Chapter 7

Name_____ Date _____

Chapter 7 Polynomial Equations and Factoring

Dear Family,

Has your family ever visited an amusement park? If so, take a family vote. What is your favorite ride in the amusement park and why? Do you all agree? Have you considered the science behind the rides at an amusement park?

Together, make a list of as many different types of amusement park rides as you can. Now, group the rides into categories. For example, put all the rides that spin into one category, and the rides that go up and down into another category, etc. Some of the rides may overlap categories.

Now consider how each ride works. Start with the roller coaster. Besides the initial pull up the first big hill, a roller coaster is driven by forces of nature. The coaster stores energy (known as potential energy) as it is climbing the first big hill preparing for descent on the other side. The energy of motion is known as kinetic energy. The coaster converts potential energy into kinetic energy back and forth as it progresses through the loops, hills, and valleys of the track. In return, the coaster accelerates and decelerates producing a strange sensation for the rider. The force of gravity also plays an important role in the thrill of the ride.

The designers of a roller coaster cannot just place the hills and loops wherever they want. They must rely on precise mathematical calculations to design a safe but awesome ride. So how do they do that?

- Using the Internet, research to find equations that roller-coaster designers use to determine potential energy and kinetic energy. Identify what the variables represent in each equation.
- Research the effects that hills, loops, and valleys have on the body. If you are a thrill seeker, is it better to sit in the front, middle, or back of the roller coaster?

Pick one other amusement park ride to research. Consider a free-fall ride or a pendulum-style ride. What types of sensations does the rider experience on these types of rides? Research the math behind the ride you chose. How does a designer make sure these rides are safe?

Find a website that would allow you to build your own amusement park ride to see if you can design a safe, thrill-seeking ride using the concepts you have learned. Amusement parks are filled with rides and attractions that utilize the concepts you will learn in this chapter. Enjoy the ride!

Nombre _____ Fecha _____

Capítulo 7 Ecuaciones polinomiales y factorización

Estimada familia:

¿Su familia ha ido alguna vez a un parque de diversiones? Si han ido, hagan una votación en familia. ¿Cuál es su juego favorito del parque de diversiones y por qué? ¿Todos están de acuerdo? ¿Han considerado las ciencias que hay detrás de los juegos de un parque de diversiones?

Juntos, hagan una lista de todos los tipos de juegos diferentes que hay en un parque de diversiones. Ahora, agrupen los juegos en categorías. Por ejemplo, coloquen todos los juegos que giran en una categoría y todos los juegos que suben y bajan en otra categoría, etc. Algunos de los juegos quizás pertenezcan a varias categorías.

Ahora, consideren cómo funciona cada juego. Comiencen con la montaña rusa. Además de la jalada inicial en la primera colina grande, una montaña rusa está accionada por las fuerzas de la naturaleza. La montaña rusa almacena energía (conocida como energía potencial) a medida que sube por la primera colina grande y se prepara para descender por el otro lado. La energía del movimiento se llama energía cinética. La montaña rusa convierte la energía potencial en energía cinética una y otra vez a medida que avanza por las vueltas, las colinas y los valles del circuito. Como consecuencia, la montaña rusa acelera y desacelera y así produce una sensación extraña para la persona que está en el juego. La fuerza de gravedad también desempeña un papel importante en la emoción del juego.

Los diseñadores de una montaña rusa no pueden poner las colinas y las vueltas donde se les ocurra. Deben hacer cálculos matemáticos precisos para diseñar un juego seguro pero sorprendente. Entonces, ¿cómo hacen eso?

- Investiguen en Internet para hallar ecuaciones que usen los diseñadores de montañas rusas para determinar la energía potencial y la energía cinética. Identifiquen qué representan las variables en cada ecuación.
- Investiguen los efectos que tienen las colinas, las vueltas y los valles en el organismo. Si buscan emoción, ¿es mejor sentarse al frente, en el medio o en la parte trasera de una montaña rusa?

Elijan otro juego de parque de diversiones para investigar. Consideren un juego en caída libre o un juego tipo péndulo. ¿Qué tipos de sensaciones tiene la persona que sube a estas clases de juegos? Investiguen las matemáticas que hay detrás del juego que eligieron. ¿De qué manera un diseñador se cerciora de que los juegos son seguros?

Hallen un sitio web que les permita construir su propio juego de parque de diversiones para ver si pueden diseñar un juego seguro y emocionante usando los conceptos que han aprendido. En los parques de diversiones, abundan los juegos y las atracciones que usan los conceptos que aprenderán en este capítulo. ¡Que disfruten el juego!

7.1 Start Thinking

Use a table to make a list of at least five different objects around you. Choose a letter of the alphabet (not "o" because it may be confused with zero) to represent each object. If you choose the same letter for more than one object, choose an exponent greater than one to accompany the letter for each duplicate. Denote how many of each object there are in the room.

Write an expression so that the number denoting how many there are of each object is in front of the letter representing that object (for example, three tables would be $3t$, if t represents tables). Separate each object with a plus sign.

7.1 Warm Up

Simplify.

1. $2 + (-17)$

2. $7 - 6$

3. $-38 + (-14)$

4. $14 - (-24)$

5. $0 + (-9)$

6. $6 - 15$

7.1 Cumulative Review Warm Up

Solve the system of linear equations by elimination. Check your solution.

1. $x + y = 8$
$x - y = 6$

2. $x - 3y = -7$
$x + 2y = 18$

3. $4x - 5y = -114$
$x + 2y = 30$

4. $x + 3y = 4$
$3x - 9y = -6$

Name _____ Date _____

7.1 Practice A

In Exercises 1–3, find the degree of the polynomial.

1. $7n^3$ **2.** $\frac{1}{3}x^5$ **3.** w^2y^5

In Exercises 4–6, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

4. $5h - 4h^3 - 2$ **5.** $10 + 4p^3$ **6.** $6v^7$

7. The expression $-16t^2 + 20t + 100$ represents the height of an object t seconds after it is dropped from a height of 100 feet. Why is this expression a trinomial? What is its degree?

In Exercises 8–11, find the sum.

8. $(7t + 6) + (-4t - 2)$ **9.** $(-12v + 3) + (8v - 7)$

10. $(3j^2 - 7j + 1) + (-6j^2 - 4j + 9)$ **11.** $(2w^2 - 7w + 3) + (2w^2 + 8w)$

In Exercises 12–15, find the difference.

12. $(p - 5) - (4p - 7)$ **13.** $(8w + 3) - (9w + 6)$

14. $(3y^2 - 6y + 9) - (6y^2 - 7y - 2)$ **15.** $(5b^2 - 6b - 9) - (-2b^2 + 8b - 1)$

16. Describe and correct the error in finding the sum.

$$\begin{aligned} (x^3 - 8x + 2) + (3x^3 + 7x + 6) &= x^3 - 8x + 2 + 3x^3 + 7x + 6 \\ &= (x^3 + 3x^3) - (8x + 7x) + (2 + 6) \\ &= 4x^3 - 15x + 8 \end{aligned}$$

In Exercises 17 and 18, find the sum or difference.

17. $(3p^2 - 6pq + 7q^2) - (p^2 - 5pq + 9q^2)$

18. $(x^2 - 4xy + 9y^2) + (-8x^2 + 6xy - y^2)$

19. Your friend says that when subtracting polynomials, the order in which you subtract does not matter. Is your friend correct? Explain.

Name_____ Date _____

7.1 Practice B

In Exercises 1–3, find the degree of the polynomial.

1. $-3.25n^8$ **2.** $\frac{1}{5}x^4yz^2$ **3.** uv^3w^9

In Exercises 4–6, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

4. $3t - 8t^2 + 10t^5$ **5.** $\frac{2}{9}n^2 - \pi n + 3n^4$ **6.** $\sqrt{14}p^5$

7. The monthly profit for a small company is represented by $250x^5 - 42x^2 + 112x$, where x is the number of beds sold. Classify the polynomial by the number of terms. What is its degree?

In Exercises 8–11, find the sum.

8. $(-2t^2 - 7t + 5) + (-8t^2 + 4t - 3)$ **9.** $(8y^2 - 2y + 4) + (5y^2 - 7y)$

10. $(3k - 5k^3 + 9) + (8k^3 - 4k + 8)$ **11.** $(3q^2 - 7q - 6) + (2q^2 - 5q^3 + 8q)$

In Exercises 12–15, find the difference.

12. $(t^3 - 5t^2 - 7) - (t - 11)$ **13.** $(-w - 13) - (-3w^3 + w^2 + 6w)$

14. $(x^4 - x^2 + 9) - (13 - 6x^2 + 8x)$ **15.** $(3g - 5g^3 + 6g^2) - (12g^3 + 9g - 10)$

16. The number of economy-size cars rented in w weeks is represented by $152 + 3w$. The number of full-size cars rented in w weeks is represented by $99 + 2w$. Write a polynomial that represents how many more economy cars are rented in w weeks than full-size cars.

In Exercises 17 and 18, find the sum or difference.

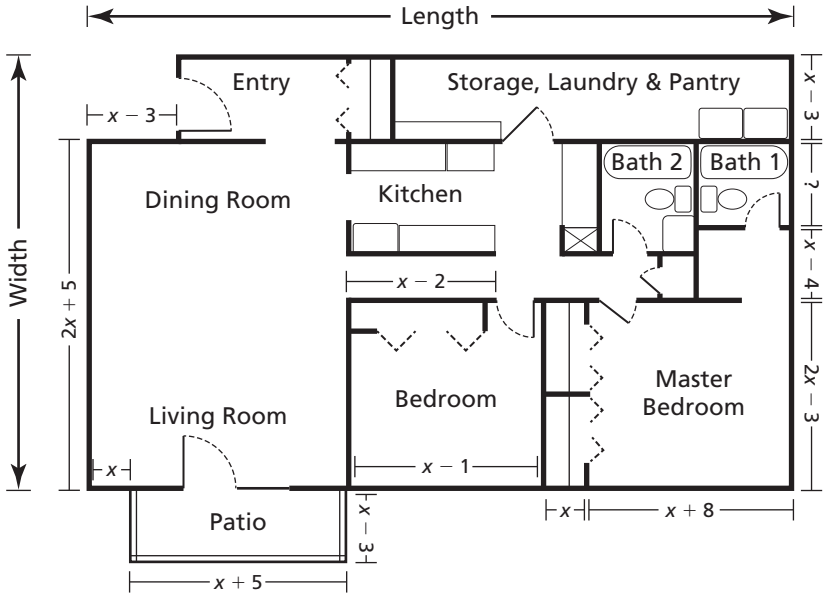
17. $(g^2 - 9h^2) + (g^2 - 15gh + 8h^2)$ **18.** $(-m^2 - 5mn) - (m^2 + 3mn - 9n^2)$

19. The polynomial $-16t^2 + v_0t + s_0$ represents the height (in feet) of an object, where v_0 is the initial vertical velocity (in feet per second), s_0 is the initial height of the object (in feet), and t is the time (in seconds). Write a polynomial that represents the height of an object that has initial velocity 25 feet per second and initial height 4 feet. Then find the height of the object after 1 second.

Name _____ Date _____

7.1 Enrichment and Extension

House Plans



Use the diagram above to answer the following questions. All measurements are in feet.

1. Find the length of the two-bedroom apartment in terms of x.
2. Find the width of the two-bedroom apartment, not including the patio, in terms of x.
3. Find the perimeter of the whole apartment, including the patio, in terms of x.
4. Find the perimeter of the living/dining area in terms of x.
5. Find the perimeter of the master bedroom, excluding the closets and sitting area, in terms of x.
6. The perimeter of the living/dining area is 60 feet. What is the value of x? Be sure to include units of measure.
7. The perimeter of the master bedroom, excluding the closets and sitting area, is 52 feet. What is the value of x?
8. The length of the apartment is 57 feet. What is the value of x?
9. Find the area of the whole apartment, excluding the patio, in terms of x.
10. Find the area of the whole apartment, including the patio, in terms of x.
11. The area of the kitchen is $x^2 - 5x + 6$. What is the width of the kitchen?

Name_____ Date _____

7.1 Puzzle Time

How Does A Flea Travel So Fast?

Write the letter of each answer in the box containing the exercise number.

Find the sum or difference.

1. $(6x + 5) + (-3x + 7)$

2. $(-9x - 13) + (8x + 3)$

3. $(2x - 8) - (4x - 2)$

4. $(5x + 8) - (6x + 2)$

5. $(3x^2 - 6x - 7) + (-2x^2 - 4x + 12)$

6. $(-x^2 - 5x + 8) - (4x^2 - 7x - 10)$

7. $(6x^2 - 3x + 10) - (-6x^2 + 11x + 9)$

8. $(-13x^3 + 15x^2 - 12x) + (-x^3 - 4x^2 - 15x + 1)$

9. $(7x^3 - x + 14) - (2x^2 - 19)$

10. $(8x - 3x^3 - 5) + (4x^3 - 6x^2 + 11)$

11. $(-5x - 16) - (-3x^3 + 2x^2 + 9x)$

Answers

Y. $-x - 10$

H. $x^3 - 6x^2 + 8x + 6$

C. $-5x^2 + 2x + 18$

I. $3x + 12$

G. $-14x^3 + 11x^2 - 27x + 1$

B. $x^2 - 10x + 5$

I. $7x^3 - 2x^2 - x + 33$

K. $12x^2 - 14x + 1$

N. $-2x - 6$

H. $21x^2 + 5x + 8$

T. $3x^3 - 2x^2 - 14x - 16$

I. $-x + 6$

12. The amount of merchandise (in millions) that store A sold can be represented by $A = 13x^2 + 8x - 3$. The amount of merchandise (in millions) that store B sold can be represented by $B = 8x^2 - 3x + 11$. Find the total amount of merchandise that stores A and B sold.

5	2		9	11	6	12	_	10	1	7	4	3	8

7.2 Start Thinking

Complete the table.

Expression	x	Answer
$(x - 2)(x - 2)$	7	
$x(x - 2) - 2(x - 2)$	7	
$(z + 4)(z - 4)$	7	
$z(z - 4) + 4(z - 4)$	7	

What do you notice about the answers for the first two expressions and the last two expressions? How are the first two expressons related to one another? Are the last two expressions related in the same way?

7.2 Warm Up

Simplify.

1. $(6 + 2b)8$

2. $2(6x - 3)$

3. $(-4x + 7)4$

4. $-2(y^3 + 6)$

5. $-\frac{3}{2}(8x + 16y)$

6. $4(x - 3x + 5)$

7.2 Cumulative Review Warm Up

Determine whether the table represents a *linear* or an *exponential* function. Explain.

1.

x	2	3	4	5	6
y	−3	−2	−1	0	1

2.

x	0	1	2	3	4
y	0.125	1	8	64	512

Name_____ Date _____

7.2 Practice A

In Exercises 1–3, use the Distributive Property to find the product.

1. $(x + 4)(x + 5)$ **2.** $(x + 1)(x - 6)$ **3.** $(x - 2)(x - 7)$

In Exercises 4–6, use a table to find the product.

4. $(y + 4)(y + 2)$ **5.** $(q + 4)(q - 7)$ **6.** $(2x - 3)(x - 1)$

7. Describe and correct the error in finding the product of the binomials.

$(x - 2)(5 - x)$

	x	5
x	x^2	$5x$
-2	$-2x$	-10

$(x - 2)(5 - x) = x^2 + 3x - 10$

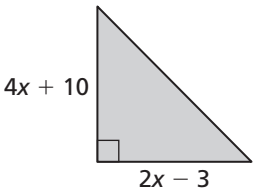
In Exercises 8–13, use the FOIL Method to find the product.

8. $(u + 2)(u + 9)$ **9.** $(w + 6)(w - 5)$ **10.** $(m - 1)(m + 8)$

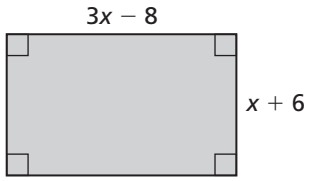
11. $(y - 6)(y - 3)$ **12.** $\left(q + \frac{1}{2}\right)\left(q - \frac{3}{2}\right)$ **13.** $(2 - 5t)(7 - t)$

In Exercises 14 and 15, write a polynomial that represents the area of the shaded region.

14.



15.



In Exercises 16–18, find the product.

16. $(x + 2)(x^2 + 5x + 1)$ **17.** $(y + 5)(y^2 + 2y - 6)$ **18.** $(h - 7)(h^2 - 3h + 2)$

19. When multiplying a binomial by a trinomial, is the degree of the product always 5? Explain.

Name _____ Date _____

7.2 Practice B

In Exercises 1–3, use the Distributive Property to find the product.

1. $(p - 5)(p - 8)$ **2.** $(5t + 1)(t - 2)$ **3.** $(4v - 3)(v + 7)$

In Exercises 4–6, use a table to find the product.

4. $(2p + 4)(5p - 1)$ **5.** $(-4 + 3r)(7r - 2)$ **6.** $(4t - 9)(-6 + 2t)$

7. Describe and correct the error in finding the product of the binomials.

$(x - 2)(5 - x)$

	5	$-x$
x	$5x$	$-x^2$
-2	-10	$2x$

$(x - 2)(5 - x) = 4x^2 + 2x - 10$

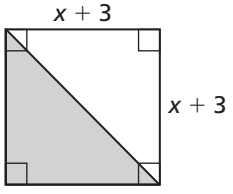
In Exercises 8–13, use the FOIL Method to find the product.

8. $(z + 9)(z - 8)$ **9.** $\left(m - \frac{2}{5}\right)\left(m + \frac{4}{5}\right)$ **10.** $(4 - x)(8 - 3x)$

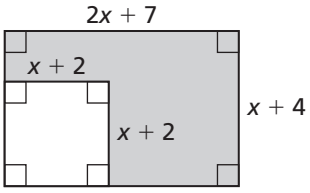
11. $(9 - 6g)(2g + 3)$ **12.** $(p + 4)(p^2 + 7p)$ **13.** $(d - 2)(d^2 - 5d)$

In Exercises 14 and 15, write a polynomial that represents the area of the shaded region.

14.



15.



In Exercises 16–18, find the product.

16. $(x + 10)(3x^2 + 5x - 2)$ **17.** $(2t^2 - 9t - 5)(3t + 7)$ **18.** $(3r^2 + 3r - 8)(5 - 2r)$

19. Write two polynomials that are not monomials, whose product is a trinomial of degree 4.

Name_____ Date _____

7.2 Enrichment and Extension

Perimeter and Area

Perimeter of a Rectangle: $P = 2\ell + 2w$

Area of a Rectangle: $A = \ell \bullet w$

Area of a Square: $A = s^2$

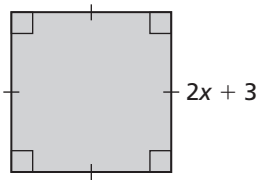
Area of a Triangle: $A = \frac{1}{2}bh$

Area of a Parallelogram: $A = b \bullet h$

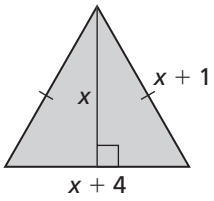
Pythagorean Theorem: $a^2 + b^2 = c^2$

In Exercises 1–3, write an algebraic expression for the area and perimeter of each shape.

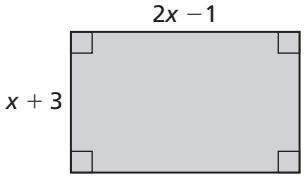
1.



2.

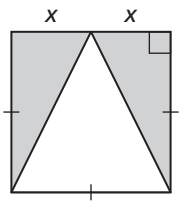


3.

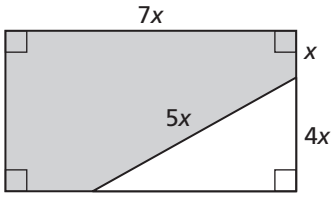


In Exercises 4–9, write an algebraic expression for the area of the shaded figure. (Recall that the height of an isosceles triangle bisects the base.)

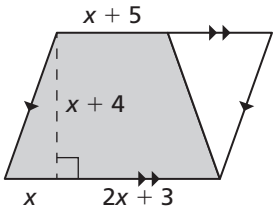
4.



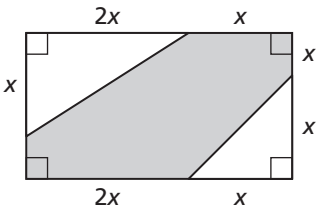
5.



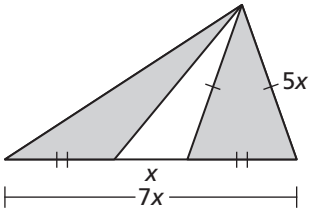
6.



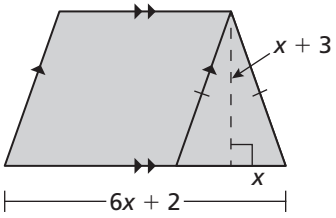
7.



8.



9.



Name _____ Date _____

7.2 Puzzle Time

How Did The Doe Win The Race?

Write the letter of each answer in the box containing the exercise number.

Find the product.

1. $(x + 7)(x + 5)$

2. $(x + 9)(x - 4)$

3. $(x - 6)(x - 3)$

4. $(x - 8)(x - 2)$

5. $(4x + 11)(x - 1)$

6. $(6x + 7)(x + 3)$

7. $(2x - 9)(-5 + 4x)$

8. $(x - 10)(x + 1)$

9. $\left(x - \frac{7}{4}\right)\left(x - \frac{1}{4}\right)$

10. $(2 - 3x)(11x + 8)$

11. $(x - 6)(x^2 + 9x)$

12. $(x + 5)(x^2 + 4x + 4)$

13. $(x - 7)(x^2 + 2x + 1)$

14. $(x - 8)(x^2 - 7x + 12)$

15. $(6x^2 - 3x + 5)(4x^2 + 3)$

16. The length of a classroom is $(10x + 6)$ feet. The width of the classroom is $(9x + 8)$ feet. Find the area of the classroom.

Answers

H. $x^2 - 9x - 10$

U. $x^2 - 10x + 16$

B. $x^3 + 9x^2 + 24x + 20$

I. $6x^2 + 25x + 21$

S. $x^3 + 3x^2 - 54x$

E. $x^2 + 12x + 35$

P. $24x^4 - 12x^3 + 38x^2 - 9x + 15$

K. $x^3 - 5x^2 - 13x - 7$

T. $4x^2 + 7x - 11$

B. $x^2 + 5x - 36$

A. $x^2 - 2x + \frac{7}{16}$

N. $x^3 - 15x^2 + 68x - 96$

Y. $-33x^2 - 2x + 16$

S. $x^2 - 9x + 18$

G. $8x^2 - 46x + 45$

C. $90x^2 + 134x + 48$

2	10		15	9	11	3	6	14	7		5	8	1		12	4	16	13

7.3 Start Thinking

Consider the expression $(x + 3)(x - 3)$. Use the FOIL method to simplify the expression. Explain why the result has only two terms rather than three, as is typical. Explain how the terms are related to the original expression. Change the number three in the expression to another number and simplify again. Is your previous explanation still true?

Consider the expressions $(x + 3)(x + 3)$ and $(x - 3)(x - 3)$. Without simplifying, how many terms would you expect in your answer? Why? How could you rewrite these expressions before simplifying?

7.3 Warm Up

Simplify.

1. $(x - 2)(x - 2)$

2. $(y - 2)(y + 9)$

3. $(z - 2)(z - 6)$

4. $(3x + 4)(x + 6)$

5. $(4x - 6)(4x - 10)$

6. $(4a + b)(3a + 6b)$

7.3 Cumulative Review Warm Up

Write an absolute value equation that has the given solutions.

1. $x = 9$ and $x = 17$

2. $x = 3$ and $x = 8$

3. $x = 5$ and $x = 16$

4. $x = -3$ and $x = 10$

5. $x = -5$ and $x = 3$

6. $x = -2$ and $x = 1$

Name _____ Date _____

7.3 Practice A

In Exercises 1–9, find the product.

1. $(x + 7)^2$

2. $(2w - 3)^2$

3. $(4q + 2)^2$

4. $(n + 4)(n - 4)$

5. $(v - 7)(v + 7)$

6. $(5x + 2)(5x - 2)$

7. $(6 + a)(6 - a)$

8. $\left(\frac{1}{3} + p\right)\left(\frac{1}{3} - p\right)$

9. $(x + 2y)(x - 2y)$

In Exercises 10–12, use special product patterns to find the product.

10. $19 \bullet 21$

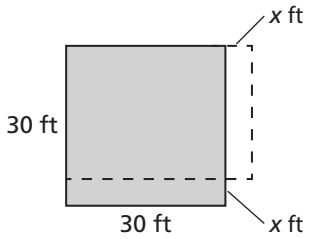
11. $49 \bullet 51$

12. 33^2

13. Describe and correct the error in finding the product.

$$\times \quad (x - 5)^2 = x^2 - 5^2$$
$$= x^2 - 25$$

14. A contractor modifies the size of a kitchen.



a. The area of the room after the modification is represented by $(30 + x)(30 - x)$. Find the product.

b. Use the polynomial in part (a) to find the area when $x = 6$. Which room has the larger area, the original room or the new room? Explain.

In Exercises 15 and 16, find the product.

15. $(x^2 + 5)(x^2 - 5)$

16. $(y^4 - 2)^2$

Name_____ Date _____

7.3 Practice B

In Exercises 1–9, find the product.

1. $(-6p + 3)^2$

2. $(3c - d)^2$

3. $(5x + 2y)^2$

4. $(9 + 4q)(9 - 4q)$

5. $\left(\frac{2}{3} + g\right)\left(\frac{2}{3} - g\right)$

6. $(3m + 8n)(3m - 8n)$

7. $(8 - 3u)(8 + 3u)$

8. $(-c + 9)(-c - 9)$

9. $(-3s - 7t)(-3s + 7t)$

In Exercises 10–12, use special product patterns to find the product.

10. 27^2

11. 40.5^2

12. $5\frac{1}{4} \bullet 4\frac{3}{4}$

13. Describe and correct the error in finding the product.

$$(x + 5)(x - 5) = x^2 + 5^2$$
$$= x^2 + 25$$

14. A circular helicopter landing pad has a radius of 200 feet. Inside the circular pad, red paint covers the outer area evenly, with a width of x feet. White paint covers the inner area.

a. Write a polynomial that represents the area of the circle that is painted white. Write your answer in terms of π.

b. Use the polynomial in part (a) to find the area of the circle that is painted white when $x = 100$.

In Exercises 15 and 16, find the product.

15. $\left(3x^2 + 7y^2\right)^2$

16. $\left(z^4 - 3w^3\right)\left(z^4 + 3w^3\right)$

17. Find k so that $25x^2 + 40x + k$ is the square of a binomial.

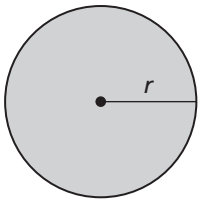
18. Find two numbers a and b such that $(a - b)^2 < (a + b)(a - b) < (a + b)^2$. Find two numbers a and b such that this is not true.

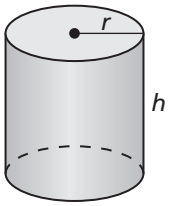
Name _____ Date _____

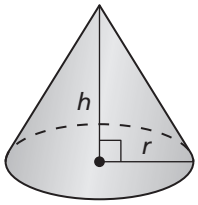
7.3 Enrichment and Extension

Area and Volumes of Spherical Shapes

Area of Circle: $A = \pi r^2$ Volume of a Cylinder: $V = \pi r^2 h$ Volume of a Cone: $V = \frac{1}{3}\pi r^2 h$

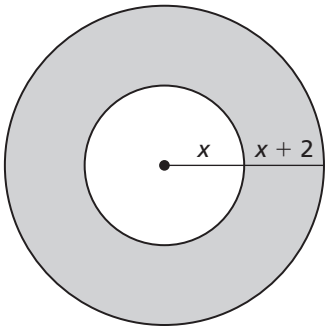




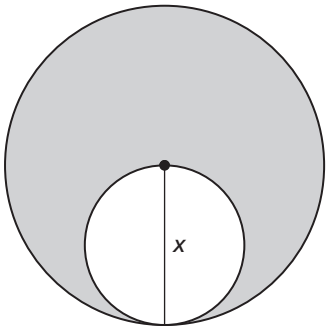


In Exercises 1–3, write an algebraic expression for the shaded area.

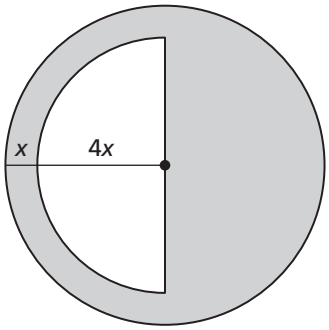
1.



2.

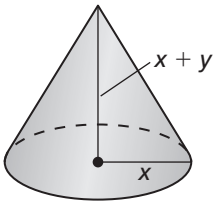


3.

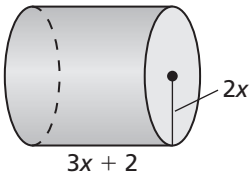


In Exercises 4–6, write an algebraic expression for the volume of the figure.

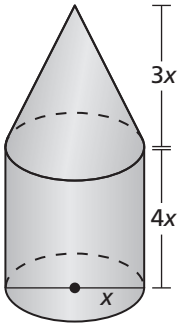
4.



5.

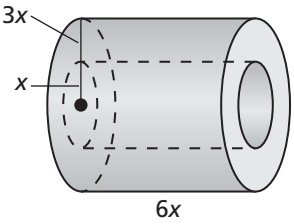


6.

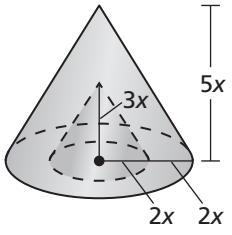


In Exercises 7 and 8, write an algebraic expression for the volume of the figure with a hole in it.

7.



8.



Name_____ Date _____

7.3 Puzzle Time

What Does The Invisible Man Rub Into His Face Before He Retires?

Write the letter of each answer in the box containing the exercise number.

Find the product.

1. $(x + 7)^2$

2. $(x - 5)^2$

3. $(-6x + 3)^2$

4. $(-14 - x)^2$

5. $(x + 9)(x - 9)$

6. $(3x - 4)(3x + 4)$

7. $\left(\frac{2}{3} + x\right)\left(\frac{2}{3} - x\right)$

8. $(x + 10y)(x - 10y)$

9. $\left(x + \frac{4}{5}\right)\left(x - \frac{4}{5}\right)$

10. $(4x - 9y)(4x + 9y)$

11. $(-6x - 7y)(-6x + 7y)$

12. $(-8x + 3y)(-8x - 3y)$

13. The area of a billboard sign is represented by $(x + 12)^2$ feet. Find this product.

14. The length of a picture frame is $(x - 6)$ inches. The width of the picture frame is $(x + 6)$ inches. Find the area of the picture frame.

Answers

R. $9x^2 - 16$

N. $x^2 - \frac{16}{25}$

A. $x^2 - 10x + 25$

C. $64x^2 - 9y^2$

I. $x^2 - 81$

E. $16x^2 - 81y^2$

N. $x^2 + 14x + 49$

V. $x^2 + 24x + 144$

S. $36x^2 - 49y^2$

G. $x^2 - 100y^2$

H. $36x^2 - 36x + 9$

M. $x^2 - 36$

A. $x^2 + 28x + 196$

I. $\frac{4}{9} - x^2$

13	4	9	7	11	3	5	1	8		12	6	10	2	14

7.4 Start Thinking

Rewrite the equation $(x + 3)(x + 5) = 0$ if $x + 3 = a$ and $x + 5 = b$. What do you know to be true about either the value of a or the value of b? Explain what this tells you about the original equation.

Use your knowledge of the Zero-Product Property to write and solve two separate equations from the original equation. Explain the significance of the solutions obtained.

7.4 Warm Up

Solve.

1. $x + 4 = -9$

2. $\frac{4}{3}x = -8$

3. $\frac{1}{2}x = -16$

4. $x - 2 = 25$

5. $\frac{1}{3}x = 11$

6. $x - 4 = 8$

7.4 Cumulative Review Warm Up

Solve the inequality. Graph the solution.

1. $-7t > 14$

2. $-12 \leq -z$

3. $\frac{n}{-4} \leq 2$

4. $-10 > -\frac{2}{3}m$

5. $12 \geq 6f$

6. $t - 8 \geq 36$

Name_____ Date _____

7.4 Practice A

In Exercises 1–9, solve the equation.

1. $x(x - 5) = 0$

2. $6d(d + 8) = 0$

3. $-3t(t + 7) = 0$

4. $(3x + 6)(2x - 10) = 0$

5. $(p + 3)(5p + 1) = 0$

6. $(3q + 2)^2 = 0$

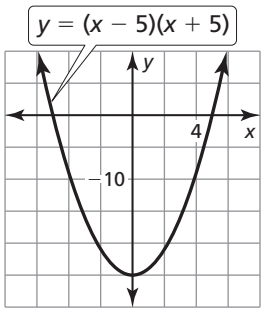
7. $(y - 10)^2 = 0$

8. $t(t + 4)(t - 5) = 0$

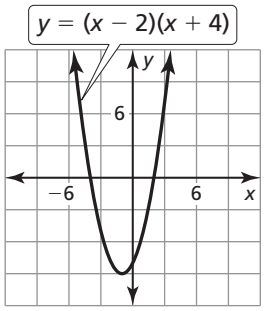
9. $7u(u - 9)(2u - 5) = 0$

In Exercises 10 and 11, find the *x*-coordinates of the points where the graph crosses the *x*-axis.

10.



11.



In Exercises 12–14, factor the polynomial.

12. $4t^2 + 12t$

13. $10k^3 - 15k^2$

14. $8x^3 - 20x^2$

In Exercises 15–17, solve the equation.

15. $3t^2 - t = 0$

16. $5y^2 + 10y = 0$

17. $21n + 12n^2 = 0$

18. Describe and correct the error in solving the equation.

$$\begin{aligned} 15t^2 + 5t &= 0 \\ 5t(3t) &= 0 \\ 5t = 0 \text{ and } 3t &= 0 \\ t = 0 \qquad t &= 0 \end{aligned}$$

19. The height y of a jumping frog can be modeled by $y = -16x^2 + 4x$, where x is the time (in seconds) since the frog jumped from the ground. Find the roots of the equation when $y = 0$. Explain what the roots mean in this situation.

Name _____ Date _____

7.4 Practice B

In Exercises 1–9, solve the equation.

1. $-3y(y - 4) = 0$

2. $(d - 6)(d + 1) = 0$

3. $(w + 3)(w - 5) = 0$

4. $(2 - 3x)(2 + 3x) = 0$

5. $9h(h - 4)(3h + 2) = 0$

6. $k(k + 2)^2 = 0$

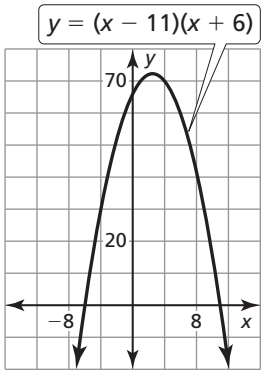
7. $(y - 7)^2(y + 9) = 0$

8. $(12 - 4n)(3n - 5)(-n + 2) = 0$

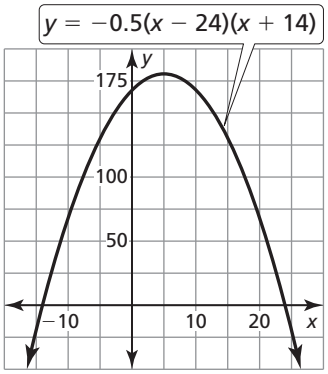
9. $(5 - n)\left(3 - \frac{1}{2}n\right)(n - 4) = 0$

In Exercises 10 and 11, find the *x*-coordinates of the points where the graph crosses the *x*-axis.

10.



11.



In Exercises 12–14, factor the polynomial.

12. $36v^2 + 24v$

13. $3r^6 - 2r^5$

14. $18a^5 + 12a$

In Exercises 15–17, solve the equation.

15. $16h^2 - 8h = 0$

16. $4w^2 = 12w$

17. $-32n = 8n^2$

18. Describe and correct the error in solving the equation.

$$15t^2 = 5t$$
$$3t = 1$$
$$t = \frac{1}{3}$$

The root is $t = \frac{1}{3}$.

19. Write a polynomial of degree 3 whose only roots are $x = 2$ and $x = \frac{2}{5}$. Is there another polynomial of degree 3 that has the same roots?

Name_____ Date _____

7.4 Enrichment and Extension

Long Division of Polynomials

Example: Divide $(x^2 - 3x + 5)$ by $(x + 1)$.

Use the rule of long division to divide polynomials.

$$x + 1 \overline{)\, x^2 - 3x + 5}$$

$$x - 4 + \frac{9}{x + 1}$$

$$\underline{-(x^2 + x)}$$ Multiply x by $x + 1$.

$$-4x + 5$$ Subtract binomial and carry down the 5.

$$\underline{-(-4x - 4)}$$ Multiply -4 by $x + 1$.

$$9$$ Subtract binomial to find remainder.

In Exercises 1–8, divide by using long division.

1. $(x^2 + 5x + 2) \div (x - 1)$
2. $(2x^2 - x + 3) \div (x - 4)$
3. $(x^2 - 5x) \div (x - 3)$
4. $(x^2 - 7x + 10)(x - 2)^{-1}$
5. $(4x^2 + 6x + 1) \div (2x - 3)$
6. $(x^2 - 4)(x + 3)^{-1}$
7. $(x^3 + x^2 - 4x + 5) \div (x - 2)$
8. $(x^3 + 5x + 3) \div (x - 2)$

Name _____ Date _____

7.4 Puzzle Time

Did You Hear About The ...

A	B	C	D	E	F
G	H	I	J	K	L
M	N	O	P	Q	

Complete each exercise. Find the answer in the answer column. Write the word under the answer in the box containing the exercise letter.

0, 9 THAT	−2, 0 SO
0, 2 OF	$-\frac{1}{6}$, 0 KEPT
−5, 9, 15 THE	$-\frac{1}{2}$, $\frac{1}{2}$ A
7 DURING	10 TRIP
$-\frac{12}{5}$, 0 A	−6, 0 HORSE
0, 4 WAS	0, 7 DIARY
−12, 12 JOCKEY	0, 5 THAT
−12, 0, $\frac{1}{3}$ RACE	−1, $-\frac{3}{5}$ SLOW
0, 11 THE	−14, 14 PENCIL

Solve the equation.

A. $y(y + 6) = 0$

B. $s(s - 9) = 0$

C. $11w(w - 4) = 0$

D. $-2u(u + 2) = 0$

E. $(5r + 3)(r + 1) = 0$

F. $(j - 7)^2 = 0$

G. $(8 - 16d)(8 + 16d) = 0$

H. $4p(3p - 1)(p + 12) = 0$

I. $b(b - 5)^2 = 0$

J. $(18 - 2e)(2e + 10)(-e + 15) = 0$

K. $(12 - m)\left(9 + \frac{3}{4}m\right)(m - 12) = 0$

L. $6q^2 + q = 0$

M. $48a + 20a^2 = 0$

N. $7n^2 = 49n$

O. $16t^2 - 32t = 0$

P. $77c - 7c^2 = 0$

Q. The archway to the entrance of an art gallery can be modeled by $y = -\frac{1}{3}(x - 5)(x + 5)$, where x and y are measured in feet. The x-axis represents the floor. Find the width of the arch at floor level.

7.5 Start Thinking

Standard Form	Factored Form
$x^2 + 8x + 12$	$(x + 6)(x + 2)$
$x^2 - 7x + 12$	$(x - 4)(x - 3)$
$x^2 - 2x - 15$	$(x + 3)(x - 5)$
$x^2 + 2x - 24$	$(x + 6)(x - 4)$

Examine the factored form of each polynomial. Find the sum of the constant terms in each set of parentheses (don't forget to use the given sign). Compare your answers to the Standard Form of each polynomial. Is there a pattern? If so, what is it? Find the product of the constant terms in each set of parentheses (again, be sure to use the correct sign). Compare your answers to the standard form of each polynomial. Is there a pattern? If so, what is it?

7.5 Warm Up

Make a list of factors for the number.

1. 42 **2.** 102 **3.** 28

4. 56 **5.** 60 **6.** 36

7.5 Cumulative Review Warm Up

Determine whether the equation represents a *linear* or *nonlinear* function. Explain.

1. $y = x^2 - 14$ **2.** $y = \sqrt{8} + x$

Name _____ Date _____

7.5 Practice A

In Exercises 1–12, factor the polynomial.

1. $x^2 + 5x + 6$

2. $x^2 + 8x + 12$

3. $z^2 + 11z + 28$

4. $w^2 - 7w + 12$

5. $y^2 - 14y + 24$

6. $x^2 - 11x + 28$

7. $x^2 + x - 20$

8. $y^2 - 6y - 16$

9. $m^2 + 8m - 9$

10. $n^2 - 3n - 40$

11. $d^2 + 5d - 24$

12. $z^2 + 3z - 28$

13. A projector displays a rectangular image on a wall. The height of the wall is x feet. The area (in square feet) of the projection is represented by $x^2 - 12x + 32$. The width of the projection is $(x - 4)$ feet.

a. Write a binomial that represents the height of the projection.

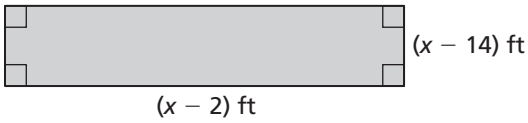
b. Find the perimeter of the projection when the height of the wall is 10 feet.

14. Describe and correct the error in factoring the polynomial.

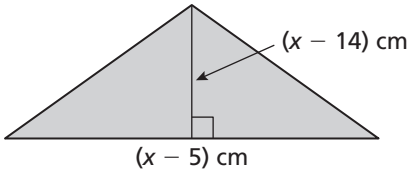
$$\times \quad x^2 - 11x + 18 = (x - 3)(x - 6)$$

In Exercises 15 and 16, find the dimensions of the polygon with the given area.

15. Area $= 45$ ft^2



16. Area $= 35$ cm^2



17. Write an equation of the form $x^2 + bx + c = 0$ that has the solutions $x = -3$ and $x = 8$. Explain how you found your answer.

Name_____ Date _____

7.5 Practice B

In Exercises 1–12, factor the polynomial.

1. $x^2 + 5x + 4$

2. $w^2 + 9w + 14$

3. $y^2 + 15y + 36$

4. $x^2 - 14x + 45$

5. $j^2 - 16j + 39$

6. $m^2 - 19m + 90$

7. $y^2 + 2y - 35$

8. $w^2 - 8w - 20$

9. $b^2 - b - 30$

10. $p^2 - 6p - 27$

11. $q + q^2 - 56$

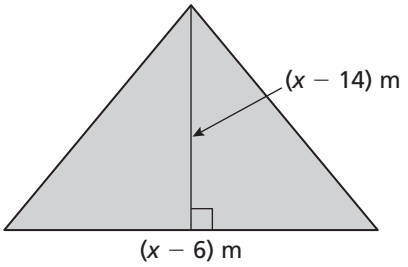
12. $-36 + t^2 + 5t$

13. Describe and correct the error in factoring the polynomial.

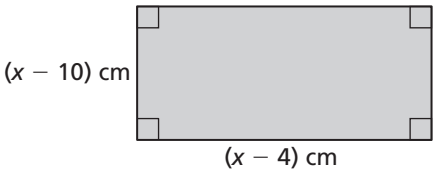
$$\times \quad x^2 + 4x - 96 = (x - 12)(x + 8)$$

In Exercises 14 and 15, find the dimensions of the polygon with the given area.

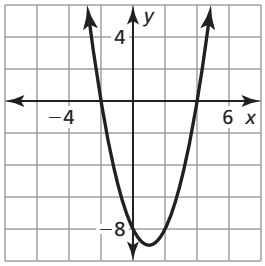
14. Area $= 120$ m^2



15. Area $= 55$ cm^2



16. The graph shows $y = x^2 - 2x - 8$.



a. Explain how you can use the graph to factor the polynomial.

b. Factor the polynomial.

Name _____ Date _____

7.5 Enrichment and Extension

Factor by Grouping

Example: Factor $2x^3 - 5x^2 - 4x + 10$.

$2x^3 - 5x^2 - 4x + 10 = (2x^3 - 5x^2) + (-4x + 10)$ Group together first two terms and second two terms.

$= x^2(2x - 5) - 2(2x - 5)$ Factor a GCF out of each pair of terms. If polynomial is factorable, there will be a binomial GCF.

$= (2x - 5)(x^2 - 2)$ Factor out $(2x - 5)$.

In Exercises 1–8, factor completely.

1. $3xy - 6x + y - 2$

2. $12ab - 4bc - 15ac + 5c^2$

3. $25y^3 + 5y^2 + 30y + 6$

4. $12x^3 + 4x^2 - 30x - 10$

5. $4v^3 - 16v^2 + v - 4$

6. $15x^2y - 10xy^2 - 3x + 2y$

7. $16yz - 5xp + 4yp - 20xz$

8. $12xy + 12x + 16x^2 + 9y$

In Exercises 9 and 10, use the Distributive Property to multiply. Then show how to factor the answer by grouping. What is the final result?

9. $(3x - y)(4y - 5)$

10. $(x - 3)(2x - 3)$

11. Create your own set of binomials. Use the Distributive Property to multiply, and then show how to factor the answer by grouping, just as you did in Exercises 9 and 10. Will this method always prove effective?

12. Do you notice a difference between your results for Exercises 9 and 10? What does having one variable do to the problem?

Name_____ Date _____

7.5 Puzzle Time

What Goes Into The Water Green But Comes Out Blue?

Write the letter of each answer in the box containing the exercise number.

Factor the polynomial.

1. $x^2 + 11x + 28$

2. $x^2 - 3x - 54$

3. $x^2 + 7x - 30$

4. $x^2 - 10x + 16$

5. $x^2 - 3x - 28$

6. $x^2 + 13x + 40$

7. $x^2 - 8x - 48$

8. $x^2 - 17x + 72$

Solve the equation.

9. $x^2 + 3x + 2 = 0$

10. $x^2 - 6x - 55 = 0$

11. $x^2 + 9x - 36 = 0$

12. $x^2 - 13x + 42 = 0$

13. $x^2 + 11x + 18 = 0$

14. $x^2 - x - 6 = 14$

15. The area of a rectangle is 63 square inches. The area of the rectangle can be represented by $A = x^2 - 16x + 126$. What are the possible values of x?

Answers

D. $(x - 12)(x + 4)$

O. $(x + 4)(x - 7)$

A. $(x + 8)(x + 5)$

N. $(x + 7)(x + 4)$

Y. $(x - 8)(x - 2)$

A. $(x - 9)(x + 6)$

R. $(x - 8)(x - 9)$

O. $(x + 10)(x - 3)$

G. 7, 9

D. $-2, -1$

L. $-4, 5$

F. $-9, -2$

C. $-5, 11$

A. 6, 7

O. $-12, 3$

6		13	8	11	15		5	1		2		10	3	14	7		9	12	4

7.6 Start Thinking

Standard Form	Factored Form
$2x^2 - 7x + 3$	$(2x - 1)(x - 3)$
$6x^2 - 13x + 6$	$(3x - 2)(2x - 3)$
$3x^2 + 7x - 6$	$(3x - 2)(x + 3)$
$3x^2 + x - 2$	$(3x - 2)(x + 1)$

Make a list of factors of the coefficient of the x^2-term in the first polynomial in the standard form column. Make a list of the factors of the constant term separately. Use the FOIL method to transform the factored form of the polynomial, showing each step. Repeat these steps for each polynomial. Explain how to use the list of factors you made to write the polynomial in factored form.

7.6 Warm Up

Factor the polynomial using the GCF.

1. $5x^2 - 5x - 5$

2. $-x^3 + 12x^2 - 4x$

3. $4z^2 - 96z - 8$

4. $81y^2 + 36y - 3$

5. $7x^2y + 10xy + 11y$

6. $15t^2 - 45t + 90$

7.6 Cumulative Review Warm Up

Write an equation in slope-intercept form of the line that passes through the given points.

1. $(8, 1), (3, 11)$

2. $(7, -2), (4, -8)$

Name_____ Date _____

7.6 Practice A

In Exercises 1–12, factor the polynomial.

1. $6x^2 - 12x - 18$

2. $5x^2 - 15x - 50$

3. $9x^2 - 36x + 27$

4. $2x^2 + 2x - 4$

5. $6x^2 - 7x - 20$

6. $2x^2 - 5x - 3$

7. $4x^2 + 21x - 18$

8. $2x^2 - 13x - 45$

9. $3x^2 + 22x - 16$

10. $-2p^2 + 7p - 6$

11. $-5v^2 + 31v - 6$

12. $-6v^2 - 11v - 4$

13. Describe and correct the error in factoring the polynomial.

$$\times \quad -2t^2 + 13t - 15 = (2t + 3)(t + 5)$$

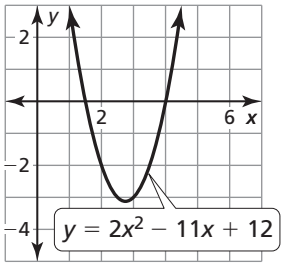
In Exercises 14 and 15, solve the equation.

14. $4x^2 - 4x - 24 = 0$

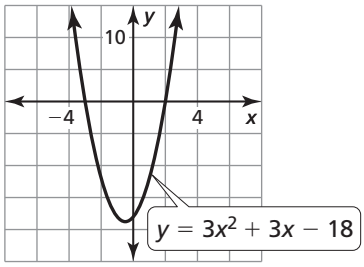
15. $3p^2 - 5p - 28 = 0$

In Exercises 16 and 17, find the *x*-coordinates of the points where the graph crosses the *x*-axis.

16.



17.



18. The height h (in feet) above the water of a cliff diver is modeled by $h = -16t^2 + 10t + 26$, where t is the time (in seconds). How long is the diver in the air?

19. For what values of t can $10x^2 + tx + 8$ be written as the product of two binomials?

In Exercises 20 and 21, factor the polynomial.

20. $6a^2 - 13ab - 5b^2$

21. $4x^2 + 11xy - 3y^2$

Name _____ Date _____

7.6 Practice B

In Exercises 1–12, factor the polynomial.

1. $5x^2 - 5x - 30$

2. $8x^2 - 16x - 192$

3. $6x^2 + 48x + 42$

4. $2x^2 + 17x - 9$

5. $12p^2 - 7p - 10$

6. $10w^2 + 24w + 8$

7. $3y^2 + y - 14$

8. $12j^2 - 32j + 5$

9. $15d^2 + 16d - 15$

10. $-9v^2 - 22v - 8$

11. $-14m^2 + 13m - 3$

12. $-20q^2 + 56q - 15$

13. Describe and correct the error in factoring the polynomial.

$$6x^2 - 4x + 2 = (2x - 2)(3x + 1)$$

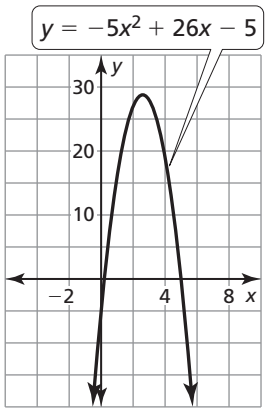
In Exercises 14 and 15, solve the equation.

14. $-12w^2 + 20w - 3 = 0$

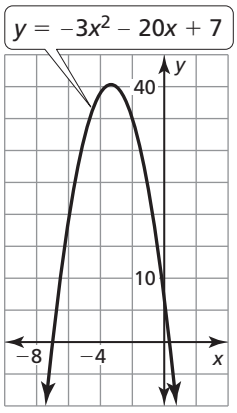
15. $18t^2 - 2 = 5t$

In Exercises 16 and 17, find the *x*-coordinates of the points where the graph crosses the *x*-axis.

16.



17.



18. The length of a rectangular patio is 8 feet less than twice its width. The area of the patio is 280 square feet. Find the dimensions of the patio.

19. For what values of t can $6x^2 + tx + 25$ be written as the product of two binomials?

In Exercises 20 and 21, factor the polynomial.

20. $-10r^2 - 11sr + 6s^2$

21. $12x^3 + 8x^2y - 20xy^2$

Name_____ Date _____

7.6 Enrichment and Extension

Factor $ax^2 + bx + c$ by Grouping

Example: Factor $5x^2 - 8x - 4$.

$5x^2 - 8x - 4 = 5x^2 - 10x + 2x - 4$	Multiply *ac* and find the factors that add to *b*.
$= (5x^2 - 10x) + (2x - 4)$	Split *b* into two factors and group terms together.
$= 5x(x - 2) + 2(x - 2)$	Factor a GCF out of each pair of terms.
$= (5x + 2)(x - 2)$	Create multiplication of two binomials.

In Exercises 1–10, factor completely.

1. $6x^2 + 11x + 3$

2. $4y^2 + 12y + 5$

3. $3p^2 + 4p - 7$

4. $4x^2 - 20x - 11$

5. $12x^2 - x - 1$

6. $40v^2 - 22v + 3$

7. $8u^2 + 34u + 15$

8. $-9d^2 - 9d + 4$

9. $45x^2 + 105x + 30$

10. $-2t^2 - t + 1$

11. Create your own trinomial by working backwards. Start with a factored set of binomials.

Name _____ Date _____

7.6 Puzzle Time

How Did The Sea Urchin Pay For His Meal?

Write the letter of each answer in the box containing the exercise number.

Factor the polynomial.

1. $2x^2 + 4x - 16$

2. $5x^2 + 21x + 18$

3. $4x^2 - 4x - 63$

4. $6x^2 - 19x + 8$

5. $-9x^2 + 31x + 20$

6. $3x^2 - 6x - 24$

7. $28x^2 + x - 15$

8. $-36x^2 + 30x + 66$

9. $-10x^2 - 34x - 12$

10. $6x^2 - 19x - 36$

Solve the equation.

11. $4x^2 + 40x + 84 = 0$

12. $-3x^2 - 15x + 72 = 0$

13. $-15x^2 + 28x - 5 = 0$

14. $24x^2 - 47x - 21 = 0$

15. The length of a rectangular platform is 2 feet longer than three times its width. The area of the platform is 56 square inches. What are the width and the length of the platform?

Answers

N. $(7x - 5)(4x + 3)$

S. $(3x + 4)(2x - 9)$

A. $(5x + 6)(x + 3)$

W. $-(9x + 5)(x - 4)$

H. $-2(5x + 2)(x + 3)$

A. $(3x - 8)(2x - 1)$

L. $2(x + 4)(x - 2)$

D. $-6(6x - 11)(x + 1)$

R. $3(x - 4)(x + 2)$

T. $(2x - 9)(2x + 7)$

A. $\frac{1}{5}, \frac{5}{3}$

I. $-7, -3$

D. $4, 14$

L. $-\frac{3}{8}, \frac{7}{3}$

O. $-8, 3$

5	11	3	9		2		10	13	7	15		8	12	1	14	4	6

7.7 Start Thinking

Consider the expression $x^2 - 16$. Rewrite the expression as the difference of two squares. The factored form of the expression is $(x + 4)(x - 4)$. Explain why the factored form of this expression is correct.

The expression $x^2 + 2x + 1$ can be factored as $(x + 1)^2$. Rewrite the factored expression as the product of two identical binomials. Use this knowledge to factor the expression $x^2 + 8x + 16$.

7.7 Warm Up

Simplify.

1. $(y - 5)^2$

2. $(x + 1)^2$

3. $(3x - 7)^2$

4. $(x - 2y)^2$

5. $(4x - 9y)^2$

6. $(2x - 7)^2$

7.7 Cumulative Review Warm Up

Solve by substitution.

1. $2x + 3y = 7$
$x = 2$

2. $y = x + 2$
$x = 6 - y$

3. $y = 4 + x$
$y = -3x + 8$

4. $x + y = 2$
$x = y + 2$

Name _____ Date _____

7.7 Practice A

In Exercises 1–3, factor the polynomial.

1. $x^2 - 36$ **2.** $49 - 4t^2$ **3.** $1 - 25y^2$

In Exercises 4–6, use a special product pattern to evaluate the expression.

4. $11^2 - 8^2$ **5.** $17^2 - 15^2$ **6.** $65^2 - 62^2$

In Exercises 7–9, factor the polynomial.

7. $k^2 + 14k + 49$ **8.** $m^2 - 18m + 81$ **9.** $x^2 + 34x + 289$

10. The area (in square centimeters) of a square thank-you card can be represented by $x^2 + 6x + 9$.

a. Write an expression that represents the side length of the card.

b. What is the perimeter of the card when $x = 4$?

In Exercises 11–14, solve the equation.

11. $v^2 - 25 = 0$ **12.** $p^2 + 8p + 16 = 0$

13. $q^2 - 14q + 49 = 0$ **14.** $16x^2 = 25$

In Exercises 15–17, factor the polynomial.

15. $5x^2 - 20$ **16.** $4x^2 - 24x + 36$ **17.** $9x^2 + 90x + 225$

18. While standing on a roof, you drop a hammer. The function $y = 16 - 16t^2$ represents the height y (in feet) of the hammer t seconds after it is dropped. After how many seconds does the hammer land on the ground?

19. Tell whether the polynomial can be factored. If not, change the constant term so that the polynomial is a perfect square trinomial.

a. $p^2 + 12p + 33$ **b.** $x^2 - 16x + 61$

20. A square picture frame has side length x inches. The square opening for a picture within the frame has side length 3 inches.

a. Write a polynomial that represents the area of the picture frame, not including the picture.

b. The area in part (a) is 55 square inches. What is the side length of the picture frame? Explain your reasoning.

Name_____ Date _____

7.7 Practice B

In Exercises 1–3, factor the polynomial.

1. $100 - 49x^2$ **2.** $121s^2 - 25t^2$ **3.** $x^2 - 144y^2$

In Exercises 4–6, use a special product pattern to evaluate the expression.

4. $86^2 - 84^2$ **5.** $44^2 - 39^2$ **6.** $28^2 - 27^2$

In Exercises 7–9, factor the polynomial.

7. $z^2 + 26z + 169$ **8.** $16x^2 - 40x + 25$ **9.** $81a^2 + 36a + 4$

10. The area (in square inches) of a square table can be represented by $25x^2 + 40x + 16$.

a. Write an expression that represents the side length of the table.

b. Will a square table cloth with side length 60 inches cover the table when $x = 12$?

In Exercises 11–14, solve the equation.

11. $100x^2 = 81$ **12.** $w^2 + 24w + 144 = 0$

13. $s^2 + 81 = 18s$ **14.** $y^2 - \frac{1}{3}y = -\frac{1}{36}$

In Exercises 15–17, factor the polynomial.

15. $8y^2 - 72$ **16.** $7p^2 + 56p + 112$ **17.** $48t^2 - 72t + 27$

18. The function $y = -16t^2 + 24t$ represents the height y (in feet) of a tennis ball bouncing straight up from the ground t seconds after it bounces. After how many seconds does the tennis ball return to the ground?

19. Tell whether the polynomial can be factored. If not, change the constant term so that the polynomial is a perfect square trinomial.

a. $q^2 + \frac{1}{2}q + \frac{1}{3}$ **b.** $4x^2 + 28x + 47$

20. A square picture frame has side length x inches. The square opening for a picture within the frame has side length 6 inches.

a. Write a polynomial that represents the area of the picture frame, not including the picture.

b. The area in part (a) is 64 square inches. What is the side length of the picture frame? Explain your reasoning.

Name _____ Date _____

7.7 Enrichment and Extension

The Binomial Theorem

In algebra, the *binomial theorem* is a technique used to simplify the power $(x + y)^n$ into a sum of terms by expansion. The formula and triangle are attributed to the mathematician *Blaise Pascal*, and is commonly known at *Pascal's Triangle*. The formula uses the coefficients from the row of the triangle, along with decreasing and increasing powers for the two variables.

1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Example: Expand $(x + 2y)^4$ by using the binomial theorem.

$$(x + 2y)^4 = x^4y^0 + 4x^3 \bullet 2y + 6x^2 \bullet 2^2y^2 + 4x \bullet 2^3y^3 + x^0 \bullet 2^4y^4$$
$$= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$$

In Exercises 1–6, expand completely.

1. $(x - y)^5$

2. $(a + 2b)^3$

3. $(x + 4y)^4$

4. $(a - 3c)^5$

5. $(a - b)^6$

6. $(x^2 + 3)^4$

In Exercises 7 and 8, find the term described.

7. 4th term of $(5y + x)^4$

8. 3rd term of $(a^2 - 2)^9$

Name_____ Date _____

7.7 Puzzle Time

When Do You Put The Cart Before The Horse?

Write the letter of each answer in the box containing the exercise number.

Factor the polynomial.

1. $x^2 - 36$

2. $81 - 25x^2$

3. $x^2 + 14x + 49$

4. $4x^2 - 44x + 121$

5. $64x^2 - 9y^2$

6. $100x^2 - 20x + 1$

7. $3x^2 - 3$

8. $-x^2 + 144$

9. $4x^2 - 16x + 16$

10. $-18x^2 + 54x - 40$

Solve the equation.

11. $x^2 - 64 = 0$

12. $49 - 4x^2 = 0$

13. $x^2 = 8x - 16$

14. $36x^2 = -12x - 1$

15. You drop a penny while taking change out of your pocket. The function $y = 49 - 9t^2$ represents the height y (in feet) of the penny t seconds after it falls from your pocket. How many seconds does it take for the penny to land on the ground?

Answers

H. $-2(3x - 4)(3x - 5)$

T. $(9 + 5x)(9 - 5x)$

N. $3(x + 1)(x - 1)$

C. $(2x - 11)^2$

E. $(8x + 3y)(8x - 3y)$

R. $4(x - 2)^2$

A. $(x - 6)(x + 6)$

N. $(10x - 1)^2$

T. $-(x + 12)(x - 12)$

I. $(x + 7)^2$ **Y.** 4

D. $\frac{7}{3}$ **I.** $-8, 8$

O. $-\frac{7}{2}, \frac{7}{2}$ **I.** $-\frac{1}{6}$

14	7		2	10	5		15	11	4	8	3	12	6	1	9	13

7.8 Start Thinking

Copy and complete the table below by putting a check mark next to the type of factoring that applies to the polynomial.

Polynomial	GCF	$x^2 + bx + c$	$ax^2 + bx + c$	Difference of Squares
$8x^2 + 20x + 48$				
$x^2 - 19x + 48$				
$x^2 - 16$				
$6x^2 + 19x + 8$				

Is it possible to use more than one factoring technique to factor a polynomial? Explain.

7.8 Warm Up

Use mental math to simplify.

1. $35 + 20 + 5$

2. $15 \bullet 7 \bullet 2$

3. $1 + 5 + 4 + 8$

4. $5 \bullet 9 \bullet 3 \bullet 2$

5. $6 \bullet 8 \bullet 5$

6. $2 + 5 + 6 + 12$

7.8 Cumulative Review Warm Up

Evaluate the expression.

1. $64^{1/6}$

2. $(-27)^{2/3}$

3. $(256)^{3/8}$

4. $\left(\sqrt{4}\right)^2$

5. $(-64)^{4/3}$

6. $216^{1/3}$

Name_____ Date _____

7.8 Practice A

In Exercises 1–4, factor the polynomial by grouping.

1. $x^3 - 3x^2 + x - 3$

2. $x^3 - 2x^2 + 9x - 18$

3. $2y^3 - 2y^2 + 3y - 3$

4. $3p^3 + 5p^2 - 12p - 20$

In Exercises 5–10, factor the polynomial completely.

5. $4y^3 - 36y$

6. $3r^2 - 8r + 7$

7. $3t^3 + 12t^2 + 12t$

8. $-6q^3 + 28q^2 + 10q$

9. $5y^5 - 5y^4 - 10y^3$

10. $7x^2 + 21x + 7$

In Exercises 11–14, solve the equation.

11. $3j^3 + 21j^2 + 30j = 0$

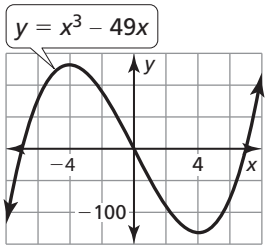
12. $w^4 - 36w^2 = 0$

13. $y^3 - 2y^2 - 9y + 18 = 0$

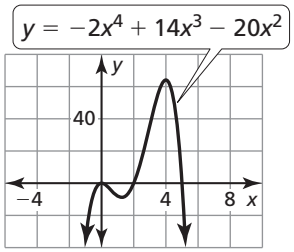
14. $5t^5 + 5t^4 - 210t^3 = 0$

In Exercises 15 and 16, find the *x*-coordinates of the points where the graph crosses the *x*-axis.

15.



16.



17. A rectangular box has a volume of 105 cubic centimeters. The width of the rectangular box is x centimeters, the length is $(2x - 3)$ centimeters, and the height is 3 centimeters.

a. Write a polynomial that represents the volume of the rectangular box.

b. What are the dimensions of the rectangular box?

In Exercises 18 and 19, factor the polynomial completely.

18. $a^3 - 4a + 3a^2b - 12b$

19. $9g^3 - g - 18g^2h + 2h$

Name _____ Date _____

7.8 Practice B

In Exercises 1–4, factor the polynomial by grouping.

1. $a^2 - 3a + ab - 3b$

2. $m^2 + 7mn + 2m + 14n$

3. $t^2 - 4t + tv - 4v$

4. $3x^2 - 4x + 9xy - 12y$

In Exercises 5–10, factor the polynomial completely.

5. $45y^4 - 20y^2$

6. $8w^5 - 48w^4 + 72w^3$

7. $p^3 - 3p^2 - 16p + 48$

8. $12z^2 - 6z + 42$

9. $-21h^4 + 77h^3 + 28h^2$

10. $x^3 + 2x^2 - 49x - 98$

In Exercises 11–14, solve the equation.

11. $p^3 + 2p^2 - 9p - 18 = 0$

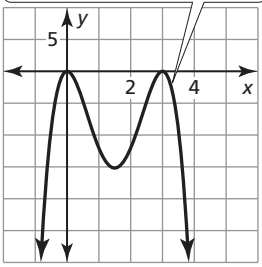
12. $3y^4 + 9y^3 - 120y^2 = 0$

13. $36t - 4t^3 = 0$

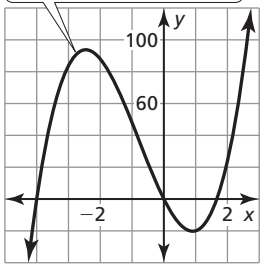
14. $3q^3 - 5q^2 - 27q + 45 = 0$

In Exercises 15 and 16, find the *x*-coordinates of the points where the graph crosses the *x*-axis.

15. $y = -3x^4 + 18x^3 - 27x^2$



16. $y = 6x^3 + 14x^2 - 40x$



17. A rectangular box has a volume of $72x$ cubic inches. The width of the rectangular box is x inches, the length is $3x$ inches, and the height is $(3x - 1)$ inches.

a. Write a polynomial that represents the volume of the rectangular box.

b. What are the dimensions of the rectangular box?

In Exercises 18 and 19, factor the polynomial completely.

18. $5x^2 + 35xy - 2x - 14y$

19. $5p^3 + p^2q - 15pq - 3q^2$

Name_____ Date _____

7.8 Enrichment and Extension

Quadratic Form $ax^2 + bx^n + c$

Factoring polynomials in quadratic form uses the same simple methods you know from factoring quadratic polynomials, only you must now take higher powers into consideration.

Example: Factor $2x^4 - x^2 - 1$.

$2x^4 - x^2 - 1 = 2x^4 - 2x^2 + x^2 - 1$	Multiply *ac* and find the factors that add to *b*.
$= (2x^4 - 2x^2) + (x^2 - 1)$	Group terms with common factors.
$= 2x^2(x^2 - 1) + 1(x^2 - 1)$	Factor a GCF out of each pair of terms.
$= (2x^2 + 1)(x^2 - 1)$	Create multiplication of two binomials.
$= (2x^2 + 1)(x + 1)(x - 1)$	Difference of two squares pattern

In Exercises 1–10, factor completely.

1. $x^6 + 11x + 30$

2. $y^4 - 5y^2 + 4$

3. $3p^8 + 4p^4 - 4$

4. $4x^4 + 3x^2 - 1$

5. $x^4 - 2x^2 + 1$

6. $-x^{10} - 7x^5 - 10$

7. $8u^6 + 10u^3 + 3$

8. $d^6 - 9d^2 + 14$

9. $-2t^8 + 7t^4 + 4$

10. $x^8 - 1$

Name _____ Date _____

7.8 Puzzle Time

What Do You Get When You Cross A Computer With A Freezer?

Write the letter of each answer in the box containing the exercise number.

Factor the polynomial completely.

1. $x^3 - 3x^2 + 4x - 12$

2. $6x^3 - 30x^2 + 7x - 35$

3. $x^2 + 2xy + 9x + 18y$

4. $x^2 - 8x + xy - 8y$

5. $4x^3 - 400x$

6. $3x^3 + 36x^2 + 108x$

7. $6x^5 + 6x^4 - 36x^3$

8. $-8x^4 + 24x^3 - 88x^2$

9. $x^3 - 8x^2 - 16x + 128$

10. $-7x^3 - 14x^2 - 7x$

Solve the equation.

11. $3x^2 - 18x + 15 = 0$

12. $x^3 + 2x^2 - 36x - 72 = 0$

13. $63x - 7x^3 = 0$

14. $8x^3 - 3x^2 = 32x - 12$

15. The volume of a box is 30 cubic inches. The width of the box is 2 inches less than the length. The height is 3 inches less than the length. Find the length of the box.

Answers

O. $4x(x + 10)(x - 10)$

L. $(x - 5)(6x^2 + 7)$

R. $(x + 4)(x - 4)(x - 8)$

S. $(x + y)(x - 8)$

W. $-7x(x + 1)^2$

E. $(x^2 + 4)(x - 3)$

A. $6x^3(x + 3)(x - 2)$

Y. $3x(x + 6)^2$

S. $-8x^2(x^2 - 3x + 11)$

E. $(x + 9)(x + 2y)$

V. $-3, 0, 3$ **R.** 5

C. $1, 5$ **O.** $-2, \frac{3}{8}, 2$

N. $-6, -2, 6$

13	1	9	6		11	5	14	2		7	12	8	10	3	15	4

Name_____ Date _____

Chapter 7 Cumulative Review

Solve the equation and check your answer.

1. $3(4w - 5) = \frac{1}{2}(4w + 10)$ **2.** $-9\pi = \pi - 2t$ **3.** $4x + 9 - 7x = 12$

Solve the inequality. Graph the solution, if possible.

4. $|4x - 8| < 44$ **5.** $\frac{|7y + 16|}{3} + 1 \leq -2$ **6.** $-3|14 - 7x| < -21$

7. A company that makes table tennis balls needs to ship bags of balls that contain 690 balls. The most a bag can be off is 6 balls.

a. Write an absolute value inequality for this situation.

b. Solve the absolute value inequality.

Graph the linear equation or linear inequality.

8. $y = \frac{5}{2}x - 4$ **9.** $y > -2x + 3$ **10.** $14x - 7y \leq 21$

Write an equation of the line in point-slope form that passes through the given point and is parallel to the given line.

11. $(5, 1); y = -7x - 14$ **12.** $(-4, 2); y = \frac{1}{5}x - 1$ **13.** $(-7, -1); 2x + 2y = 2$

Solve the system of linear equations by graphing, substitution, or elimination.

14. $-4x + 9y = 9$
$x - 3y = -6$

15. $y = 4x + 3$
$y = -x - 2$

16. $-3x - 3y = 3$
$y = -5x - 17$

17. The difference of two numbers is 3. Their sum is 13. Find the numbers.

18. You are saving money for your first car. You have \$2000 in your account. You can deposit \$100 every week. You want at least \$5000.

a. Write an inequality that represents this situation.

b. What is the minimum number of weeks for which you must save?

Simplify the expression. Write your answer using only positive exponents.

19. $\frac{2^3 x^{-2}}{z^{-8}}$ **20.** $\frac{10^{-3} r^3}{z^{-6}}$ **21.** $\frac{1^{-3} x^{-2} y^0}{7^{-1} z^{-4}}$

Name _____ Date _____

Chapter 7 Cumulative Review (continued)

Evaluate the expression.

22. $\sqrt[3]{512}$ **23.** $\sqrt[4]{1}$ **24.** $\sqrt[5]{-16{,}807}$

25. $216^{2/3}$ **26.** $81^{3/4}$ **27.** $243^{7/5}$

Use the formula $r = \left(\frac{F}{P}\right)^{1/n} - 1$ to find the annual inflation rate to the nearest tenth of a percent.

28. A classic car increases in value from \$5000 to \$20,000 over a period of 30 years.

29. The cost of a gallon of gas increases from \$0.49 to \$2.99 over a period of 50 years.

Evaluate the function for the given value of x.

30. $y = 2^x; x = -3$ **31.** $y = -2(5)^x; x = 0$ **32.** $f(x) = \frac{1}{3}(5)^x; x = -2$

Write a function that represents the situation.

33. A \$2000 computer decreases in value by 40% every year.

34. Your savings account has \$2500 and increases 3.5% every year.

35. A population of 900 deer is decreasing by 12% every year.

Solve the equation. Check your solution.

36. $7^{x+3} = 7^9$ **37.** $11^{3x+24} = 11^{15}$ **38.** $4^{5x-6} = 16^{x+3}$

Write the next three terms of the geometric sequence.

39. 648, 216, 72, … **40.** 1, 2, 4, 8, … **41.** 24, 12, 6, …

Write the first six terms of the sequence.

42. $a_1 = 2,\ a_n = a_{n-1} - 2$ **43.** $a_1 = 4,\ a_n = 7a_{n-1}$

Write the polynomial in standard form. Then classify the polynomial by the number of terms.

44. $7x^2 - 12x^3$ **45.** $7w + 16w^7 - 3w^4$ **46.** $\frac{3}{8}z^2 + \pi z^3 - 8z^8$

Name_____ Date _____

Chapter 7 Cumulative Review (continued)

Find the sum.

47. $(-12g - 4) + (14g + 7)$

48. $(4h + 3) + (-7h - 8)$

49. $(3v^2 + 4v - 5) + (-4v^2 + 7v - 12)$

50. $(t^3 - 2t^2 + 7) + (7t^2 - 13t^3 - 5t)$

Find the difference.

51. $(x - 7) - (3x + 8)$

52. $(10y + 4) - (-3y - 34)$

53. $(x^2 + 2x - 8) - (2x^2 - 5x + 9)$

54. $(-w - 7) - (-7w^4 + 3w^2 + 9w)$

Use the Distributive Property, a table, or the FOIL Method to find the product.

55. $(x - 5)(x + 3)$

56. $(y - 5)(y + 2)$

57. $(n - 10)(n - 3)$

58. $(2r - 5)(r + 9)$

59. A rectangular flower bed has a width of $(2x + 5)$ and a length of $(x + 10)$.

a. Write a polynomial that represents the area of the flower bed.

b. Find the area of the flower bed when the length is 25 feet.

Find the product.

60. $(x + 4)^2$

61. $(3y - 5)^2$

62. $(-7x - 3y)^2$

63. $(w - 2)(w + 2)$

64. $(2m - 4)(2m - 4)$

65. $(9h + 2t)(9h - 2t)$

66. A square has a side length of $3x + 2$.

a. Write a polynomial that represents the area of the square.

b. Find the area if $x = 3$.

Solve the equation.

67. $(3x - 9)(2x + 10) = 0$

68. $(7x - 14)(5x + 25) = 0$

69. $(7x - 8)^2 = 0$

70. $(2 - 5g)(2 + 5g) = 0$

Solve the equation.

71. $5x^2 - 15x = 0$

72. $21p^2 + 14p = 0$

73. $18g - 6g^2 = 0$

Name _____ Date _____

Chapter 7 Cumulative Review (continued)

Factor the polynomial.

74. $y^2 + 7y + 10$

75. $x^2 + 6x + 8$

76. $w^2 + 11w + 18$

77. $x^2 - 6x + 8$

78. $d^2 - 7d + 12$

79. $z^2 - 12z + 20$

80. $m^2 + 2m - 15$

81. $z^2 + 2z - 24$

82. $x^2 - 10x - 11$

Solve the equation.

83. $t^2 + 5t + 4 = 0$

84. $y^2 - 3y - 44 = 10$

Factor the polynomial.

85. $2x^2 + 22x + 60$

86. $5y^2 + 25y + 20$

87. $6w^2 + 66w + 60$

88. $2t^2 + 5t + 2$

89. $3u^2 - 8u + 4$

90. $-6z^2 - 25z - 25$

91. $4x^2 - 17x + 4$

92. $4r^2 - 35r + 49$

93. $5g^2 - 18g + 9$

94. You throw a ball off the top of a building. The height h (in feet) of the ball above the ground is modeled by $h = -16t^2 + 76t + 20$, where t is the time (in seconds). How long is the ball in the air?

Factor the polynomial.

95. $x^2 - 100$

96. $h^2 - 36$

97. $9b^2 - 25$

98. $k^2 + 8k + 16$

99. $a^2 - 30a + 225$

100. $100g^2 + 180g + 81$

Solve the equation.

101. $z^2 - 64 = 0$

102. $y^2 - 14y + 49 = 0$

Factor the polynomial completely.

103. $25x^3 + 5x^2 + 30x + 6$

104. $28y^3 + 16y^2 - 21y - 12$

105. $8w^3 - 64w^2 + w - 8$

106. $15x^3 + 21x^2 - 10x - 14$