity: time (minutes)
- dependent quantity: distance traveled (miles)

Jelly Bean Challenge

Mr. Wright judges the annual Jelly Bean Challenge at the summer fair. Every year, he encourages the citizens in his town to guess the number of jelly beans in a jar. He keeps a record of everyone's guesses and the number of jelly beans that each person's guess was off by.

- independent quantity: number of jelly beans guessed
- dependent quantity: number of jelly beans the guess is off by
Problem 2
Students are given eight different numberless graphs and will match each graph with the appropriate scenario from Problem 1. They then label each axis on every graph using the independent and dependent quantities, including the units of measurement.

Grouping
- Ask a student to read the introduction before Question 1. Discuss as a class.
- Have students complete Question 1 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Question 1
- Why did you decide to use this graph to describe this scenario?
- What words in the scenario helped you to decide this was the appropriate graph?
- Could more than one graph model this scenario? Why or why not?
- Did you need to use any graph twice?
- Is there any scenario that cannot be modeled using one of the graphs?
- How did you decide the label for the x-axis of the graph?
- How did you decide the label for the y-axis of the graph?

PROBLEM 2 Matching Graphs and Scenarios

While a person can describe the monthly cost to operate a business, or talk about a marathon pace a runner ran to break a world record, graphs on a coordinate plane enable people to see the data. Graphs relay information about data in a visual way. If you read almost any newspaper, especially in the business section, you will probably encounter graphs.

| Points on a coordinate plane that are or are not connected with a line or smooth curve model, or represent, a relationship in a problem situation. In some problem situations, all the points on the coordinate plane will make sense. In other problem situations, not all the points will make sense. So, when you model a relationship on a coordinate plane, it is up to you to consider the situation and interpret the meaning of the data values shown. |

1. Cut out each graph on the following pages. Then, analyze each graph, match it to a scenario, and tape it next to the scenario it matches. For each graph, label the x- and y-axes with the appropriate quantity and unit of measure. Then, write the title of the problem situation on each graph.

- Is the independent quantity located on the x-axis or the y-axis? Does it make a difference? Explain.
- Is the dependent quantity located on the x-axis or the y-axis? Does it make a difference? Explain.
1.1 Understanding Quantities and Their Relationships

**Graph A**

- **Music Club**
- **Variables:** Number of Songs (x), Cost (dollars) (y)

**Graph B**

- **Smart Phone, but Is It a Smart Deal**
- **Variables:** Time (weeks) (x), Interest (dollars) (y)

**Graph C**

- **Jelly Bean Challenge**
- **Variables:** Number of Jelly Beans Guessed (x), Number of Jelly Beans the Guess Is off By (y)

**Graph D**

- **It’s Magic**
- **Variables:** Number of Cuts (x), Length of Each Piece of Rope (feet) (y)
Graph E
A Trip to School

Graph F
Baton Twirling

Graph G
Can’t Wait to Hit the Slopes!

Graph H
Something’s Fishy
Problem 3
Students will examine each graph and answer questions related to similarities, differences, the placement of labels on each axis, and discrete or continuous data patterns.

Grouping
• First, have students complete Questions 1 through 4 with a partner. Then share the responses as a class.
• Next, have students complete Question 5 with a partner. Then share the responses as a class.
• Finally, have students complete Question 6 independently. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4
• Is the independent quantity always located on the same axis?
• Is the dependent quantity always located on the same axis?
• Which graphs contained straight lines?
• Which graphs contained curved lines?
• How would you describe the behavior of the graph from left to right?
• Which graphs could be described as increasing? Why?

PROBLEM 3 Oh, Say, Can You See (in the Graphs)!
Now that you have matched a graph with the appropriate problem situation, let’s go back and examine all the graphs.

1. What similarities do you notice in the graphs?
   Answers will vary.
   • The independent quantity is graphed on the x-axis while the dependent quantity is graphed on the y-axis.
   • All the graphs are continuous.

2. What differences do you notice in the graphs?
   Answers will vary.
   • Some graphs contain straight lines, while some contain curves.
   • Some graphs seem to move up as they go from left to right, some move down from left to right.
   • Some graphs are made of pieces that go up, go down, or stay constant from left to right.

3. How did you label the independent and dependent quantities in each graph?
   I labeled the independent quantity on the x-axis and the dependent quantity on the y-axis in each graph.

4. Analyze each graph from left to right. Describe any graphical characteristics you notice.
   Answers will vary.
   • Some graphs only increase.
   • Some graphs only decrease.
   • Some graphs both increase and decrease.
   • Some graphs have a minimum or maximum value.
   • Some graphs increase or decrease at a constant rate.

• Which graphs could be described as decreasing? Why?
• Are any graphs both increasing and decreasing?
• Is it possible for a graph to be both increasing and decreasing at the same time?
• Can the curves on the graph be described as smooth curves? Why or why not?
• Which graphs have a maximum value?
Guiding Questions for Share Phase, Question 5

- Based on the scenarios, why do both the Smart Phone, but Is It a Smart Deal? and Music Club graphs increase?
- Based on the scenarios, why is the Smart Phone, but Is It a Smart Deal? a smooth curve, but the Music Club graph is a straight line?
- Based on the scenarios, why do both the Something’s Fishy and It’s Magic graphs decrease?
- Based on the scenarios, why is the Something’s Fishy graph a straight line, but the It’s Magic graph is a smooth curve?
- Based on the scenarios, why do both the Baton Twirling and Jelly Bean Challenge graphs increase then decrease?
- Based on the scenarios, why is the Baton Twirling graph a smooth curve, but the Jelly Bean Challenge graph a straight line?

Guiding Questions for Share Phase, Question 6

- Can your graph be described as increasing or decreasing?
- Is your graph curved or linear in nature?
- Does your graph contain any horizontal line segments? If so, what does this represent in the scenario?

- How many different pieces are on your graph?
- Does your graph contain any parallel line segments? What does this imply with respect to the scenario?
- If your graph contained a line segment having a negative slope, what would this imply with respect to the scenario?
- What point on your graph represents Myra’s home?
- What point on your graph indicates that Myra arrived at school?
Check for Students’ Understanding

Two graphs are shown.

- One graph describes Molly’s height in inches over a period of years.
- One graph describes Molly’s weight in pounds over a period of years.

1. Match each graph with the appropriate scenario and explain your reasoning.
   Graph 1 describes Molly’s weight over a period of years because weight can increase and decrease.
   Graph 2 describes Molly’s height over a period of years because height eventually reaches a maximum and then remains the same.

2. Identify the independent and dependent quantities in Graph 1.
   The independent quantity in Graph 1 is time in terms of years. The dependent quantity in Graph 1 is the weight in terms of pounds.

3. Identify the independent and dependent quantities in Graph 2.
   The independent quantity in Graph 2 is time in terms of years. The dependent quantity in Graph 2 is the height in terms of inches.

4. Label each axis with the appropriate quantity and unit.
A Sort of Sorts
Analyzing and Sorting Graphs

LEARNING GOALS
In this lesson, you will:
• Review and analyze graphs.
• Determine similarities and differences among various graphs.
• Sort graphs by their similarities and rationalize the differences between the groups of graphs.
• Use the Vertical Line Test to determine if the graph of a relation is a function.

KEY TERMS
• relation
• domain
• range
• function
• Vertical Line Test
• discrete graph
• continuous graph

ESSENTIAL IDEAS
• A relation is the mapping between a set of inputs and a set of outputs.
• A function is a relation between a given set of elements, called the domain and the range, for which each element in the domain there exists exactly one element in the range.
• The domain of a function is the set of all input values.
• The range of a function is the set of all output values.
• The Vertical Line Test is used to determine if the graph of a relation is a function.

MATHEMATICS COMMON CORE STANDARDS
F-IF Interpreting Functions

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

Interpret functions that arise in applications in terms of the context

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
Overview
Students begin this lesson by cutting out twenty-two different graphs. They will sort the graphs into different groups based on their own rationale, compare their groupings with their classmates, and discuss the reasoning behind their choices. Next, four different groups of graphs are given and students analyze the groupings and explain possible rationales behind the choices made. The terms relation, function, domain, and range are defined. The Vertical Line Test is introduced and used to determine if various graphs are or are not functions.
Warm Up

1. Use the points on each graph to complete the corresponding table of values.

   **Graph A**
   
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
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<td>2</td>
<td>3</td>
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<td>3</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

   **Graph B**
   
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
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<tr>
<td>2</td>
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<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Which graph has a different $x$-value for each $y$-value?
   
   **Graph B** has a different $x$-value for each $y$-value.

3. Which graph has the same $x$-values for different $y$-values?
   
   **Graph A** has the same $x$-values for different $y$-values.

4. If the equation $x = 2$ were drawn on both graphs, how many points on each graph would intersect this line?
   
   The line would intersect two points on the graph in Question 1 and the line would intersect one point on the graph in Question 2.
A Sort of Sorts
Analyzing and Sorting Graphs

In this lesson, you will:
- Review and analyze graphs.
- Determine similarities and differences among various graphs.
- Sort graphs by their similarities and rationalize the differences between the groups of graphs.
- Use the Vertical Line Test to determine if the graph of a relation is a function.

Key Terms
- relation
- domain
- range
- function
- Vertical Line Test
- discrete graph
- continuous graph

Are you getting the urge to start driving? Chances are that you’ll be studying for your driving test before you know it. But how much will driving cost you? For all states in the U.S., auto insurance is a must before any driving can take place. For most teens and their families, this more than likely means an increase in auto insurance costs.

So how do insurance companies determine how much you will pay? The fact of the matter is that auto insurance companies sort drivers into different groups to determine their costs. For example, they sort drivers by gender, age, marital status, and driving experience. The type of car is also a factor. A sports vehicle or a luxury car is generally more expensive to insure than an economical car or a family sedan. Even the color of a car can affect the cost to insure it!

Do you think it is good business practice to group drivers to determine auto insurance costs? Or do you feel that each individual should be reviewed solely on the merit of the driver based on driving record? Do you think auto insurance companies factor in where a driver lives when computing insurance costs?
Problem 1
Students cut out 22 graphs and will sort the graphs into different groupings of their choice. They then compare their groupings with their classmates’ groupings and explain their reasoning. Students are asked to create a list of all the different graphical behaviors used to sort the graphs.

Grouping

• Ask a student to read the introduction before Question 1. Discuss as a class.
• Have students complete Questions 1 and 2 in groups. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2
• How many groups do you have?
• How did you decide what graph goes in each group?
• Do any of your groups contain a single graph?
• Did you and your group members disagree about any particular grouping?
• Were there any graphs that didn’t fit into a grouping?
1.2 Analyzing and Sorting Graphs

A. \[ f(x) = \begin{cases} 
-2x + 10, & -\infty \leq x < 3 \\
4, & 3 \leq x < 7 \\
-2x + 18, & 7 \leq x \leq +\infty 
\end{cases} \]
Domain: all real numbers

B. \[ f(x) = -\frac{1}{2}x^2 + 2 \]
Domain: all real numbers

C. \[ f(x) = \begin{cases} 
\frac{1}{2}x + 4, & -\infty \leq x < 2 \\
-3x + 11, & 2 \leq x < 3 \\
\frac{1}{2}x + \frac{1}{2}, & 3 \leq x \leq +\infty 
\end{cases} \]
Domain: all real numbers

D. \[ f(x) = |x| \]
Domain: all real numbers

E. \[ x^2 + y^2 = 16 \]
Domain: \(-4 \leq x \leq 4\)

F. \[ f(x) = -3x^3 + 4, \text{ where } x \text{ is an integer} \]
Domain: all integers
1.2 Analyzing and Sorting Graphs

- **G**
  - \( f(x) = x \)
  - Domain: all real numbers

- **H**
  - \( f(x) = \left( \frac{1}{2} \right)^x - 5 \)
  - Domain: all real numbers

- **I**
  - \( f(x) = -|x| \)
  - Domain: all real numbers

- **J**
  - \( x = y(y - 3)(y + 3) \)
  - Domain: \(-10 \leq x \leq 10\)

- **K**
  - \( f(x) = 2^x \), where \( x \) is an integer
  - Domain: all integers

- **L**
  - \( f(x) = -\frac{2}{3}x + 5 \)
  - Domain: all real numbers
1.2 Analyzing and Sorting Graphs

- **M**: 
  - $f(x) = x^2$ 
  - Domain: all real numbers

- **N**: 
  - $f(x) = \pm \sqrt{x}$ 
  - Domain: $x \geq 0$

- **O**: 
  - $f(x) = -x + 3$, where $x$ is an integer 
  - Domain: all integers

- **P**: 
  - $f(x) = \frac{1}{|x|}$ 
  - Domain: all real numbers

- **Q**: 
  - $f(x) = -2|x + 2| + 4$ 
  - Domain: all real numbers

- **R**: 
  - $x = 2$ 
  - Domain: $x = 2$
1.2 Analyzing and Sorting Graphs

**Graph S**
- \( f(x) = \begin{cases} -2, & -\infty < x < 0 \\ \frac{1}{2}x - 2, & 0 \leq x \leq \infty \end{cases} \)
- Domain: all real numbers

**Graph T**
- \( f(x) = x^2 + 8x + 12 \)
- Domain: all real numbers

**Graph U**
- \( f(x) = 2, \) where \( x \) is an integer
- Domain: all integers

**Graph V**
- \( f(x) = |x - 3| - 2 \)
- Domain: all real numbers
2. Compare your groupings with your classmates’ groupings. Create a list of the different graphical behaviors you noticed.

Answers may vary.

Possible graphical behaviors:
- always increasing from left to right
- always decreasing from left to right
- the graph both increases and decreases
- straight lines
- smooth curves
- discrete data values
- the graph has a maximum value
- the graph has a minimum value
- the graph is a function
- the graph is not a function
- the graph goes through the origin
- the graph forms a U shape
- the graph forms a V shape

Are any of the graphical behaviors shared among your groups? Or, are they unique to each group?
Problem 2

Four different scenarios that show groups of graphs are given and students will explain the rationale behind the groupings, and errors in the reasoning behind a grouping. Rationales for grouping include graphs being discrete, having vertical symmetry, located in a single quadrant, and not being a function.

Grouping

Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Question 1

- How are Matthew’s graphs different than other graphs you may have seen?
- Which of Matthew’s graphs are linear in appearance?
- Which of Matthew’s graphs are not linear in appearance?
- What kind of scenarios can you think of for each of Matthew’s graphs?

PROBLEM 2 I Like the Way You Think

1. Matthew grouped these graphs together.

Why do you think Matthew put these graphs in the same group?
Answers will vary.
These graphs are made up of dots (discrete data).
Guiding Questions for Share Phase, Question 2

- Where can you place the line of symmetry on each of Ashley’s graphs?
- Do any of Ashley’s graphs have horizontal symmetry?
- Is the y-axis always the line of vertical symmetry?

Ashley

I grouped these graphs together because they all show vertical symmetry. If I draw a vertical line through the middle of the graph, the image is the same on both sides.

a. Show why Ashley’s reasoning is correct.
   See graphs.
   Notice that for Graph V, the student may need to extend the graph to see the symmetry.

b. If possible, identify other graphs that show vertical symmetry.
   B, E, I, Q, and U
Guiding Questions for Share Phase, Question 3

- Are Duane's graphs only what they appear to be or is only part of each graph visible?
- What part of each of Duane's graphs is not visible?
- Why do you think parts of each graph are not visible?
- Do the lines and curves on Duane's graph continue to infinity?

3.

**Duane**

I grouped these graphs together because each graph only goes through two quadrants.

**a.** Explain why Duane's reasoning is not correct.

Even though it is not visible, Graph T continues into the first quadrant. Therefore, the graph goes through three quadrants. Each of the other graphs D, M, and P satisfy Duane's reasoning.

**b.** If possible, identify other graphs that only go through two quadrants.

**Guiding Questions for Share Phase, Question 4**

- How are Josephine's graphs different from other graphs you have seen?
- For each x-value on one of Josephine's graphs, how many y-values are there?

4. Judy grouped these four graphs together, but did not provide any rationale.

![Graphs](image)

a. What do you notice about the graphs?
   
   Answers will vary.
   
   In each graph, for at least one value of x, there is more than one value of y.

b. What rationale could Judy have provided?
   
   Answers will vary.
   
   The graphs are not functions.
Problem 3
The terms relation, domain, range, and function are defined. The Vertical Line Test is introduced as a visual method used to determine whether a relation represented as a graph is a function. Discrete and continuous graphs are also defined. The Vertical Line Test is then used to sort the graphs from Problem 1 into two groups.

Grouping
• Have a student read the definitions. Discuss and complete Question 1 as a class.
• Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Guiding Questions for Discuss Phase, Question 1
• If a graph passes the Vertical Line Test, what does that mean?
• If a graph fails the Vertical Line Test, what does that mean?
• Do Josephine’s graphs pass or fail the Vertical Line Test?
• For each x-value on one of Josephine’s graphs, how many y-values are there?

PROBLEM 3 Function Junction

A relation is the mapping between a set of input values called the domain and a set of output values called the range. A function is a relation between a given set of elements, such that for each element in the domain there exists exactly one element in the range.

The Vertical Line Test is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

A discrete graph is a graph of isolated points. A continuous graph is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

The Vertical Line Test applies for both discrete and continuous graphs.

1. Analyze the four graphs Judy grouped together. Do you think that the graphs she grouped are functions? Explain how you determined your conclusion.
   No. The graphs Judy grouped together are not functions. I used the Vertical Line Test for each graph and saw that the vertical line intersected the graph at more than one point in each graph.

2. Use the Vertical Line Test to sort the graphs in Problem 1 into two groups: functions and non-functions. Record your results by writing the letter of each graph in the appropriate column in the table shown.

<table>
<thead>
<tr>
<th>Functions</th>
<th>Non-Functions</th>
</tr>
</thead>
</table>

So all functions are relations, but only some relations are functions. I guess it all depends on the domain and range.
Guiding Questions for Share Phase, Questions 2 and 3

- How many graphs did you sort into the function group?
- How many graphs did you sort into the non-function group?
- Did all of the graphs fit into one of the two groups? Can a graph be neither?
- What do graphs of non-functions look like?
- What do graphs of functions look like?
- Are all curved graphs considered graphs of non-functions?
- Can you think of a graph that is curved and is a function? What does it look like?
- Are all linear graphs considered graphs of functions?
- Can you think of a graph that is linear and is a non-function? What does it look like?

3. Each graph in this set of functions has a domain that is either:
   - the set of all real numbers, or
   - the set of integers.

For each graph, remember that the x-axis and the y-axis display values from -10 to 10 with an interval of 2 units.

Label each function graph with the appropriate domain.
The graphs with the domain as the set of all real numbers are: A, B, C, D, G, H, I, K, L, M, P, Q, S, T, U, and V.
The graphs with the domain as the set of integers are: F, O, R, and U.

4. Clip all your graphs together and keep them for the next lesson.
Talk the Talk

Students will sketch a graph that is a function and one that is not a function to demonstrate their understanding.

Grouping

Have students complete Questions 1 and 2 independently. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2

• Does your graph pass or fail the Vertical Line Test?
• What is the significance of your graph if it passes the Vertical Line Test?
• What is the significance of your graph if it fails the Vertical Line Test?
• How many $y$-values does each $x$-value have on your graph?
• Is your graph discrete or continuous?

1. Sketch a graph of a function. Explain how you know that it is a function.
   
   Answers will vary.

   For each value of $x$, there is exactly one value of $y$.

2. Sketch a graph that is not a function. Explain how you know that it is not a function.
   
   Answers will vary.

   For at least one value of $x$, there are two values of $y$.

Be prepared to share your solutions and methods.
1. Sketch a relation that is an example of a continuous function.

2. Sketch a relation that is an example of a discrete function.
3. Sketch a relation that is an example of a continuous non-function.

![Continuous Non-Function Graph]

4. Sketch a relation that is an example of a discrete non-function.

![Discrete Non-Function Graph]
There Are Many Ways to Represent Functions

Recognizing Algebraic and Graphical Representations of Functions

LEARNING GOALS
In this lesson, you will:

- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

ESSENTIAL IDEAS

- The function notation \( f(x) \) indicates that \( x \) is the independent variable.
- When using a graphing calculator, equations must be written in function notation.
- A function is said to be increasing when both the independent and the dependent variables are increasing.
- A function is said to be decreasing when the dependent variable decreases as the independent variable increases.
- A function is said to be constant when the dependent variable of the function does not change or remains constant over the entire domain.
- A family of functions is a group of functions that share certain attributes.
- The family of linear functions includes functions of the form \( f(x) = mx + b \), where \( m \) and \( b \) are real numbers and \( m \) is not equal to 0.
- The family of exponential functions includes functions of the form \( f(x) = a \cdot b^x \), where \( a \) and \( b \) are real numbers and \( b \) is greater than 0 but not equal to 1.
- The family of quadratic functions includes functions of the form \( f(x) = ax^2 + bx + c \), where \( a \), \( b \), and \( c \) are real numbers, and \( a \) is not equal to 0.
- The family of linear absolute value functions includes functions of the form \( f(x) = a|x + b| + c \), where \( a \), \( b \), and \( c \) are real numbers, and \( a \) is not equal to 0.
- Linear piecewise functions include functions that have equation changes for different parts, or pieces, of the domain.
- A function has an absolute minimum if there is a point whose \( y \)-coordinate is less than the \( y \)-coordinates of every other point on the graph.
- A function has an absolute maximum if there is a point whose \( y \)-coordinate is greater than the \( y \)-coordinates of every other point on the graph.

KEY TERMS

- function notation
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- linear absolute value functions
- linear piecewise functions
**MATHEMATICS COMMON CORE STANDARDS**

**F-IF Interpreting Functions**

Interpret functions that arise in applications in terms of the context

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

**Analyze functions using different representations**

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

**A-REI Reasoning with Equations and Inequalities**

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**F-IF Interpreting Functions**

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

**Overview**

Function notation is introduced. Steps are provided to guide students through entering and viewing the graph of a linear function on a graphing calculator. The terms increasing function, decreasing function, and constant function are defined. Students will sort the graphs from the previous lesson into groups using these terms and match each graph with its appropriate equation written in function notation. The terms function family, linear function, and exponential function are then defined.

Next, the terms absolute minimum and absolute maximum are defined. Students will continue to sort the remaining graphs into groups using these terms and match each graph with its appropriate equation written in function notation. The terms quadratic function and linear absolute value function are then defined. Finally, linear piecewise functions are defined and students match the remaining graphs to their appropriate functions.

In the final activity, students will complete a graphic organizer for each function family that describes the graphical behavior and displays the various graphical examples.
Warm Up

1. Graph the equation $y = x$
   
   **Graph A**

2. Graph the equation $y = -x$
   
   **Graph B**

3. Describe the impact the negative sign has on the graph of the equation $y = x$.
   
   The negative sign reverses the direction of the graph, or flips the graph upside down.
4. Graph the equation $y = x^2$

Graph A

5. Graph the equation $y = -x^2$

Graph B

6. Describe the impact the negative sign has on the graph of the equation $y = x^2$.

The negative sign reverses the direction of the graph, or flips the graph upside down.
Recognizing Algebraic and Graphical Representations of Functions

**LEARNING GOALS**

In this lesson, you will:
- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

**KEY TERMS**

- function notation
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- linear absolute value functions
- linear piecewise functions

Just about everything you see, hear, or own has a name. It’s not just people who have names—streets have names, cars have names, even trees have names. So where do these names come from and why were they chosen? There are many naming conventions we use in our society. The purpose of these naming conventions is to provide useful information about the object being named. For example, just saying “I live on a street” does not provide much information. However, saying “I live on East Main Street” makes it much more clear where you live.

Think about other objects and their names. Why do you think they were named the way they were? What information is provided by their names? Would another name suit the object better?
Problem 1
Function notation is defined. A scenario is described and then expressed in function notation. Step by step procedures are given to help students enter a function into a graphing calculator and adjust the window to view a complete graph.

Grouping
Ask a student to read the information. Discuss as a class.

Guiding Questions for Discuss Phase, Problem 1
- Can all equations be written in function notation? Why not?
- What is an example of an equation that cannot be written in function form?

Problem 1

A New Way to Write Something Familiar
Functions can be represented in a number of ways. An equation representing a function can be written using function notation. Function notation is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function \( f(x) \) is read as “\( f \) of \( x \)” and indicates that \( x \) is the independent variable.

Let's look at the relationship between an equation and function notation.

Consider orders for a custom T-shirt shop. U.S. Shirts charges $8 per shirt plus a one-time charge of $15 to set a T-shirt design. The equation \( y = 8x + 15 \) can be written to model this situation. The independent variable \( x \) represents the number of shirts ordered, and the dependent variable \( y \) represents the total cost of the order, in dollars.

You know this is a function because for each number of shirts ordered (independent value) there is exactly one total cost (dependent value) associated with it.

Because this situation is a function, you can write \( y = 8x + 15 \) in function notation.

\[ f(x) = 8x + 15 \]

The cost, defined by \( f \), is a function of \( x \), the number of shirts ordered.

A common way to name a function is \( f(x) \). However, you can choose any variable to name a function. You could write the T-shirt cost function as \( C(s) = 8s + 15 \), where the cost, defined as \( C \), is a function of \( s \), the number of shirts ordered.
1.3 Recognizing Algebraic and Graphical Representations of Functions

Guiding Questions for Discuss Phase, Problem 1

- Do you think a non-function can be graphed on the graphing calculator? Explain.
- What will happen if the Ymin is not changed to 220? What impact will this have on the graph of the function?
- What will happen if the Ymax is not changed to 20? What impact will this have on the graph of the function?
- What will happen if the Yscl is not changed to 2? What impact will this have on the graph of the function?
- When viewing a graph on the graphing calculator, how will you know the Xscl or the Yscl needs to be adjusted?
- When viewing a graph on the graphing calculator, how will you know the Xmin or the Xmax needs to be adjusted?
- When viewing a graph on the graphing calculator, how will you know the Ymin or the Ymax needs to be adjusted?
- What does it mean to view a complete graph?
- When viewing a graph on the graphing calculator, how will you know the graph is not a complete graph?
- Why is it important to view a complete graph?

You can input equations written in function notation into your graphing calculator. Your graphing calculator will list different functions as $Y_1$, $Y_2$, $Y_3$, etc.

Let’s graph the function $f(x) = 8x + 15$ on a calculator by following the steps shown.

Step 1: Press $\text{Y} =$. Your cursor should be blinking on the line $Y1 =$. Enter the equation. To enter a variable like $x$, press the key with $X$, $T$, $\theta$, $n$ once.

Step 2: Press $\text{WINDOW}$ to set the bounds and intervals you want displayed.

Step 3: Press $\text{GRAPH}$ to view the graph.

The $\text{Xmin}$ represents the least point on the $x$-axis that will be seen on the screen. The $\text{Xmax}$ represents the greatest point that will be seen on the $x$-axis. Lastly, the $\text{Xscl}$ represents the intervals. Similar names are used for the $y$-axis ($\text{Ymin}$, $\text{Ymax}$, and $\text{Yscl}$).

A convention to communicate the viewing $\text{WINDOW}$ on a graphing calculator is shown.

- $\text{Xmin} = -10$
- $\text{Xmax} = 10$
- $\text{Ymin} = -20$
- $\text{Ymax} = 20$
- $\text{Xscl} = 1$
- $\text{Yscl} = 2$
### Guiding Questions for Share Phase, Questions 1 through 3

- How would you describe the graph of an increasing function?
- Are some graphs increasing at a faster rate than others? How can you tell?
- How would you describe the graph of a decreasing function?
- Are some graphs decreasing at a faster rate than others? How can you tell?
- How would you describe the graph of a constant function?
1.3 Recognizing Algebraic and Graphical Representations of Functions

2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Enter each function into a graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

• \( f(x) = x \)
  Graph G
• \( f(x) = \frac{\sqrt{2}}{2} - 5 \)
  Graph H
• \( f(x) = 2^x \), where \( x \) is an integer
  Graph K
• \( f(x) = -\frac{2}{3} x + 5 \)
  Graph L
• \( f(x) = -x + 3 \), where \( x \) is an integer
  Graph O
• \( f(x) = \frac{\sqrt{2}}{2} \)
  Graph P
• \( f(x) = 2 \), where \( x \) is an integer
  Graph U

3. Consider the seven graphs and functions that are increasing functions, decreasing functions, or constant functions.
   a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>G, L, O, U</td>
<td>H, K, P</td>
</tr>
<tr>
<td>Linear/Constant</td>
<td>Exponential</td>
</tr>
</tbody>
</table>

   b. What is the same about all the functions in each group?
   Answers may vary.
   All the functions in Group 1 form straight lines.
   All the functions in Group 2 form smooth curves.
   All the functions in Group 2 involve exponents.

---

- Did you have trouble entering any of the functions in the graphing calculator? If so, which ones?
- When entering the functions into the graphing calculator, which functions required the use of parenthesis?
- When entering the functions into the graphing calculator, how do you know when you need to use parenthesis?
- What criteria did you use to sort the graphs into two groups?
Problem 3

Students will sort the graphs from the combination category in Problem 2 into three groups having the characteristics of absolute minimum, absolute maximum, and having no absolute minimum or absolute maximum. Focusing only on the eight graphs containing absolute minimums, absolute maximums, or having no absolute minimums or absolute maximums, they match each graph with the appropriate equation written in function notation, and then sort these graphs again into two groups based on containing absolute minimums and absolute maximums. The terms quadratic functions and linear absolute value functions are defined and students identify which graphs are best represented using these terms.

PROBLEM 3 Least, Greatest, or Neither?

A function has an absolute minimum if there is a point that has a y-coordinate that is less than the y-coordinates of every other point on the graph. A function has an absolute maximum if there is a point that has a y-coordinate that is greater than the y-coordinates of every other point on the graph.

1. Sort the graphs from the Combination category in Problem 2 into three groups:
   - those that have an absolute minimum value,
   - those that have an absolute maximum value, and
   - those that have no absolute minimum or maximum value.

Then record the function letter in the appropriate column of the table shown.

<table>
<thead>
<tr>
<th>Absolute Minimum</th>
<th>Absolute Maximum</th>
<th>No Absolute Minimum or Absolute Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>D, M, T, V</td>
<td>B, F, I, Q</td>
<td>A, C, S</td>
</tr>
</tbody>
</table>

2. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

3. If \( f(x) = mx + b \), represents a linear function, describe the \( m \) and \( b \) values that produce a constant function.

If \( m = 0 \) and \( b \) is any real number, then the result will be a constant function.

Congratulations! You have just sorted the graphs into their own function families. A function family is a group of functions that share certain characteristics.

The family of linear functions includes functions of the form \( f(x) = mx + b \), where \( m \) and \( b \) are real numbers.

The family of exponential functions includes functions of the form \( f(x) = a \cdot b^x \), where \( a \) and \( b \) are real numbers, and \( b \) is greater than 0 but is not equal to 1.

4. Think about the graphical behavior of the function over its entire domain.

5. Ask a student to read the definitions. Discuss as a class.

6. Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.
Guiding Questions for Share Phase, Questions 1 through 3

- What impact does the negative sign in front of the lead terms coefficient have on the graph of the function?
- Where can the absolute value function be found on the graphing calculator?
- Did you have trouble entering any of the functions in the graphing calculator? If so, which ones?
- When entering the functions into the graphing calculator, which functions required the use of parenthesis?
- What criteria did you use to sort the graphs into two groups?
- How would you describe the graph of a quadratic function?
- How would you describe the graph of an absolute value function?
- How would you describe the graph of a linear piecewise function?
- Do you think all piecewise functions are linear in nature? Explain.

2. Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Enter each function into your graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- \( f(x) = x^2 + 8x + 12 \)  
  Graph T
- \( f(x) = |x - 3| - 2 \)  
  Graph V
- \( f(x) = x^3 \)  
  Graph M
- \( f(x) = |x| \)  
  Graph D
- \( f(x) = -|x| \)  
  Graph I
- \( f(x) = -3x^2 + 4 \), where \( x \) is integer  
  Graph F
- \( f(x) = -\frac{1}{2}x^2 + 2x \)  
  Graph B
- \( f(x) = -2|x + 2| + 4 \)  
  Graph Q

3. Consider the graphs of functions that have an absolute minimum or an absolute maximum. (Do not consider Graphs A and C yet.)

a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>B, F, M, T</td>
<td>D, I, Q, V</td>
</tr>
<tr>
<td>Quadratic</td>
<td>Linear Absolute Value</td>
</tr>
</tbody>
</table>

b. What is the same about all the functions in each group? Answers may vary.

- All the functions in Group 1 are made of smooth curves.
- All the functions in Group 2 are made of two straight lines. All the functions in Group 2 involve absolute value.
Grouping
Ask a student to read the information before Question 4 aloud. Discuss and complete Question 4 as a class.

Problem 4
The remaining three graphs are defined as linear piecewise functions. Graphing calculator instructions are provided to demonstrate how to enter linear piecewise functions. Students will match the functions to the corresponding graphs.

Grouping
Ask a student to read the information aloud. Discuss as a class.

Congratulations! You have just sorted functions into two more function families. The family of quadratic functions includes functions of the form \( f(x) = ax^2 + bx + c \), where \( a \), \( b \), and \( c \) are real numbers, and \( a \) is not equal to 0.

The family of linear absolute value functions includes functions of the form \( f(x) = a|x + b| + c \), where \( a \), \( b \), and \( c \) are real numbers, and \( a \) is not equal to 0.

4. Go back to your table in Question 3 and identify which group represents quadratic functions and which group represents linear absolute value functions. See table.

PROBLEM 4  Piecing Things Together
Analyze each of the functions shown. These functions represent the last three graphs of functions from the no absolute minimum and no absolute maximum category.

- \( f(x) = \begin{cases} -2x + 10, & -\infty \leq x < 3 \\ 4, & 3 \leq x < 7 \\ -2x + 18, & 7 \leq x \leq +\infty \end{cases} \)
  - Graph A
- \( f(x) = \begin{cases} -2, & -\infty < x < 0 \\ \frac{1}{2}x - 2, & 0 \leq x < \infty \end{cases} \)
  - Graph S
- \( f(x) = \begin{cases} \frac{1}{2}x + 4, & -\infty \leq x < 2 \\ -3x + 11, & 2 \leq x < 3 \\ \frac{1}{2}x + \frac{1}{2}, & 3 \leq x \leq +\infty \end{cases} \)
  - Graph C

These functions are part of the family of linear piecewise functions. Linear piecewise functions include functions that have equation changes for different parts, or pieces, of the domain.

Because these graphs each contain compound inequalities, there are additional steps required to use a graphing calculator to graph each function.

If your graphing calculator does not have an infinity symbol, you can enter the biggest number your calculator can compute using scientific notation. On mine, this is \( 9 \times 10^{9} \). I enter this by pressing 9 2nd E/E 9/nine.Alt, which is shown on my calculator as 9E9/nine.Alt.

So then, for negative infinity, would I use \( -9 \times 10^{9} \) to the \( 9^n \) power? How do I enter that?
### Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

### Guiding Questions for Share Phase, Questions 1 and 2

- How would you describe the graph of a linear piecewise function?
- Do you think all piecewise functions are linear in nature? Explain.

Let's graph the piecewise function:

\[
\begin{align*}
-2x + 10, & \quad -\infty \leq x < 3 \\
f(x) = 4, & \quad 3 \leq x < 7 \\
f(x) = -2x + 18, & \quad 7 \leq x \leq +\infty
\end{align*}
\]

You can use a graphing calculator to graph piecewise functions.

**Step 1:** Press \(Y=\). Enter the first section of the function within parentheses. Then press the division button.

**Step 2:** Press the ( key twice and enter the first part of the compound inequality within parentheses.

**Step 3:** Enter the second part of the compound inequality within parentheses and then type two closing parentheses. Press GRAPH here to see the first section of the piecewise function.

**Step 4:** Enter the remaining sections of the piecewise functions as \(Y_2\) and \(Y_3\).

By completing the first piecewise function, you can now choose the graph that matches your graphing calculator screen.

1. Enter the remaining functions into your graphing calculator to determine the shapes of their graphs.
2. Match each function to its corresponding graph by writing the function directly on the graph that it represents.
Congratulations! You have just sorted the remaining functions into one more function family.

The family of linear piecewise functions includes functions that have equation changes for different parts, or pieces, of the domain.

You will need these graphs again in Problem 5. Wait for it...
Problem 5
Students will paste their equations and linear, exponential, quadratic, linear absolute value, and linear piecewise graphs into appropriate graphic organizers and describe the graphical behavior of each function.

Grouping
Have students complete Question 1 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Question 1
• Which families of functions contain curves?
• Which families of functions contain straight lines?
• Does a linear function contain an absolute minimum or absolute maximum? Explain.
• Does an exponential function contain an absolute minimum or absolute maximum? Explain.

You've done a lot of work up to this point! You've been introduced to linear, exponential, quadratic, linear absolute value, and linear piecewise functions. Don't worry—you don't need to know everything there is to know about all of the function families right now. As you progress through this course, you will learn more about each function family.

Be prepared to share your solutions and methods.
**Definition**

The family of linear functions includes functions of the form \( f(x) = mx + b \), where \( m \) and \( b \) are real numbers.

**Graphical Behavior**

Increasing / Decreasing:
Linear functions can be increasing or decreasing over the entire domain.

Maximum / Minimum:
Linear functions have no maximum or minimum point.

Curve / Line:
Linear functions are made up of straight lines.

**Examples**

- **G**
  - \( f(x) = x \)
  - Domain: all real numbers

- **L**
  - \( f(x) = \frac{2}{3}x + 5 \)
  - Domain: all real numbers

- **O**
  - \( f(x) = -x + 3 \), where \( x \) is an integer
  - Domain: all integers

- **U**
  - \( f(x) = 2 \), where \( x \) is an integer
  - Domain: all integers
Definition
The family of exponential functions includes functions of the form \( f(x) = a \cdot b^x \), where \( a \) and \( b \) are real numbers, and \( b \) is greater than 0 but not equal to 1.

Graphical Behavior
Increasing / Decreasing:
Exponential functions can be increasing or decreasing over the entire domain.

Maximum / Minimum:
Exponential functions do not have a maximum or minimum.

Curve / Line:
Exponential functions are made of smooth curves.

Examples

\[
H \quad f(x) = \left(\frac{1}{2}\right)^x - 5
\]
Domain: all real numbers

\[
K \quad f(x) = 2^x,
\]
where \( x \) is an integer
Domain: all integers

\[
P \quad f(x) = \left(\frac{1}{2}\right)^x
\]
Domain: all real numbers
1.3 Recognizing Algebraic and Graphical Representations of Functions

**Definition**
The family of **quadratic functions** includes functions of the form, \( f(x) = ax^2 + bx + c \) where \( a, b, \) and \( c \) are real numbers, and \( a \) is not equal to 0.

**Graphical Behavior**
Increasing / Decreasing:
Quadratic functions can increase then decrease or decrease then increase over the domain.

Maximum / Minimum:
Quadratic functions can have either a maximum or a minimum depending on the shade of the parabola.

Curve / Line:
Quadratic functions are made of smooth curves.

**Examples**

- **B**
  \( f(x) = -\frac{1}{2}x^2 + 2x \)
  Domain: all real numbers

- **F**
  \( f(x) = -3x^2 + 4 \), where \( x \) is an integer
  Domain: all integers

- **M**
  \( f(x) = x^2 \)
  Domain: all real numbers

- **T**
  \( f(x) = x^2 + 8x + 12 \)
  Domain: all real numbers
**Definition**

The family of **linear absolute value functions** includes functions of the form

\[ f(x) = a|x + b| + c \]

where \( a, b, \) and \( c \) are real numbers, and \( a \) is not equal to 0.

**Graphical Behavior**

**Increasing / Decreasing:**

Linear absolute value function can increase then decrease, or decrease then increase over the domain.

**Maximum / Minimum:**

Linear absolute value functions can have either an absolute maximum or an absolute minimum.

**Curve / Line:**

Linear absolute value functions are made of straight lines.

---

**Examples**

- **D**
  \[ f(x) = |x| \]
  Domain: all real numbers

- **I**
  \[ f(x) = -|x| \]
  Domain: all real numbers

- **Q**
  \[ f(x) = -2|x + 2| + 4 \]
  Domain: all real numbers

- **V**
  \[ f(x) = |x - 3| - 2 \]
  Domain: all real numbers
1.3 Recognizing Algebraic and Graphical Representations of Functions

**Definition**
The family of linear piecewise functions includes functions that have equation changes for different parts, or pieces, of the domain.

**Graphical Behavior**

**Increasing / Decreasing:**
Linear piecewise functions can have pieces that are increasing, decreasing, or constant.

**Maximum / Minimum:**
Linear piecewise functions may or may not have a maximum or a minimum.

**Curve / Line:**
Linear piecewise functions are made up of pieces that are straight lines and line segments.

### Examples

#### A
- \( f(x) = \begin{cases} -2x + 10, & -\infty \leq x < 3 \\ 4, & 3 \leq x < 7 \\ -2x + 18, & 7 \leq x \leq +\infty \end{cases} \)
- **Domain:** all real numbers

#### B
- \( f(x) = \begin{cases} \frac{1}{2}x + 4, & -\infty \leq x < 2 \\ -3x + 11, & 2 \leq x < 3 \\ \frac{1}{2}x^2 + \frac{1}{2}, & 3 \leq x \leq +\infty \end{cases} \)
- **Domain:** all real numbers

#### C
- \( f(x) = \begin{cases} -2, & -\infty < x < 0 \\ \frac{3}{2}x - 2, & 0 \leq x < +\infty \end{cases} \)
- **Domain:** all real numbers
Molly thinks the two graphs shown belong to the linear absolute value function family. Do you agree or disagree? Explain your reasoning.

I disagree with Molly. The graphs are not functions. They do not pass the Vertical Line Test.
1.4

Function Families for 200, Alex...

Recognizing Functions by Characteristics

LEARNING GOALS

In this lesson, you will:
- Recognize similar characteristics among function families.
- Recognize different characteristics among function families.
- Determine function types given certain characteristics.

ESSENTIAL IDEAS

- Graphs described as smooth curves are associated with an exponential function or a quadratic function.
- Graphs described as straight lines are associated with a linear function or a linear absolute value function.
- Graphs described as containing an absolute maximum or an absolute minimum are associated with a quadratic function or a linear absolute value function.

MATHEMATICS COMMON CORE STANDARDS

F-IF Interpreting Functions

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-LE Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

A-CED Creating Equations

Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Overview

Students are given characteristics of specific functions describing their graphical behaviors and they will name the possible function family/families to fit each description. Next, they are given characteristics of functions and will write an equation and sketch a graph to fit each description. Finally, using the scenarios from the first lesson of the chapter, they complete a table by listing the function family associated with each graph, and describing all of the attributes of the function with respect to increasing or decreasing, containing absolute minimums or absolute maximums, and a domain that is either discrete or continuous.
Warm Up

1. Sketch a graph that can be described as a linear function.
   
   **Graph A**

2. Write an equation to fit this graph.
   
   \[ y = x \]

3. Sketch a graph that can be described as a decreasing linear function.
   
   **Graph B**

4. Write an equation to fit this graph.
   
   \[ y = -x \]
5. Sketch a graph that can be described as a decreasing linear function with a slope of 3.

**Graph C**

![Graph C](image)

6. Write an equation to fit this graph.

   $y = -3x$

7. Sketch a graph that can be described as a decreasing linear function with a slope of 3 and a $y$-intercept of $-5$.

**Graph D**

![Graph D](image)

8. Write an equation to fit this graph.

   $y = -3x - 5$
since the debut of television in the early 1950s, americans have had a love/hate relationship with the game show. one of the original game shows that aired was name that tune. the game was played when two contestants were given a clue about a song. then, one opponent would “bid” that the song could be named in a certain number of notes played. the other opponent could either beat the number of notes “bid” from the opponent, or they could tell their opponent to “name that tune!”

Do you like game shows? If so, what are your favorite game shows?
Problem 1
Several functions are described and students are asked to identify the function family/families best associated with each description.

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2
- What does the graph of a smooth curve look like?
- What is does a graph that is not a smooth curve look like?
- What does a function that increases over the entire domain look like?
- What does a function that decreases over the entire domain look like?
- What does a function that has a maximum look like?
- What does a function that has an absolute maximum look like?
- What is the difference between the appearance of a function that has a maximum and a function that has an absolute maximum?
- What does a function that has an absolute minimum look like?

PROBLEM 1 Name That Function!

1. Use the characteristic(s) provided to choose the appropriate function family or families from the word box shown.

   - linear function family
   - exponential function family
   - quadratic function family
   - linear absolute value function family

   a. The graph of this function family:
      - is a smooth curve.
      - exponential function or quadratic function

   b. The graph of this function family:
      - is made up of one or more straight lines.
      - linear absolute value function or linear function

   c. The graph of this function family:
      - increases or decreases over the entire domain.
      - linear function or exponential function

   d. The graph of this function family:
      - has a maximum or a minimum.
      - quadratic function or linear absolute value function

2. A second characteristic has been added to the graphical description of each function. Name the possible function family or families given the graphical characteristics.

   a. The graph of this function family:
      - has an absolute minimum or absolute maximum, and
      - is a smooth curve.
      - quadratic function

   b. The graph of this function family:
      - either increases or decreases over the entire domain, and
      - is made up of a straight line.
      - linear function
c. The graph of this function family:
   • is a smooth curve, and
   • either increases or decreases over the entire domain.
   exponential function

d. The graph of this function family:
   • has either an absolute minimum or an absolute maximum
   • has symmetry, and
   • is made up of 2 straight lines
   linear absolute value function

Each function family has certain graphical behaviors with some behaviors common among different function families. Notice, the more specific characteristics that are given, the more specifically you can Name that Function!
Problem 2

Students are given several characteristics of a function and asked to write an equation and sketch a graph for each set of criteria. Students should work in pairs in such a way that both partners are able to simultaneously sketch their own graph and only after the graphs are sketched and the equations are written should they share their responses with each other to discuss similarities and differences.

Grouping

• Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2

• If the equation is described as a function, what does that imply?
• If the equation is described an exponential function, what does that imply?
• If the equation is described as a continuous function, what does that imply?
• If the equation is described as a decreasing function, what does that imply?
• Is there more than one correct equation and graph that fits this list of criteria?
• If the equation is described contains a minimum, what does that imply?

PROBLEM 2 Graph That Function!

1. Use the given characteristics to create an equation and sketch a graph. Use the equations given in the box as a guide. Then share your graph with your partner. Discuss similarities and differences between your graphs.

When creating your equation, use a, b, and c values that are any real numbers between −3 and 3. Do not use any functions that were used previously in this chapter.

Answers may vary.

Linear function
\( f(x) = mx + b \)

Exponential function
\( f(x) = a \cdot b^x \)

Quadratic function
\( f(x) = ax^2 + bx + c \)

Linear Absolute Value Function
\( f(x) = a|x + b| + c \)

Equation: \( f(x) = \frac{1}{2}x \)

Don’t forget about the function family graphic organizers you created if you need some help.
• If the equation is described is discrete, what does that mean to you?
• If the equation is described as a linear absolute value function, what does that imply?
• If the equation is described as linear, what does that imply? Is it a function?
• If the equation is described as increasing, what does that imply?
• If the equation is described as continuous, what does that imply? Is it a function?
• If the equation is described as quadratic what does that imply? Is it a function?
• How many characteristics did you list for your function?
• Is it possible to compose a list of characteristics that do not describe a function? Explain.
• Do any of the characteristics on your list contradict each other?
• What is an example of two characteristics that contradict each other?
• Is there more than one correct sketch that matches all of the characteristics on your list?
• How is your list of characteristics different than your partner’s list of characteristics?

b. Create an equation and sketch a graph that:
   • has a minimum,  
   • is discrete, and  
   • is a linear absolute value function.
Equation: $f(x) = |x - 1| + 2$ where $x$ is an integer

Is the domain the same or different for each function?

c. Create an equation and sketch a graph that:
   • is linear,  
   • is discrete,  
   • is increasing, and  
   • is a function.
Equation: $f(x) = 2x - 1$ where $x$ is an integer
d. Create an equation and sketch a graph that:
   • is continuous,
   • has a maximum,
   • is a function, and
   • is quadratic.
Equation: \( f(x) = -x^2 \)

---

e. Create an equation and sketch a graph that:
   • is not a function,
   • is continuous, and
   • is a straight line.
Equation: \( x = 3 \)
1.4 Recognizing Functions by Characteristics

Guiding Questions for Share Phase, Question 2
- How many of the scenarios are associated with a linear function?
- How many of the scenarios are associated with a quadratic function?
- How many of the scenarios are associated with an exponential function?
- How many of the scenarios are associated with a linear piecewise function?
- How many of the scenarios are associated with an absolute value function?
- How many of the scenarios can be described as continuous?

Talk the Talk
Throughout this chapter, you were introduced to five function families: linear, exponential, quadratic, linear absolute value, and linear piecewise. Let’s revisit the first lesson in this chapter: A Picture Is Worth a Thousand Words. Each of the scenarios in this lesson represents one of these function families.

1. Describe how each scenario represents a function.
   For every input value there is exactly one output value.

2. Complete the table to describe each scenario.
   a. Identify the appropriate function family.
   b. Based on the problem situation, identify whether the graph of the function should be discrete or continuous.
   c. Create a sketch of the mathematical model.
   d. Identify the graphical behavior.

Recall that each of the graphs representing the scenarios was drawn with either a continuous line or a continuous smooth curve to model the problem situation.
- How many of the scenarios can be described as discrete?
- How many of the scenarios contain a maximum?
- How many of the scenarios contain a minimum?
- How many of the scenarios can be described as increasing?
- How many of the scenarios can be described as decreasing?

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Function Family</th>
<th>Domain of the Real-World Situation: Continuous or Discrete</th>
<th>Sketch of the Mathematical Model</th>
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<td>Continuous</td>
<td>Graph H</td>
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<th>Sketch of the Mathematical Model</th>
<th>Graphical Behavior</th>
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<td>Jelly Bean Challenge</td>
<td>Linear Absolute Value Function</td>
<td>Discrete</td>
<td></td>
<td>Absolute minimum</td>
</tr>
</tbody>
</table>

Be prepared to share your solutions and methods.
Check for Students’ Understanding

List possible characteristics for each of the following graphs.

1. Graph A

- The graph is not a function.
- The graph is continuous.
- The graph is piecewise.
- The domain of the graph is all \( x \)-values greater than or equal to 1 and less than or equal to 4.
- The range of the graph is all \( y \)-values greater than or equal to \(-1\) and less than or equal to 5.
2. Graph B

- The graph is not a function.
- The graph is continuous.
- The graph is piecewise.
- The domain of the graph is all x-values greater than or equal to 4 and less than or equal to 7.
- The range of the graph is all y-values greater than or equal to −1 and less than or equal to 5.
Chapter 1 Summary

KEY TERMS

- dependent quantity (1.1)
- independent quantity (1.1)
- relation (1.2)
- domain (1.2)
- range (1.2)
- function (1.2)
- Vertical Line Test (1.2)
- discrete graph (1.2)
- continuous graph (1.2)
- function notation (1.3)
- increasing function (1.3)
- decreasing function (1.3)
- constant function (1.3)
- function family (1.3)
- linear functions (1.3)
- exponential functions (1.3)
- absolute minimum (1.3)
- absolute maximum (1.3)
- quadratic functions (1.3)
- linear absolute value functions (1.3)
- linear piecewise functions (1.3)

1.1 Identifying the Dependent and Independent Quantities for a Problem Situation

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the dependent quantity. The quantity that the dependent quantity depends upon is called the independent quantity.

Example

Caroline makes $8.50 an hour babysitting for her neighbors' children after school and on the weekends.

The dependent quantity is the total amount of money Caroline earns based on the independent quantity. The independent quantity is the total number of hours she babysits.
Labeling and Matching a Graph to an Appropriate Problem Situation

Graphs relay information about data in a visual way. Connecting points on a coordinate plane with a line or smooth curve is a way to model or represent relationships. The independent quantity is graphed on the horizontal or x-axis, while the dependent quantity is graphed on the vertical, or y-axis. Graphs can be straight lines or curves, and can increase or decrease from left to right. When matching with a problem situation, consider the situation and the quantities to interpret the meaning of the data values.

Example

Pedro is hiking in a canyon. At the start of his hike, he was at 3500 feet. During the first 20 minutes of the hike, he descended 500 feet at a constant rate. Then he rested for half an hour before continuing the hike at the same rate.

Time is the independent quantity and elevation is the dependent quantity.

1.2 Analyzing and Comparing Types of Graphs

Looking for patterns can help when sorting and comparing graphs. A discrete graph is a graph of isolated points. A continuous graph is a graph of points with no breaks in it. The points are connected by a straight line or smooth curve. Some graphs show vertical symmetry (if a vertical line were drawn through the middle of the graph the image is the same on both sides). Other possible patterns to look for include: only goes through two quadrants, always increasing from left to right, always decreasing from left to right, straight lines, smooth curves, the graph goes through the origin, the graph forms a U shape, the graph forms a V shape.
1.2 Using the Vertical Line Test When Determining Whether a Relation Is a Function

A relation is the mapping between a set of input values called the domain and a set of output values called the range. A function is a relation between a given set of elements for which each element in the domain has exactly one element in the range. The Vertical Line Test is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

Examples

A line drawn vertically through the graph touches more than one point. The graph does not represent a function. A line drawn vertically through the graph only touches one point. The graph represents a function.
Writing Equations Using Function Notation

Functions can be represented in a number of ways. An equation representing a function can be written using function notation. Function notation is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function \( f(x) \) is read as “\( f \) of \( x \)” and indicates that \( x \) is the independent variable. Remember, you know an equation is a function because for each independent value there is exactly one dependent value associated with it.

Example

Write this equation using function notation:
\[ y = 2x + 5 \]

The dependent variable (\( y \)), defined by \( f \), is a function of \( x \), the independent variable.

\( f(x) = 2x + 5 \)

Determining Whether a Graph Represents a Function That Is Increasing, Decreasing, or Constant

A function is described as increasing when both the independent and dependent variables are increasing. If a function increases across the entire domain, then the function is called an increasing function. A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a decreasing function. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a constant function.

Example

The function shown in the graph is a decreasing function because the dependent variable (\( y \)) decreases as the independent variable (\( x \)) increases.
Determining Whether a Graph Represents a Function with an Absolute Maximum or Absolute Minimum

A function has an absolute minimum if there is a point that has a $y$-coordinate that is less than the $y$-coordinates of every other point on the graph. A function has an absolute maximum if there is a point that has a $y$-coordinate that is greater than the $y$-coordinates of every other point on the graph.

Example

The function shown in the graph has an absolute maximum because the $y$-coordinate of the point $(0, 5)$ is greater than the $y$-coordinates of every other point on the graph.

Distinguishing Between Function Families

A function family is a group of functions that share certain characteristics.

The family of linear functions includes functions of the form $f(x) = ax + b$, where $a$ and $b$ are real numbers.

The family of exponential functions includes functions of the form $f(x) = a \cdot b^x$, where $a$ and $b$ are real numbers, and $b$ is greater than 0, but not equal to 1.

The family of quadratic functions includes functions of the form $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

The family of linear absolute value functions includes functions of the form $f(x) = a|x + b| + c$, where $a$, $b$, and $c$ are real numbers, and $a$ is not equal to 0.

The family of linear piecewise functions includes functions that have an equation that changes for different parts, or pieces, of the domain.
Examples

The function is quadratic.

The function is linear.

The function is exponential.

The function is linear absolute value.

The function is linear piecewise.
1.4 Identifying a Function Given Its Characteristics

Certain characteristics of a graph such as whether it increases or decreases over its domain, has an absolute minimum or maximum, is a smooth curve or not, or other characteristics, can help when determining if a function is linear, exponential, quadratic, or linear absolute value.

**Example**

The graph of a function \( f(x) \) is a smooth curve and has an absolute minimum. Thus, the function is quadratic.

1.4 Graphing a Function Given Its Characteristics

Use the given characteristics to create an equation and sketch a graph.

Linear function \( f(x) = mx + b \)

Exponential function \( f(x) = a \cdot b^x \)

Quadratic function \( f(x) = ax^2 + bx + c \)

Linear Absolute Value Function \( f(x) = a|x + b| + c \)

**Example**

Create an equation and sketch a graph that has:

- an absolute maximum
- and is a linear absolute value function.

\( f(x) = -|x| \)
Identifying a Function Given Its Graph

Certain characteristics of a graph such as whether it increases or decreases over its domain, has an absolute minimum or maximum, is a smooth curve or not, or other characteristics, can help when determining if a graph represents a linear, exponential, quadratic, or linear absolute value function.

Example

The graph shown is a linear absolute value function. It is discrete. The graph decreases and then increases. It has an absolute minimum.