Name (Last, First): _____

Period: #5 Date: _____

9.4: Solving Quadratic Equations by Completing the Square

Review (9.1) **Write the radical in the simplest form:**

1) $\sqrt{162}$

2) $\sqrt{\frac{13}{16}}$

3) $-\sqrt{\frac{7}{121}}$

4) $\frac{1}{4+\sqrt{5}}$

5) $\frac{\sqrt{10}}{\sqrt{2}-4}\left(\frac{\sqrt{2}+4}{\sqrt{2}+4}\right) = \frac{\sqrt{20}+4\sqrt{10}}{\sqrt{2}^2-4^2}$

$= \frac{\sqrt{20}+4\sqrt{10}}{2-16}$

$= \frac{\sqrt{20}+4\sqrt{10}}{-14}$

6) $\frac{7}{\sqrt{2}-\sqrt{6}}$

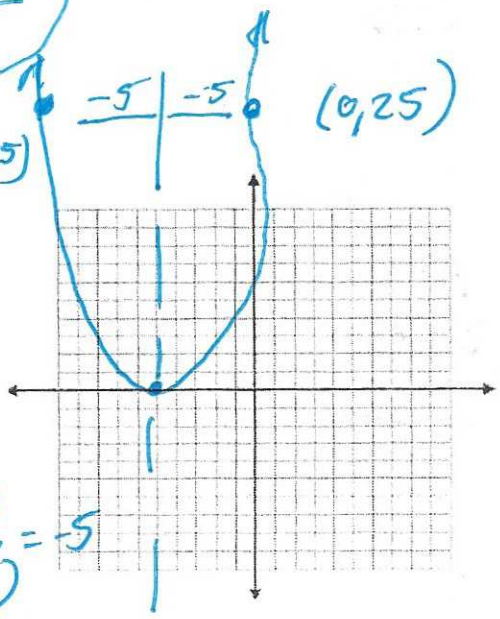
Review (9.2) **Graph the equation and identify the intercepts**

$-x^2 = 10x + 25$

$0 = x^2 + 10x + 25$

y-intercept: (0, 25)

Find the x-intercepts: (−5, 0) and (−5, 0)



A: 1
B: 10
C: 25

$A \cdot C = 25$

5 5

$(x+5)^2$

$\frac{-b}{2a}$ $\frac{-(10)}{2(1)} = -5$

Review (9.3) **Solve using square roots**

8) $9(x+1)^2 = 144$

$\frac{9}{9}$ $\frac{144}{9}$

$(x+1)^2 = 16$

$x+1 = \pm 4$

9) $4x^2 - 371 = -29$

10) $x^2 + 9 = 0$

Completing the Square: For the expression of the form $Ax^2 + bx$, you can add a constant "c" to the expression so that $x^2 + bx + c$ is a perfect square trinomial.

Core Concept: Completing the Square

1) Find half of the "b" value (the coefficient of x in $x^2 + bx$)
2) Square the result from step 1. 4
3) Add the result from step 2 to $x^2 + bx$
4) Factor the resulting expression as a square of a binomial:

B = 4

$\frac{1}{2}b = 2$

$x^2 + 4x + 4$

$(x + \ )^2$

$x^2 + bx + \left(\frac{b}{2}\right)^2$

$= \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right) = \left(x + \frac{b}{2}\right)^2$

x	$\frac{b}{2}$
x^2	$\frac{b}{2}x$
$\frac{b}{2}x$	$\left(\frac{b}{2}\right)^2$

Example 1 – Completing the Square:

Complete the square for each expression. Then factor the trinomial

a) $x^2 + 6x$.

B: 6

$x^2 + 6x + \left(\frac{6}{2}\right)^2$

$x^2 + 6x + 9$

$(x+3)^2$

Step 1: Find one-half of "b": 3

Step 2: Square the result of step 1: 9

Step 3: Add the result of Step 2 to the binomial: $x^2 + 6x + 9$

Step 4: Factor the resulting expression as a square of a binomial:

A 1

B 6

C 9

$A \cdot C = 9$

$3 \quad 3 = 6$

	x	3
x	x^2	$3x$
3	$3x$	9

$(x+3)(x+3)$

$(x+3)^2$

b) $x^2 - 9x$

Step 1: Find one-half of "b": -4.5

Step 2: Square the result of step 1: 20.25

Step 3: Add the result of Step 2 to the binomial: $x^2 - 9x + 20.25$

Step 4: Factor the resulting expression as a square of a binomial: $(x - 4.5)^2$

$(a-b)^2 = a^2 - 2ab + b^2$

	a	$-b$
a	a^2	$-ab$
$-b$	$-ab$	b^2

$a^2 - ab - ab + b^2$

	x	-4.5
x	x^2	$-4.5x$
-4.5	$-4.5x$	20.25

Your Turn: Solve the equation using square roots:

1) $x^2 + 10x + \left(\frac{10}{2}\right)^2$

A: 1
B: 10
C:

$(x+5)^2$

$x^2 + 10x + 25$

2) $x^2 - 4x + \left(\frac{-4}{2}\right)^2$

A: 1
B: -4
C:

$(x-2)^2$

$x^2 - 4x + 4$

3) $x^2 + 7x$

$\frac{1}{2}b = 3.5$ $\quad (x+3.5)^2$

$x^2 + 7x + (3.5)^2$

$x^2 + 7x + 12.25$

Example 2 – Solving a Quadratic Equation $x^2 + bx = d$

Solve $x^2 - 16x = -15$ by completing the square

$x^2 - 16x + \left(\frac{-16}{2}\right)^2 = -15 + 64$

$(x-8)^2 = 49$

Step 1: Find half of "b": -8

Step 2: Add the result of step 1 to both sides of the equation:

$x^2 - 16x + (-8)^2 = -15 + (-8)^2$

Step 3: Write the left side as a factored binomial:

$x^2 - 16x + 64 = -15 + 64$

$(x-8)^2 = 49$

$(x-8)^2 = 49$

Step 4: Factor the right side of the equation:

$\sqrt{(x-8)^2} = \sqrt{49}$

Step 5: Take the square root of each side of the equation:

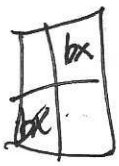
$x - 8 = \pm 7$

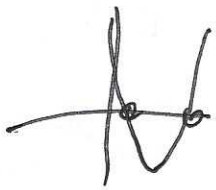
Step 6: Add 8 to each side: $x - 8 = 7$ $\quad +8 \quad +8$ $\qquad x - 8 = -7$ $\quad +8 \quad +8$

The solutions are $x = 15$ and $x = 1$

or (15, 0) and (1, 0)

ROOTS





Your Turn: Solve the equation by completing the square:

4) $x^2 + 14x = 15$

$x^2 + 14x + (7)^2 = 15 + 7^2$

$x^2 + 14x + 49 = 15 + 49$

$(x+7)^2 = 64$

$\sqrt{(x+7)^2} = \sqrt{64}$

$x + 7 = \pm 8$

$x + 7 = 8$ $\quad -7 \quad -7$

$x = 1$

$(1, 0)$

$x + 7 = -8$ $\quad -7 \quad -7$

$x = -15$

$(-15, 0)$

5) $x^2 - 4x = -2$

$x^2 - 4x + (-2)^2 = -2 + (-2)^2$

$x - 4x + 4 = -2 + 4$

$(x-2)^2 = 2$

$\sqrt{(x-2)^2} \quad \sqrt{2}$

$x - 2 = \pm\sqrt{2}$

$x = 2 \pm \sqrt{2}$

$x = 2 + 1.41 \quad 2 - 1.41$

$x \approx 3.41 \quad x \approx .59$

$x^2 - 6x + \left(\frac{-6}{2}\right)^2$

6) $x^2 - 6x = 16$

$x^2 - 6x + 9 = 16 + 9$

$(x-3)^2 = 25$

$x - 3 = \pm 5$

$x - 3 = 5 \qquad x - 3 = -5$

$x = 8 \qquad x = -2$

$x^2+10x-4=0$ $\quad a^2+2ab+b^2=(a+b)^2-4$ → 1 −4, 2 −2, −1 4

Example 3 – Solving a Quadratic Equation $x^2 + bx + c = 0$

Solve $2x^2 + 20x - 8 = 0$ / 2

$x^2+10x-4=0$
$+4 \quad +4$
$x^2+10x=4$

$x^2+10x+\left(\frac{10}{2}\right)^2=4+\left(\frac{10}{2}\right)^2$
$x^2+10x+25=4+25$
$(x+5)^2=29$

$\sqrt{(x+5)^2}=\sqrt{29}$
$x+5=\pm 5.3$

$x+5=5.3$ / $-5 \quad -5$
$x=.3$

$x+5=-5.3$ / $-5 \quad -5$
$x=-10.3$

Your Turn: Solve the equation by completing the square:

7) $x^2 - 8x + 15 = 0$

$x^2-8x=-15$
$x^2-8x+\left(\frac{-8}{2}\right)^2=-15+\left(\frac{-8}{2}\right)^2$
$x^2-8x+16=-15+16$
$(x-4)^2=1$
$x-4=\pm 1$
$x=5$ $\quad x=3$

8) $2x^2 + 20x + 44 = 0$

$x^2+10x+22=0$
$x^2+10x=-22$
$x^2+10x+(5)^2=-22+5^2$
$(x+5)^2=3$
$x+5=\pm\sqrt{3}$
$x=-5\pm\sqrt{3}$

9) $-3x^2 - 24x + 18 = -40$

$\frac{-3x^2}{-3}\ \frac{-24x}{-3}=\frac{-58}{-3}$
$x^2+8x=19.33$
$x^2+8x+\left(\frac{8}{2}\right)^2=19.33+\left(\frac{8}{2}\right)^2$
$x^2+8x+16=19.33+16$
$(x+4)^2=35.33$
$x+4=\pm 5.94$

$x+4=5.94$ / $-4 \quad -4$
$x=1.94$

$x+4=-5.94$ / $-4 \quad -4$
$x=-9.94$

Example 4 – Finding the maximum or minimum value of a quadratic by completing the square:

a) $y = x^2 + 4x - 1$

A: 1
B: 4

$\frac{-(4)}{2(1)}=-2$ $\quad \frac{-(b)}{2(a)}$

$y=(-2)^2+4(-2)-1$
$4-8-1$
$y=-5$

VERTEX. $(-2, -5)$

MIN. VALUE

b) $y = -x^2 + 2x + 7$

MAX VALUE $(1, 8)$

A: −1
B: 2

$\frac{-(2)}{2(-1)}=1$

$y=-(1)^2+2(1)+7$
$-1+2+7$

Your Turn: Solve the equation by completing the square:

MIN

10) $y = x^2 - 4x + 4$

A: 1
B: −4

$\frac{-(-4)}{2(1)}=(2)$

$(2)^2-4(2)+4$
$4-8+4$

MIN

11) $y = x^2 + 12x + 40$

A: 1
B: 12

$\frac{-(12)}{2(1)}=(-6)$

$(-6)^2+12(-6)+40$
$36-72+40$
$76-72=4$

MIN

12) $y = x^2 - 2x - 2$

A: 1
B: −2

$\frac{-(-2)}{2(1)}=(1)$

$(1)^2-2(1)-2$
$1-2-2$

$(2, 0)$ $\quad (-6, 4)$ $\quad (1, -3)$