NOTES – SYSTEMS OF LINEAR EQUATIONS

System of Equations – a set of equations with the same variables
(two or more equations graphed in the same coordinate plane)

Solution of the system – an ordered pair that is a solution to all equations
is a solution to the equation.

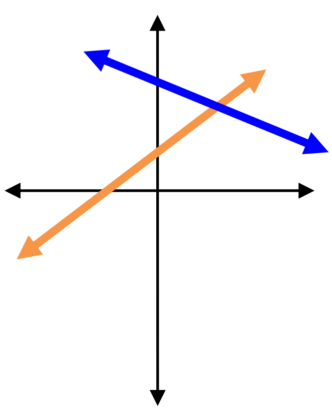
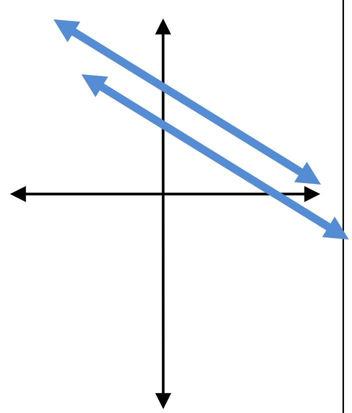
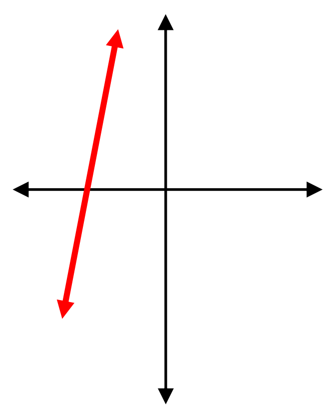
a. one solution
b. no solution
c. an infinite number of solutions

Other terminology

consistent – a system that has at least one solution

a. **independent** – has exactly one solution
b. **dependent** – an infinite number of solutions

inconsistent – a system that has no solution

Number of Solutions (solutions are where they intersect)	exactly one solution	no solution	Infinitely many solutions
Definitions	consistent and independent	inconsistent	consistent and dependent
Graph			

****To solve a system of equations by graphing simply graph both equations on the same coordinate plane and find where they intersect.*

THREE METHODS FOR SOLVING SYSTEMS OF EQUATIONS

1. **Graphing**

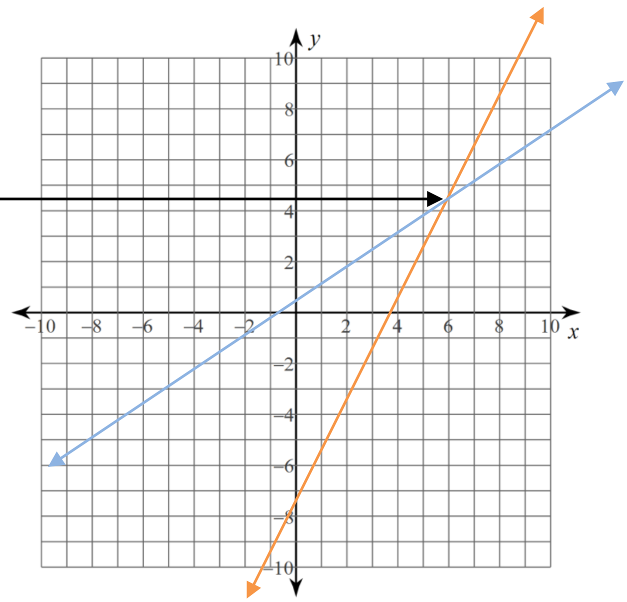
   a. Graph one equation

   b. Graph the other equation on the same plane.

   c. Find the point, or points, or intersection.

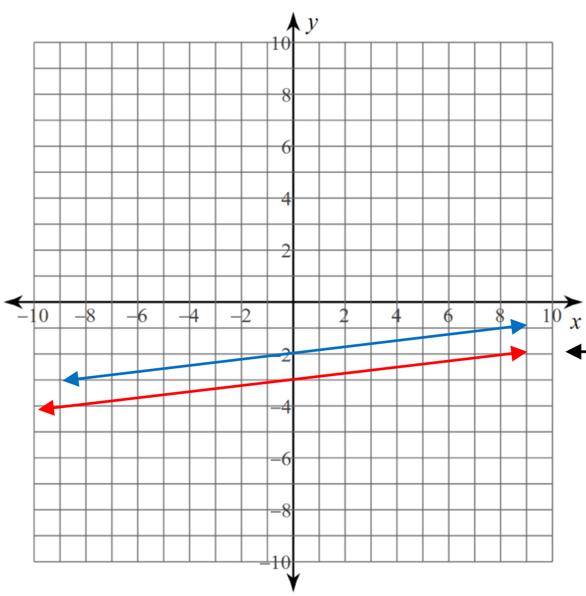
$$y = 2x - 7$$
$$y = \frac{2}{3}x + 1$$

Ex 1:



(6, 5) is the solution to the system.
It is consistent and independent.

Ex 2: $x - 27 = 9y$
$18 = x - 9y$



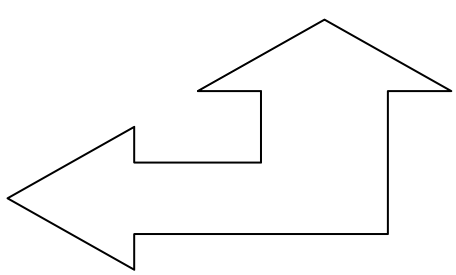
$$x - 27 = 9y$$
$$\frac{x}{9} - \frac{27}{9} = \frac{9y}{9}$$
$$\frac{1}{9}x - 3 = y$$

$$18 = x - 9y$$
$$\underline{-x} \quad \underline{-x}$$
$$-x + 18 = -9y$$
$$\frac{-x}{-9} + \frac{18}{-9} = \frac{-9y}{-9}$$
$$\frac{1}{9}x - 2 = y$$

The lines are parallel.
*There is **no solution** to this system.*
It is inconsistent.

Ex 3: $2x + 3y = 15$ and $y = -\frac{2}{3}x + 5$

$$\underline{-2x} \qquad \underline{-2x}$$
$$3y = -2x + 15$$
$$\frac{3y}{3} = \frac{-2x}{3} + \frac{15}{3}$$
$$y = \frac{-2}{3}x + 5$$



They are the same equation so they would graph into the same line.

*There are **infinitely many solutions**.*

This system is consistent and dependent.

2. **Substitution**

a. If possible, solve at least one equation for one variable.
b. Substitute the result into the other equation to replace one of the variables.
c. Solve the equation.
d. Substitute the value you just found into the first equation.
e. Solve for the other variable.
f. Write the solution as an ordered pair.

I'm choosing to solve for this y.

{Quick steps: Solve, Substitute, Solve, Substitute, Solve, Write the solution}

Ex 1: $8x + 5y = 2$ *and* $-2x + y = 4$

$-2x + y = 4$
$\underline{+2x \quad\quad +2x}$
$y = 2x + 4$

$8x + 5(2x + 4) = 2$
$8x + 10x + 20 = 2$
$18x + 20 = 2$
$\underline{-20 \quad -20}$
$18x = -18$

$\frac{18x}{18} = \frac{-18}{18}$

$x = -1$

$y = 2(-1) + 4$
$y = -2 + 4$
$y = 2$

The solution is $(-1,\ 2)$.
It is consistent and independent.

Ex 2: $-2x + 2y = -4$ *and* $-x = -y - 4$ *(let's solve this one for x first)*

$\frac{-x}{-1} = \frac{-y}{-1} - \frac{4}{-1}$
$x = y + 4$

$-2(y + 4) + 2y = -4$

$-2y - 8 + 2y = -4$

$-\cancel{2y} - 8 + \cancel{2y} = -4$

$-8 = -4$ *This is a false statement, therefore this system has no solution.*
The lines are parallel and are inconsistent.

Ex 3: $2x + y = 5$ *and* $-6x - 3y = -15$

$2x + y = 5$
$\underline{-2x \quad\quad -2x}$
$y = -2x + 5$

$-6x - 3(-2x + 5) = -15$

$-6x + 6x - 15 = -15$

$-\cancel{6x} + \cancel{6x} - 15 = -15$

$-15 = -15$

This is a true statement, therefore this system has infinitely many solutions. It is consistent and dependent.

3. **Elimination**

a. Write both equations so that like terms are aligned vertically.
b. Multiply one or both equations by a constant to get two equations that contain at least one set of exactly opposite terms.
c. Add the equations, eliminating one variable.
d. Solve for the remaining variable.
e. Substitute the value from (d) into one of the equations and solve for the other variable.
f. Write the solution as an ordered pair.

Ex 1: $6x + 14y = 6 \longrightarrow 6x + 14y = 6 \longrightarrow 6x + 14y = 6$

$-4x - 7y = -11 \longrightarrow 2[-4x - 7y = -11] \longrightarrow \underline{-8x - 14y = -22}$

$-2x = -16$

$\frac{-2x}{-2} = \frac{-16}{-2}$

$x = 8$

I choose the first equation to substitute x = 8 into.

$6(8) + 14y = 6$

$48 + 14y = 6$

$\underline{-48} \quad \underline{-48}$

$14y = -42$

$\frac{14y}{14} = \frac{-42}{14}$

$y = -3$

The solution is $(8, -3)$. *It is consistent and independent.*

Ex 2: $-8x + 2y = -10 \longrightarrow -8x + 2y = -10 \longrightarrow -8x + 2y = -10$

$-4x + y = -2 \longrightarrow -2[-4x + y = -2] \longrightarrow \underline{8x - 2y = 4}$

$0 = -6$

This is a false statement and has no solution. The lines are parallel. It is inconsistent.

Ex 3: $6x + 8y = -28 \longrightarrow 6x + 8y = -28 \longrightarrow 6x + 8y = -28$

$-3x - 4y = 14 \longrightarrow 2[-3x - 4y = 14] \longrightarrow \underline{-6x - 8y = 28}$

$0 = 0$

This is a **true** statement and has infinitely many solutions. The equations are the exact same line. It is consistent and dependent.