MYP Alg II Unit 8 Polynomial Functions Exam review

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. Rewrite the polynomial $12x^2 + 6 - 7x^5 + 3x^3 + 7x^4 - 5x$ in standard form. Then, identify the leading coefficient, degree, and number of terms. Name the polynomial.
   a. $-7x^5 + 7x^4 + 3x^3 + 12x^2 - 5x + 6$
      leading coefficient: $-7$; degree: 5; number of terms: 6; name: quintic polynomial
   b. $6 - 5x + 12x^2 + 7x^3 + 3x^4 - 7x^5$
      leading coefficient: 6; degree: 0; number of terms: 6; name: quintic polynomial
   c. $6 - 5x + 12x^2 + 3x^3 + 7x^4 - 7x^5$
      leading coefficient: 6; degree: 0; number of terms: 6; name: quintic polynomial
   d. $-7x^5 + 7x^4 + 12x^3 + 3x^2 - 5x + 6$
      leading coefficient: $-7$; degree: 5; number of terms: 6; name: quintic polynomial

2. Add. Write your answer in standard form.
   $(4d^5 - d^3) + (d^5 + 6d^3 - 4)$
   a. $5d^5 + 5d^3 - 4$
   b. $5d^5 + 5d^3$
   c. $5d^{10} + 5d^6 - 4$
   d. $4d^5 + 6d^3 - 4$

4. Graph the polynomial function $f(x) = -x^3 + 3x^2 + 2x^2 - 5x - 4$ on a graphing calculator. Describe the graph, and identify the number of real zeros.
   a. From left to right, the graph alternately increases and decreases, changing direction two times. It crosses the $x$-axis three times, so there appear to be three real zeros.
   b. From left to right, the graph increases and then decreases. It crosses the $x$-axis twice, so there appear to be two real zeros.
   c. From left to right, the graph alternately increases and decreases, changing direction three times. It crosses the $x$-axis four times, so there appear to be four real zeros.
   d. From left to right, the graph alternately increases and decreases, changing direction three times. It crosses the $x$-axis two times, so there appear to be two real zeros.

6. Find the product $2c \cdot d^4 (-4c^4d^5 - c^3d^2)$.
   a. $-2c^7d^9 + c^4d^{15}$
   b. $-8c^6d^{20} - 2c^3d^4$
   c. $2c^8d^{10} + 2c^5d^6$
   d. $-8c^7d^9 - 2c^4d^6$

7. Find the product $(5x - 3)(x^3 - 5x + 2)$.
   a. $5x^4 - 3x^3 - 25x^2 + 25x - 6$
   b. $5x^3 + 22x^2 - 5x - 6$
   c. $5x^3 - 28x^2 + 25x - 6$
   d. $5x^4 - 3x^3 + 25x^2 - 5x - 6$

9. Find the product $(x - 2y)^3$.
   a. $x^3 - 6x^2y + 12xy^2 - 8y^3$
   b. $x^3 + 8y^3$
   c. $x^3 - 8y^3$
   d. $x^3 + 6x^2y + 12xy^2 + 8y^3$
11. The right triangle shown is enlarged such that each side is multiplied by the value of the hypotenuse, $3y$. Find the expression that represents the perimeter of the enlarged triangle.

![Right Triangle Diagram]

a. $6y + 24xy$

b. $9y^2 + 8xy$

c. $9y^2 + 24xy$

d. $6y^2 + 24xy$

13. Divide by using synthetic division.

$(x^2 - 9x + 10) \div (x - 2)$

a. $x - 9 + \frac{6}{x - 2}$

b. $x - 11 + \frac{32}{x - 2}$

c. $x - 7 + \frac{4}{x - 2}$

d. $2x - 18 + \frac{10}{x - 2}$

14. Use synthetic substitution to evaluate the polynomial $P(x) = x^3 - 4x^2 + 4x - 5$ for $x = 4$.

a. $P(4) = 11$

c. $P(4) = -149$

b. $P(4) = -53$

d. $P(4) = 149$

15. Write an expression that represents the width of a rectangle with length $x + 5$ and area $x^3 + 12x^2 + 47x + 60$.

a. $x^2 + 7x + 12$

b. $x^2 + 17x + 132 + \frac{720}{x + 3}$

c. $x^2 + 17x - 38 - \frac{50}{x + 3}$

d. $x^3 + 7x^2 + 12x$

16. Determine whether the binomial $(x - 4)$ is a factor of the polynomial $P(x) = 5x^3 - 20x^2 - 5x + 20$.

a. $(x - 4)$ is not a factor of the polynomial $P(x) = 5x^3 - 20x^2 - 5x + 20$.

b. $(x - 4)$ is a factor of the polynomial $P(x) = 5x^3 - 20x^2 - 5x + 20$.

c. Cannot determine.

17. Factor $x^3 + 5x^2 - 9x - 45$.

a. $(x - 5)(x^2 + 9)$

b. $(x - 5)(x - 3)(x + 3)$

c. $(x + 5)(x - 3)(x + 3)$

d. $(x + 5)(x^2 + 9)$

18. Factor the expression $81x^6 + 24x^3y^3$.

a. $3x^3(3x + 2y)(9x^2 - 6xy + 4y^2)$

b. $3x^3(3x + 2y)^3$

c. $3x^3(3x + 2y)(9x^2 + 6xy + 4y^2)$

d. $3x^3(27x^3 + 8y^3)$

19. Solve the polynomial equation $3x^5 + 6x^4 - 72x^3 = 0$ by factoring.

a. The roots are $-6$ and $4$.

b. The roots are $0$, $-6$, and $4$.

c. The roots are $0$, $6$, and $-4$.

d. The roots are $-18$ and $12$. 

22. Identify the roots of $-3x^3 - 21x^2 + 72x + 540 = 0$. State the multiplicity of each root.
   a. $x - 5$ is a factor once, and $x + 6$ is a factor twice.
      The root 5 has a multiplicity of 1, and the root $-6$ has a multiplicity of 2.
   b. $x + 5$ is a factor once, and $x - 6$ is a factor twice.
      The root 5 has a multiplicity of 1, and the root $-6$ has a multiplicity of 2.
   c. $x - 5$ is a factor once, and $x + 6$ is a factor twice.
      The root $-5$ has a multiplicity of 1, and the root 6 has a multiplicity of 2.
   d. $x + 5$ is a factor once, and $x - 6$ is a factor twice.
      The root $-5$ has a multiplicity of 1, and the root 6 has a multiplicity of 2.

24. Identify all of the real roots of $4x^4 + 31x^3 - 4x^2 - 89x + 22 = 0$.
   a. $-2$ and $\frac{1}{4}$
   b. $-2, \frac{1}{4}, -3 + 2\sqrt{5}, -3 - 2\sqrt{5}$
   c. $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm2, \pm11, \pm\frac{11}{2}, \pm\frac{11}{4}$
   d. $-\frac{1}{4}, 3 + 2\sqrt{5}, 3 - 2\sqrt{5}$

25. Write the simplest polynomial function with zeros $-2$, $7$, and $\frac{-1}{2}$.
   a. $P(x) = x^3 + 9x^2 + 9x - 7$
   b. $P(x) = x^3 + 9x^2 + 9x + 7$
   c. $P(x) = x^3 - \frac{9}{2}x + \frac{9}{2}x - 7$
   d. $P(x) = x^3 - 2x^2 + 7x - \frac{1}{2}$

26. Solve $x^4 - 3x^3 - x^2 - 27x - 90 = 0$ by finding all roots.
   a. The solutions are 5 and $-2$.
   b. The solutions are 5, $-2$, $3i$, and $-3i$.
   c. The solutions are $-3, -1, -27$, and $-90$.
   d. The solutions are $-5, 2, 3i$, and $-3i$.

27. Write the simplest polynomial function with the zeros $2 - i$, $\sqrt{5}$, and $-2$.
   a. $P(x) = x^5 - 2x^4 - 3x^3 + 20x^2 + 15x - 50 = 0$
   b. $P(x) = x^5 - 2x^4 - 10x^3 + 16x^2 + 25x - 30 = 0$
   c. $P(x) = x^5 - 2x^4 - 3x^3 - 20x^2 - 65x - 50 = 0$
   d. $P(x) = x^6 - 4x^5 - 4x^4 + 36x^3 - 25x^2 - 80x + 100 = 0$

29. What polynomial function has zeros 1, $1 + i$, and $1 - i$?
   a. $P(x) = x^3 - 4x^2 + 3x - 1$
   b. $P(x) = x^3 + 2x^2 - 3x - 3$
   c. $P(x) = x^3 - 3x^2 + 4x - 2$
   d. $P(x) = x^3 - x^2 + 2x + 1$

30. Identify the leading coefficient, degree, and end behavior of the function $P(x) = -5x^4 - 6x^2 + 6$.
   a. The leading coefficient is $-5$. The degree is 4.
      As $x \to -\infty, P(x) \to -\infty$ and as $x \to +\infty, P(x) \to -\infty$
   b. The leading coefficient is $-5$. The degree is 6.
      As $x \to -\infty, P(x) \to -\infty$ and as $x \to +\infty, P(x) \to -\infty$
   c. The leading coefficient is $-5$. The degree is 4.
      As $x \to -\infty, P(x) \to +6$ and as $x \to +\infty, P(x) \to +6$
   d. The leading coefficient is $-5$. The degree is 6.
      As $x \to -\infty, P(x) \to +6$ and as $x \to +\infty, P(x) \to +6$
32. Graph the function \( f(x) = x^3 + 3x^2 - 6x - 8 \).
   a. 
   b. 
   c. 
   d. 

33. Graph \( g(x) = 4x^3 - 24x + 9 \) on a calculator, and estimate the local maxima and minima.
   a. The local maximum is about \(-13.627417\). The local minimum is about \(31.627417\).
   b. The local maximum is about \(31.627417\). The local minimum is about \(-13.627417\).
   c. The local maximum is about \(13.627417\). The local minimum is about \(-31.627417\).
   d. The local maximum is about \(22.627417\). The local minimum is about \(-22.627417\).
MULTIPLE CHOICE

1. ANS: A
The standard form is written with the terms in order from highest to lowest degree. In standard form, the degree of the first term is the degree of the polynomial. The polynomial has 6 terms. It is a quintic polynomial.

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<tr>
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<tbody>
<tr>
<td>A Correct!</td>
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<tr>
<td>B The standard form is written with the terms in order from highest to lowest degree.</td>
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<td>C The standard form is written with the terms in order from highest to lowest degree.</td>
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<tr>
<td>D Find the correct coefficient of the x-cubed term.</td>
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PTS: 1 DIF: Average REF: Page 407 OBJ: 6-1.2 Classifying Polynomials

2. ANS: A
\[(4d^2 - d^3) + (d^5 + 6d^3 - 4)\]
\[= (4d^2 + 6d^3) + (-d^3 + d^5) + (-4)\] Identify like terms. Rearrange terms to get like terms together.
\[= 5d^5 + 5d^3 - 4\] Combine like terms.

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<td>A Correct!</td>
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<td>B Check that you have included all the terms.</td>
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<td>C When adding polynomials, keep the same exponents.</td>
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<td>D First, identify the like terms and rearrange these terms so they are together. Then, combine the like terms.</td>
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PTS: 1 DIF: Basic REF: Page 407 OBJ: 6-1.3 Adding and Subtracting Polynomials NAT: 12.5.3.c STA: 2A.2.A

3. ANS: A
\[C(6) = 0.04(6)^3 - 0.65(6)^2 + 3.5(6) + 9 = 15.24\]
\[C(11) = 0.04(11)^3 - 0.65(11)^2 + 3.5(11) + 9 = 22.09\]
\[C(6)\] represents the cost, \$15.24, of delivering flowers to a destination that is 6 miles from the shop. \[C(11)\] represents the cost, \$22.09, of delivering flowers to a destination that is 11 miles from the shop.

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<td>A Correct!</td>
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<td>B You reversed the values of (C(6)) and (C(11)).</td>
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<td>C You added all the terms. There is a minus sign before 0.65.</td>
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<td>D Square the number of miles before multiplying by 0.65.</td>
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From left to right, the graph alternately increases and decreases, changing direction three times. The graph crosses the x-axis two times, so there appear to be two real zeros.

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**Feedback**

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<td>A</td>
<td>How many times does the graph change direction? How many times does the graph cross the x-axis?</td>
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<td>B</td>
<td>How many times does the graph change direction? How many times does the graph cross the x-axis?</td>
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<tr>
<td>C</td>
<td>How many times does the graph cross the x-axis?</td>
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<tr>
<td>D</td>
<td>Correct!</td>
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5. **ANS: A**

\[
h(x) - 2k(x) \\
= 2x^2 + 6x - 9 - 2(3x^2 - 8x + 8) \quad \text{Substitute the given values.} \\
= 2x^2 + 6x - 9 - 6x^2 + 16x - 16 \quad \text{Distribute.} \\
= -4x^2 + 22x - 25 \quad \text{Simplify.} \\
\]

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<td>A</td>
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<td>B</td>
<td>Check for algebra mistakes. Multiply every term in (k(x)) by (-2).</td>
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<td>C</td>
<td>Check for algebra mistakes. Multiply every term in (k(x)) by (-2).</td>
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<td>D</td>
<td>Check for algebra mistakes. Multiplying by (-2) changes the sign of every term in (k(x)).</td>
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6. **ANS: D**

Use the Distributive Property to multiply the monomial by each term inside the parentheses. Group terms to get like bases together, and then multiply.

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Multiply the coefficients for each term; don't add.

When multiplying like bases, add the exponents.

Don't forget to multiply the coefficients for each term.

Correct!


7. ANS: A

\[(5x - 3)(x^3 - 5x + 2)\]
\[= 5x(x^3 - 5x + 2) - 3(x^3 - 5x + 2)\]
\[= 5x^4 - 25x^2 + 10x - 3x^3 + 15x - 6\]
\[= 5x^4 - 3x^3 - 25x^2 + 25x - 6\]

Distribute 5x and -3.

Distribute 5x and -3 again.

Multiply.

Combine like terms.

Feedback

A Correct!

B Combine only like terms.

C Combine only like terms.

D Check the signs.


8. ANS: A

Total revenue is the product of the number of engines and the revenue per engine. \[T(x) = M(x)R(x)\]. Multiply the two polynomials using the distributive property.

\[6x^2 - 4x + 300\]

\[\times \quad 30x^2 + 70x + 1,000\]

\[6,000x^2 - 4,000x + 300,000\]

\[420x^3 - 280x^2 + 21,000x\]

\[180x^4 - 120x^3 + 9,000x^2\]

\[180x^4 + 300x^3 + 14,720x^2 + 17,000x + 300,000\]

Feedback

A Correct!

B Multiply each of the terms in the first polynomial by each of the terms in the second polynomial.

C First, multiply the coefficients. Then add the coefficients of like terms.

D First, multiply the coefficients. Then add the coefficients of like terms.

PTS: 1  DIF: Average  REF: Page 415  OBJ: 6-2.3 Application  NAT: 12.5.3.c  TOP: 6-2 Multiplying Polynomials

9. ANS: A
Write in expanded form.
\[(x-2y)(x-2y)(x-2y)\]

Multiply the last two binomial factors.
\[(x-2y)(x^3-4xy+4y^2)\]

Distribute the first term, distribute the second term, and combine like terms.
\[x^3-6x^2y+12xy^2-8y^3\]

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PTS: 1  DIF: Average  REF: Page 416  OBJ: 6-2.4 Expanding a Power of a Binomial  NAT: 12.5.3.c

10. ANS: B
The coefficients for \(n=4\) or row 5 of Pascal’s Triangle are 1, 4, 6, 4, and 1.
\[
(4x + 3)^4 = \left[1(4x)^4(+3)^0\right] + \left[4(4x)^3(+3)^1\right] + \left[6(4x)^2(+3)^2\right] + \left[4(4x)^1(+3)^3\right] + \left[1(4x)^0(+3)^4\right] \\
= 256x^4 + 768x^3 + 864x^2 + 432x + 81
\]

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PTS: 1  DIF: Average  REF: Page 417  OBJ: 6-2.5 Using Pascal’s Triangle to Expand Binomial Expressions

11. ANS: C
measure of leg 1 = 3y(4x) = 12xy
measure of leg 2 = 3y(4x) = 12xy
measure of hypotenuse = 3y(3y) = 9y^2

\[P = 9y^2 + 12xy + 12xy \]
\[P = 9y^2 + 24xy\]

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12. ANS: B
To divide, first write the dividend in standard form. Include missing terms with a coefficient of 0. 
\[6x^3 + 0x^2 + 5x - 8\]

Then write out in long division form, and divide.

\[
\begin{array}{c|cc}
\multicolumn{3}{c}{x - 2} \\
\hline
5x^3 & 0x^2 & 5x & -8 \\
- \(6x^3 - 12x^2\) & \hline
12x^2 & 5x & \hline
- \(12x^2 - 24x\) & \hline
29x - 8 & \hline
- \(29x - 58\) & \hline
50 & \hline
\end{array}
\]

Write out the answer with the remainder to get \(6x^2 + 12x + 29 + \frac{50}{x-2}\).

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<td>A Remember to include the remainder in the answer.</td>
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<td>B Correct!</td>
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<td>C Be careful when subtracting the terms.</td>
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<td>D Remember to divide by the &quot;(-2)&quot;.</td>
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13. ANS: C
For \((x - 2), a = 2\).

\[
\begin{array}{c|c|c|c}
2 & 1 & -9 & 10 \\
\hline & 2 & -14 & \\
1 & -7 & -4 & \\
\end{array}
\]
Write the coefficients of the expression. Bring down the first coefficient. Multiply and add each column.

Write the remainder as a fraction to get \(x - 7 + \frac{4}{x-2}\).

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<tr>
<td>A Multiply each column by the value 'a'.</td>
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<td>B The value 'a' occurs in the divisor as 'x – a'.</td>
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<tr>
<td>C Correct!</td>
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<td>D Begin synthetic division at the second coefficient.</td>
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14. ANS: A
Write the coefficients of the dividend. Use $\alpha = 4$.

$$
\begin{array}{cccc}
4 & 1 & -4 & 4 & -5 \\
\hline
4 & 0 & 16 \\
\hline
1 & 0 & 4 & 11 \\
\end{array}
$$

$P(4) = 11$

**Feedback**

| A | Correct! |
| B | Bring down the first coefficient. |
| C | Add each column instead of subtracting. |
| D | Write the coefficients in the synthetic division format. Some of them are negative numbers. |

**PTS:** 1  **DIF:** Basic  **REF:** Page 424  **OBJ:** 6-3.3 Using Synthetic Substitution  
**NAT:** 12.5.3.c  **TOP:** 6-3 Dividing Polynomials

15. **ANS:** A  

Width = \( \frac{\text{Area}}{\text{Length}} \)

\[
\text{width} = \frac{x^3 + 12x^2 + 47x + 60}{x + 5} \quad \text{Substitute.}
\]

Use synthetic division.

$$
\begin{array}{cccc}
-5 & 1 & 12 & 47 & 60 \\
\hline
-5 & -35 & -60 \\
\hline
1 & 7 & 12 & 0 \\
\end{array}
$$

The width can be represented by $x^2 + 7x + 12$.

**Feedback**

| A | Correct! |
| B | When dividing by $x + 5$, divide by $-5$ in synthetic division. |
| C | Add each column instead of subtracting. |
| D | The degree of the polynomial quotient is always one less than the degree of the dividend. |

**PTS:** 1  **DIF:** Average  **REF:** Page 425  **OBJ:** 6-3.4 Application  
**NAT:** 12.5.3.c  **TOP:** 6-3 Dividing Polynomials

16. **ANS:** B  

Find $P(4)$ by synthetic substitution.

$$
\begin{array}{cccc}
4 & 5 & -20 & -5 & 20 \\
\hline
20 & 0 & 0 \\
\hline
5 & 0 & -5 & 0 \\
\end{array}
$$

Since $P(4) = 0$, $x - 4$ is a factor of the polynomial $P(x) = 5x^3 - 20x^2 - 5x + 20$. 
A 

\[(x - r)\] is a factor of \(P(x)\) if and only if \(P(r) = 0\). Find \(P(r)\) by synthetic substitution.

B 
Correct!

C 

\[(x - r)\] is a factor of \(P(x)\) if and only if \(P(r) = 0\). Find \(P(r)\) by synthetic substitution.

PTS: 1
DIF: Average
REF: Page 430
OBJ: 6-4.1 Determining Whether a Linear Binomial is a Factor
NAT: 12.5.3.d
STA: 2A.2.A
TOP: 6-4 Factoring Polynomials

17. ANS: C

\[(x^3 + 5x^2) + (-9x - 45)\] 

Group terms.

\[= x^2(x + 5) - 9(x + 5)\] 

Factor common monomials from each group.

\[= (x + 5)(x^2 - 9)\] 

Factor out the common binomial.

\[= (x + 5)(x - 3)(x + 3)\] 

Factor the difference of squares.

PTS: 1
DIF: Average
REF: Page 431
OBJ: 6-4.2 Factoring by Grouping
NAT: 12.5.3.d
STA: 2A.2.A
TOP: 6-4 Factoring Polynomials

18. ANS: A

Factor out the GCF.

\[3x^3(27x^3 + 8y^3)\]

Write as a sum of cubes.

\[3x^3((3x)^3 + (2y)^3)\]

Factor.

\[3x^3(3x + 2y)((3x)^2 - 6xy + (2y)^2) = 3x^3(3x + 2y)(9x^2 - 6xy + 4y^2)\]

PTS: 1
DIF: Basic
REF: Page 431
OBJ: 6-4.3 Factoring the Sum or Difference of Two Cubes
NAT: 12.5.3.d
STA: 2A.2.A
TOP: 6-4 Factoring Polynomials

19. ANS: A

The graph indicates \(f(x)\) has zeroes at \(x = -1\) and \(x = 2\). By the Factor Theorem, \((x + 1)\) and \((x - 2)\) are factors of \(f(x)\). Use either root and synthetic division to factor the polynomial. Choose the root \(x = -1\).
Write $f(x)$ as a product.

$$f(x) = (x + 1)(-x^2 + 4x - 4)$$  

Factor out $-1$ from the quadratic.

$$f(x) = -(x + 1)(x^2 - 4x + 4)$$  

Factor the perfect-square quadratic.

$$f(x) = -(x + 1)^2$$

<table>
<thead>
<tr>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Correct!</td>
</tr>
<tr>
<td>B After identifying the roots, use synthetic division to factor the polynomial.</td>
</tr>
<tr>
<td>C The graph decreases as $x$ increases. How is this represented in the function?</td>
</tr>
<tr>
<td>D The Factor Theorem states that if $r$ is a root of $f(x)$, then $x - r$, not $x + r$, is a factor of $f(x)$.</td>
</tr>
</tbody>
</table>

**PTS:** 1  
**DIF:** Average  
**REF:** Page 432  
**OBJ:** 6-4.4 Application

### Example 20

Rewrite the expression as a difference of cubes.

$$\frac{1}{30} - \frac{1}{3}$$

Use $a^3 - b^3 = (a - b)(a^2 + ab + b^3)$. Simplify.

$$= (2x - 1)^3 - 3^3$$

Combine like terms.

$$= [(2x - 1) - 3][(2x - 1)^2 + 3(2x - 1) + 3^2]$$

$$= (2x - 4)(4x^2 - 4x + 1 + 6x - 3 + 9)$$

$$= (2x - 4)(4x^2 + 2x + 7)$$

<table>
<thead>
<tr>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Correct!</td>
</tr>
<tr>
<td>B Use the formula for factoring a difference of two cubes.</td>
</tr>
<tr>
<td>C Use the formula for factoring a difference of two cubes.</td>
</tr>
<tr>
<td>D Check your answer by multiplying the factors.</td>
</tr>
</tbody>
</table>

**PTS:** 1  
**DIF:** Advanced  
**NAT:** 12.5.3.d  
**TOP:** 6-4 Factoring Polynomials

### Example 21

Factor out the GCF, $3x^3$.

$$3x^5 + 6x^4 - 72x^3 = 0$$

Factor the quadratic.

$$3x^3(x + 6)(x - 4) = 0$$

Set each factor equal to 0.

$$3x^3 = 0, x + 6 = 0, x - 4 = 0$$

Solve for $x$.

$$x = 0, x = -6, x = 4$$

<table>
<thead>
<tr>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Set the GCF equal to zero.</td>
</tr>
<tr>
<td>B Correct!</td>
</tr>
<tr>
<td>C Set each factored expression equal to zero and solve.</td>
</tr>
</tbody>
</table>
Factor out the GCF first.

PTS: 1  DIF: Average  REF: Page 438  
OBJ: 6-5.1 Using Factoring to Solve Polynomial Equations  STA: 2A.2.A  
TOP: 6-5 Finding Real Roots of Polynomial Equations

22. ANS: A  
\[-3x^3 - 21x^2 + 72x + 540 = 0\]  
\[-3x^3 - 21x^2 + 72x + 540 = -3(x - 5)(x + 6)(x + 6)\]  
The root 5 has a multiplicity of 1.  
The root -6 has a multiplicity of 2.

<table>
<thead>
<tr>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

PTS: 1  DIF: Average  REF: Page 439  OBJ: 6-5.2 Identifying Multiplicity  
TOP: 6-5 Finding Real Roots of Polynomial Equations

23. ANS: A  
Let \( x \) be the width in inches. The length is \( x + 2 \), and the height is \( x - 1 \).

**Step 1** Find an equation.  
\[x(x + 2)(x - 1) = 140\]  
Volume is the product of the length, width, and height.  
\[x^3 + x^2 - 2x = 140\]  
Multiply the left side.  
\[x^3 + x^2 - 2x - 140 = 0\]  
Set the equation equal to 0.

**Step 2** Factor the equation, if possible.  
Factors of -140: \( \pm 1, \pm 2, \pm 4, \pm 5, \pm 7, \pm 10, \pm 14, \pm 20, \pm 28, \pm 35 \), Rational Root Theorem 
\( \pm 120, \pm 140 \).

Use synthetic substitution to test the positive roots (length can’t be negative) to find one that actually is a root.  
\[(x - 5)(x^2 + 6x + 28) = 0\]  
The synthetic substitution of 5 results in a remainder of 0. 5 is a root.  
\[x = \frac{-6 \pm \sqrt{36 - 4(1)(28)}}{2(1)} = \frac{-6 \pm \sqrt{-76}}{2}\]  
Use the Quadratic Formula to factor \( x^2 + 6x + 28 \).  
The roots are complex.

Width = 5 in.  
Width must be a positive real number.

<table>
<thead>
<tr>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
</tbody>
</table>
Be careful using synthetic substitution.

6 is not a possible root.

PTS: 1  DIF: Average  REF: Page 440  OBJ: 6-5.3 Application
TOP: 6-5 Finding Real Roots of Polynomial Equations

24. ANS: B

The possible rational roots are ±1, ±\(\frac{1}{2}\), ±\(\frac{1}{4}\), ±2, ±11, ±\(\frac{11}{2}\), ±\(\frac{11}{4}\).

Test −2.

\[
\begin{array}{c|cccc}
-2 & 4 & 31 & -4 & -89 & 22 \\
 & & -8 & -46 & 100 & -22 \\
\hline
& 4 & 23 & -50 & 11 & 0
\end{array}
\]

The remainder is 0, so −2 is a root.

Now test \(\frac{1}{4}\).

\[
\begin{array}{c|cccc}
\frac{1}{4} & 4 & 23 & -50 & 11 \\
 & & 1 & 6 & -11 \\
\hline
& 4 & 24 & -44 & 0
\end{array}
\]

The remainder is 0, so \(\frac{1}{4}\) is a root.

The polynomial factors to \((x + 2)(x - \frac{1}{4})(4x^2 + 24x - 44)\).

To find the remaining roots, solve \(4x^2 + 24x - 44 = 0\).

Factor out the common factor to get \(4(x^2 + 6x - 11) = 0\).

Use the quadratic formula to find the irrational roots.

\[
x = \frac{-6 \pm \sqrt{36 + 44}}{2} = -3 \pm 2\sqrt{5}
\]

The fully factored equation is \((x + 2)(x - \frac{1}{4})(x - (−3 + 2\sqrt{5}))(x - (−3 - 2\sqrt{5}))\).

The roots are −2, \(\frac{1}{4}\), \((−3 + 2\sqrt{5})\), \((−3 - 2\sqrt{5})\).

Feedback

A  These are the two rational roots. There are also irrational roots.
B  Correct!
C  These are the possible rational roots. Use these to find the rational roots.
D  Be careful when finding the irrational roots.

PTS: 1  DIF: Average  REF: Page 441  OBJ: 6-5.4 Identifying All of the Real Roots of a Polynomial Equation
TOP: 6-5 Finding Real Roots of Polynomial Equations

25. ANS: A

\[P(x) = 0\]

\[P(x) = (x + 2)(x - 7)(x + \frac{1}{2})\]  If \(r\) is a zero of \(P(x)\), then \(x - r\) is a factor of \(P(x)\).

\[P(x) = (x^2 - 5x - 14)(x + \frac{1}{2})\]  Multiply the first two binomials.

\[P(x) = x^3 - \frac{7}{2}x^2 + \frac{9}{2}x - 7\]  Multiply the trinomial by the binomial.
Feedback

A
Correct!

B
If \( r \) is a zero of \( P(x) \), then \((x - r)\), not \((x + r)\), is a factor of \( P(x) \).

C
The simplest polynomial with zeros \( r_1, r_2, \) and \( r_3 \) is \((x - r_1)(x - r_2)(x - r_3)\).

D
If \( r \) is a zero of \( P(x) \), then \((x - r)\) is a factor of \( P(x) \).

PTS: 1
DIF: Average
REF: Page 445
OBJ: 6-6.1 Writing Polynomial Functions Given Zeros
TOP: 6-6 Fundamental Theorem of Algebra

26.

ANS: B

The polynomial is of degree 4, so there are four roots for the equation.

**Step 1:** Identify the possible rational roots by using the Rational Root Theorem.

\[ \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 9, \pm 10, \pm 15, \pm 18, \pm 30, \pm 45, \pm 90 \]

\[ p = -90 \text{ and } q = 1 \]

**Step 2:** Graph \( x^4 - 3x^3 - x^2 - 27x - 90 = 0 \) to find the locations of the real roots.

The real roots are at or near 5 and -2.

**Step 3:** Test the possible real roots.

<table>
<thead>
<tr>
<th>Test the possible root of 5:</th>
<th>Test the possible root of -2:</th>
</tr>
</thead>
</table>
| \( \begin{array}{cccc}
5 & 1 & -3 & -1 \\
\hline
& 5 & 10 & 45 & 90
\end{array} \) | \( \begin{array}{cccc}
-2 & 1 & -3 & -1 \\
\hline
& -2 & 10 & -18 & 90
\end{array} \) |

| 1 & 2 & 9 & 18 & 0 | 1 & -5 & 9 & -45 & 0 |

The polynomial factors into \((x - 5)(x + 2)(x^2 + 9) = 0\).

Step 4: Solve \( x^2 + 9 = 0 \) to find the remaining roots.

\( x^2 = -9 \)

\( x = \pm 3i \)
The fully factored equation is \((x - 5)(x + 2)(x + 3i)(x - 3i) = 0\).
The solutions are 5, -2, -3i, and 3i.

### Feedback

| A | The polynomial is of degree 4, so there are 4 roots. |
| B | Correct! |
| C | Graph the equation to find the locations of the real roots. |
| D | Set each factored expression equal to zero and solve! |

PTS: 1   DIF: Average   REF: Page 446
OBJ: 6-6.2 Finding All Roots of a Polynomial Equation
TOP: 6-6 Fundamental Theorem of Algebra

27. ANS: A

There are five roots: \(2 - i, 2 + i, \sqrt{5}, -\sqrt{5}, \) and \(-2\). (By the Irrational Root Theorem and Complex Conjugate Root Theorem, irrational and complex roots come in conjugate pairs.) Since it has 5 roots, the polynomial must have degree 5.

Write the equation in factored form, and then multiply to get standard form.

\[ P(x) = 0 \]
\[ (x - (2 - i))(x - (2 + i))(x - \sqrt{5})(x - (-\sqrt{5}))(x - (-2)) = 0 \]
\[ (x^2 - 4x + 5)(x^2 - 5)(x + 2) = 0 \]
\[ (x^4 - 4x^3 + 20x - 25)(x + 2) = 0 \]
\[ P(x) = x^5 - 2x^4 - 8x^3 + 20x^2 + 15x - 50 = 0 \]

### Feedback

| A | Correct! |
| B | \(i\) squared is equal to \(-1\), so the opposite is equal to 1. |
| C | \(-4x(-5) = 20x\) |
| D | Only the irrational roots and the complex roots come in conjugate pairs. There are five roots in total. |

PTS: 1   DIF: Average   REF: Page 447
OBJ: 6-6.3 Writing a Polynomial Function with Complex Zeros
TOP: 6-6 Fundamental Theorem of Algebra

28. ANS: B

Write an equation to represent the volume of ice cream. Note that the hemisphere and the cone have the same radius, \(x\).

\[ V = V_{\text{cone}} + V_{\text{hemisphere}} \]

\[ V_{\text{cone}} = \frac{1}{3} \pi x^2 h \]
\[ = \frac{1}{3} \pi x^2 (10) \]
\[ = \frac{10}{3} \pi x^2 \]

\[ V_{\text{hemisphere}} = \frac{1}{2} V_{\text{sphere}} \]
\[ = \frac{1}{2} \left( \frac{4}{3} \pi x^3 \right) \]
\[ = \frac{2}{3} \pi x^3 \]

So,
\[ V(x) = \frac{10}{3} \pi x^2 + \frac{2}{3} \pi x^3 \]

\[ 96 \pi = \frac{10}{3} \pi x^2 + \frac{2}{3} \pi x^3 \]

Set the volume equal to 96π.

\[ 0 = \frac{2}{3} \pi x^3 + \frac{10}{3} \pi x^2 - 96 \pi \]

Write in standard form.

\[ 0 = 2x^3 + 10x^2 - 288 \]

Multiply both sides by \( \frac{3}{\pi} \).

The graph indicates a possible positive root of 4. Use synthetic division to verify that 4 is a root, and write the equation as \((x - 4)(2x^2 + 13x + 72)\). Since the discriminant of \(2x^2 + 18x + 72\) is \(-252\), the roots of \(2x^2 + 18x + 72\) are complex. The radius must be a positive real number, so the radius of the sugar cone is 4 cm.

### Feedback

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>Write the total volume as the sum of the volume of a cone of height 10 cm and the volume of a hemisphere. Then solve for the radius.</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>Correct!</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>Write the total volume as the sum of the volume of a cone of height 10 cm and the volume of a hemisphere. Then solve for the radius.</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>Write the total volume as the sum of the volume of a cone of height 10 cm and the volume of a hemisphere. Then solve for the radius.</td>
</tr>
</tbody>
</table>

PTS: 1  DIF: Average  REF: Page 447  OBJ: 6-6.4 Problem-Solving Application

29.  ANS: C

\( P(x) = 0 \)

\((x - 1)(x - (1 + i))[x - (1 - i)] = 0 \)

If \( r \) is a root of \( P(x) \), then \( x - r \) is a factor of \( P(x) \).

\((x - 1)(x - 1 - i)(x - 1 + i) = 0 \)

Distribute.

\((x - 1)(x^2 - x + x + 1 - i + i + x + 1) = 0 \)

Multiply the trinomials. Use \(-i^2 = 1\).

\((x - 1)(x^2 - 2x + 2) = 0 \)

Combine like terms.

\( x^2 - 2x^2 + 2x - x^2 + 2x - 2 = 0 \)

Multiply the binomial and trinomial.

\( x^2 - 3x^2 + 4x - 2 = 0 \)

Combine like terms.

### Feedback

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>If ( r ) is a root of ( P(x) ), then ( x - r ) is a factor of ( P(x) ).</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>First, multiply the factors. Then, combine like terms to get a polynomial function.</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>Correct!</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>If ( r ) is a root of ( P(x) ), then ( x - r ) is a factor of ( P(x) ).</td>
</tr>
</tbody>
</table>

PTS: 1  DIF: Advanced  TOP: 6-6 Fundamental Theorem of Algebra

30.  ANS: A

The leading coefficient is \(-5\), which is negative. The degree is 4, which is even.

So, as \( x \to -\infty \), \( P(x) \to -\infty \) and as \( x \to +\infty \), \( P(x) \to -\infty \).

### Feedback

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>Correct!</td>
</tr>
</tbody>
</table>
31. ANS: D

As \( x \to -\infty \), \( P(x) \to -\infty \) and as \( x \to \infty \), \( P(x) \to \infty \).

\( P(x) \) is of odd degree with a positive leading coefficient.

### Feedback

<table>
<thead>
<tr>
<th></th>
<th>The degree is the greatest exponent.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>For polynomials, the function always approaches positive infinity or negative infinity as ( x ) approaches positive infinity or negative infinity.</td>
</tr>
<tr>
<td>D</td>
<td>The degree is the greatest exponent.</td>
</tr>
</tbody>
</table>

PTS: 1 DIF: Basic REF: Page 454

OBJ: 6-7.1 Determining End Behavior of Polynomial Functions

TOP: 6-7 Investigating Graphs of Polynomial Functions

32. ANS: D

**Step 1:** Identify the possible rational roots by using the Rational Root Theorem. \( p = -8 \) and \( q = 1 \), so roots are positive and negative values in multiples of 2 from 1 to 8.

**Step 2:** Test possible rational zeros until a zero is identified.

<table>
<thead>
<tr>
<th>Test ( x = 1 ).</th>
<th>Test ( x = -1 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( 3 )</td>
<td>( 3 )</td>
</tr>
<tr>
<td>( -6 )</td>
<td>( -6 )</td>
</tr>
<tr>
<td>( -8 )</td>
<td>( -8 )</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>( -2 )</td>
<td>( -2 )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( 4 )</td>
<td>( 2 )</td>
</tr>
<tr>
<td>( -10 )</td>
<td>( 8 )</td>
</tr>
</tbody>
</table>

\( x = -1 \) is a zero, and \( f(x) = (x + 1)(x^2 + 2x - 8) \).

**Step 3:** Factor: \( f(x) = (x + 1)(x - 2)(x + 4) \).

The zeros are \(-1, 2, \) and \(-4\).

**Step 4:** Plot other points as guidelines.

\( f(0) = -8 \) so the y-intercept is \(-8\). Plot points between the zeros.

\( f(1) = -10 \) and \( f(-3) = 10 \)

**Step 5:** Identify end behavior.
The degree is odd and the leading coefficient is positive, so as \( x \to -\infty, P(x) \to -\infty \) and as \( x \to +\infty, P(x) \to +\infty \).

**Step 6:** Sketch the graph by using all of the information about \( f(x) \).

<table>
<thead>
<tr>
<th>Feedback</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>The leading coefficient is positive, so ( x ) should go to negative infinity as ( P(x) ) goes to negative infinity.</td>
</tr>
<tr>
<td>B</td>
<td>The ( y )-intercept should be the same as the last term in the equation.</td>
</tr>
<tr>
<td>C</td>
<td>The function is cubic, so should have 3 roots.</td>
</tr>
<tr>
<td>D</td>
<td>Correct!</td>
</tr>
</tbody>
</table>

**Feedback**

A | You reversed the values of the maximum and minimum. |
B | Correct! |
C | The constant is a positive number. |
D | You forgot to add the constant of the function to the calculator. |

**Step 1** Graph \( g(x) \) on a calculator.

The graph appears to have one local maximum and one local minimum.

**Step 2** Use the maximum feature of your graphing calculator to estimate the local maximum. The local maximum is about 31.627417.

**Step 3** Use the minimum feature of your graphing calculator to estimate the local minimum. The local minimum is about –13.627417.

**Feedback**

A | Correct! |
B | Find the \( x \)-value for the local maximum. |
C | Find the \( x \)-value for the local maximum. |
D | Find the \( x \)-value for the local maximum. |

**33.**

**ANS:** B

**Step 1** Graph \( g(x) \) on a calculator.

The graph appears to have one local maximum and one local minimum.

**Step 2** Use the maximum feature of your graphing calculator to estimate the local maximum. The local maximum is about 31.627417.

**Step 3** Use the minimum feature of your graphing calculator to estimate the local minimum. The local minimum is about –13.627417.

**Feedback**

A | Correct! |
B | Find the \( x \)-value for the local maximum. |
C | Find the \( x \)-value for the local maximum. |
D | Find the \( x \)-value for the local maximum. |

**34.**

**ANS:** A

Find a formula to represent the volume. Use \( x \) as the side length for the squares you are cutting out.

\[
V(x) = x(8.5 - 2x)(11 - 2x)
\]

Graph \( V(x) \). Note that values of \( x \) less than 0 or greater than 4.25 do not make sense for this problem. The graph has a local maximum of about 66.1 when \( x \approx 1.6 \). So, the largest open box will have a volume of about 66.1 inches cubed when the sides of the squares are about 1.6 inches long.

**Feedback**

A | Correct! |
B | Find the \( x \)-value for the local maximum. |
C | Find the \( x \)-value for the local maximum. |
D | Find the \( x \)-value for the local maximum. |
35. **ANS:** A

\[ g(x) = f(x) + 2 \]

\[ g(x) = (x^3 + 1) + 2 \]

\[ g(x) = x^3 + 3 \]

To graph \( g(x) = f(x) + 2 \), translate the graph of \( f(x) \) up 2 units. This is a vertical translation.

### Feedback

<table>
<thead>
<tr>
<th>A</th>
<th>Correct!</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>( f(x) + c ) represents a vertical translation of ( f(x) ).</td>
</tr>
<tr>
<td>C</td>
<td>( f(x) + c ) represents a vertical translation of ( f(x) ).</td>
</tr>
<tr>
<td>D</td>
<td>The sign of ( c ) determines whether ( f(x + c) ) represents a vertical translation of ( f(x) ) (</td>
</tr>
</tbody>
</table>

36. **ANS:** D

For a function \( g(x) \) that reflects \( f(x) \) across the y-axis:

\[ g(x) = f(-x) \]

\[ g(x) = 5(-x)^3 + 7(-x)^2 + 4(-x) - 5 \]

\[ g(x) = -5x^3 + 7x^2 - 4x - 5 \]

### Feedback

<table>
<thead>
<tr>
<th>A</th>
<th>This is a reflection of ( f(x) ) across the x-axis. To reflect across the y-axis, replace ( x ) with ( (-x) ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>A negative number squared is a positive number.</td>
</tr>
<tr>
<td>C</td>
<td>The constant remains the same.</td>
</tr>
<tr>
<td>D</td>
<td>Correct!</td>
</tr>
</tbody>
</table>

37. **ANS:** A

\[ g(x) = f(2x) \]

\[ g(x) = (2x)^4 - 3(2x)^2 - 1 \]

\[ g(x) = 16x^4 - 12x^2 - 1 \]

### Feedback

<table>
<thead>
<tr>
<th>A</th>
<th>Correct!</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>The transformation is inside the function; this makes a horizontal transformation.</td>
</tr>
<tr>
<td>C</td>
<td>The transformation is inside the function; this makes a horizontal transformation.</td>
</tr>
</tbody>
</table>
The function makes a different type of horizontal transformation.

PTS: 1  DIF: Average  REF: Page 461
OBJ: 6-8.3 Compressing and Stretching Polynomial Functions
TOP: 6-8 Transforming Polynomial Functions

38. ANS: A
\[ g(x) = 6f(x+5) \]
\[ g(x) = 6(2(x+5)^3 + 4) \]
\[ g(x) = 12(x+5)^3 + 24 \]

Feedback
A Correct!
B The left shift value is added to the x value before it is cubed.
C A shift to the left involves adding, not subtracting.
D The vertical stretch factor will effect the y-intercept.

PTS: 1  DIF: Average  REF: Page 462  OBJ: 6-8.4 Combining Transformations
TOP: 6-8 Transforming Polynomial Functions

39. ANS: D
\[ g(x) = f(x+4) \]
\[ g(x) = (x+4)^3 - 5(x+4)^2 + 2(x+4) + 2 \]
\[ g(x) = x^3 + 12x^2 + 48x + 64 - 5x^2 - 40x - 80 + 2x + 8 + 2 \]
\[ g(x) = x^3 + 7x^2 + 10x - 6 \]

The transformation represents a horizontal shift left of 4 units, which corresponds to making the same profit for selling 4 fewer bicycles.

Feedback
A The transformation is \( f(x+4) \), not \( f(x) + 4 \).
B Correct!
C The transformation is a horizontal shift left.
D The transformation is not a vertical shift.

PTS: 1  DIF: Average  REF: Page 462  OBJ: 6-8.5 Application
TOP: 6-8 Transforming Polynomial Functions

40. ANS: B
The x-intercepts are constant, so the transformation is not a horizontal shift or a horizontal stretch.

The graph of \( g(x) \) is symmetric about the x-axis, so the transformation is not a vertical shift. \( g(x) \) has a higher maximum and a lower minimum than \( f(x) \), showing a vertical stretch. So the transformation is a vertical stretch.

Feedback
A The transformed function is symmetric about the x-axis, so the transformation is not a vertical shift.
B Correct!
C The x-intercepts are constant, so the transformation is not a horizontal shift.
D The x-intercepts are constant, so the transformation is not a horizontal stretch.
41. ANS: A
The x-values increase by a constant, 2. Find the differences of the y-values.

<table>
<thead>
<tr>
<th>( y )</th>
<th>(-12)</th>
<th>(-7)</th>
<th>(-21)</th>
<th>(-51)</th>
<th>(-93)</th>
<th>(-142)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First differences</td>
<td>5</td>
<td>-14</td>
<td>-30</td>
<td>-42</td>
<td>-49</td>
<td>Not constant</td>
</tr>
<tr>
<td>Second differences</td>
<td>-19</td>
<td>-16</td>
<td>-12</td>
<td>-7</td>
<td>Not constant</td>
<td></td>
</tr>
<tr>
<td>Third differences</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td>Not constant</td>
<td></td>
</tr>
<tr>
<td>Fourth differences</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>Constant</td>
<td></td>
</tr>
</tbody>
</table>

The fourth differences are constant. A quartic polynomial best describes the data.

**Feedback**

**A** Correct!

**B** Check your work. The third differences are not constant.

**C** Check your work. The second differences are not constant.

**D** To find the differences in the y-values, subtract each y-value from the y-value that follows it.

42. ANS: A
Find the finite differences for the y-values.

<table>
<thead>
<tr>
<th>Population</th>
<th>280</th>
<th>437</th>
<th>571</th>
<th>781</th>
<th>1164</th>
</tr>
</thead>
<tbody>
<tr>
<td>First differences</td>
<td>157</td>
<td>134</td>
<td>210</td>
<td>383</td>
<td></td>
</tr>
<tr>
<td>Second differences</td>
<td>-23</td>
<td>76</td>
<td>173</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third differences</td>
<td>99</td>
<td>97</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The third differences of these data are not exactly constant, but because they are relatively close, a cubic function would be a good model. Using the cubic regression feature on a calculator, the function is found to be:

\[ f(x) \approx 0.13x^3 - 2.39x^2 + 40x + 280 \]
43. **ANS:** A

Let \( x \) represent the number of weeks before the election. Make a scatter plot of the data. The function appears to be cubic or quartic. Use the regression feature to check the \( R^2 \)-values.

\[
\text{cubic: } R^2 \approx 0.7402 \quad \text{quartic: } R^2 \approx 0.8214
\]

The quartic function is a more appropriate choice. The data can be modeled by

\[
f(x) = 8.16x^4 - 126.60x^3 + 466.66x^2 + 16.83x + 2649.93
\]

Substitute 5 for \( x \) in the quartic model.

\[
f(5) = 8.16(5)^4 - 126.60(5)^3 + 466.66(5)^2 + 16.83(5) + 2649.93 = 3675.58
\]

Based on the model, the number of supporters 5 weeks before the election was 3676.

<table>
<thead>
<tr>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

**PTS:** 1  **DIF:** Average  **REF:** Page 468  **OBJ:** 6-9.3 Application  **TOP:** 6-9 Curve Fitting by Using Polynomial Models

44. **ANS:** A

\[
f(x) = (x - a_1)(x - a_2)(x - a_3)(x - a_4)
\]

If \( r \) is a root of \( P(x) \), then \( x - r \) is a factor of \( P(x) \).

\[
f(x) = [x - (-2)] [x - (-\frac{1}{2})] [x - (\frac{1}{2})] [x - (\frac{3}{2})]
\]

Substitute the roots from the graph.

\[
f(x) = (x + 2)(x + \frac{1}{2})(x - \frac{1}{2})(x - \frac{3}{2})
\]

Simplify.

\[
f(x) = (x + 2)(2x + 1)(2x - 1)(2x - 3)
\]

Multiply by 8 and simplify.

<table>
<thead>
<tr>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

**PTS:** 1  **DIF:** Advanced  **TOP:** 6-9 Curve Fitting by Using Polynomial Models

**NUMERIC RESPONSE**

45. **ANS:** 16

**PTS:** 1  **DIF:** Average  **TOP:** 6-3 Dividing Polynomials

46. **ANS:** 45
47. ANS: 2

PTS: 1  DIF: Advanced  TOP: 6-5 Finding Real Roots of Polynomial Equations

PTS: 1  DIF: Advanced  TOP: 6-7 Investigating Graphs of Polynomial Functions