Warm-up

Simplify the rational expression and state excluded values (what can the domain not be?).

1. $\frac{6x+9}{3x-15} \cdot \frac{x-5}{4x+6}$

$\frac{1}{2}; x \neq 5, -\frac{3}{2}$

2. $\frac{x^2-4}{x-2} \div \frac{x+2}{4x-8}$

$4(x-2)\ or\ 4x-8; x \neq -2, 2$

P6C Unlike Denominators

- Add and subtract rational expressions with unlike denominators

Find the Least Common Denominator and Combining Terms

1. Factor each denominator completely.
2. List the factors of the first denominator.
3. Add to the list in step 2 any factors of the second denominator that do not appear on the list.
4. Form the product of each different factor from the list in step 3. This product is the LCD.

Example 5:

Finding the Least Common Denominator

$$\frac{1}{4x^3} + \frac{2}{10x^2}$$

Step 1: Find the LCD of $4x^3$ and $10x^2$

Prime Factors of $4x^3$: $2 \cdot 2 \cdot x \cdot x \cdot x$

Prime Factors of $10x^2$: $2 \cdot 5 \cdot x \cdot x$

Step 2: LCD =product of the HIGHEST number of occurrences for EACH FACTOR

$$2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x = 20x^3$$

You Try: Find the LCD, then simplify.

$$\frac{1}{8x^5y^4}+\frac{2}{36x^2y^7}$$

$$\longrightarrow 8 \cdot 9 \cdot x^5 \cdot y^7 = \boxed{72x^5y^7}$$

Simplify with common denominators:

$$\frac{1}{8x^5y^4} + \frac{2}{36x^2y^7}$$

Determine the LCD = $72x^5y^7$

$$\frac{9y^3}{9y^3} \cdot \frac{1}{8x^5y^4} + \frac{2x^3}{2x^3} \cdot \frac{2}{36x^2y^7}$$

Multiply both fractions by the needed factors to get the LCD

$$\frac{9y^3}{72x^5y^7} + \frac{4x^3}{72x^5y^7}$$

Simplify each fraction

$$\frac{9y^3 + 4x^3}{72x^5y^7}$$

Add the numerators

EX 8: $\dfrac{11x}{x^2+3x-28}+\dfrac{x}{x+7}$

$$\frac{11x}{(x+7)(x-4)}+\frac{x}{x+7}$$

$$\frac{11x}{(x+7)(x-4)}+\frac{x-4}{x-4}\cdot\frac{x}{x+7}$$

$$\frac{11x}{(x+7)(x-4)}+\frac{x(x-4)}{(x+7)(x-4)}$$

$$\frac{11x+x^2-4x}{(x+7)(x-4)}$$

$$\frac{x^2+7x}{(x+7)(x-4)}$$

$$\frac{x(x+7)}{(x+7)(x-4)}$$

$$\frac{x}{(x-4)};x\neq-7,4$$

Factor denominators & determine LCD

LCD = $(x+7)(x-4)$

EX 9: $\frac{x+1}{x^2-9}+\frac{4}{x+3}-\frac{x-1}{x-3}$

$$\frac{x+1}{(x+3)(x-3)}+\frac{4}{x+3}-\frac{x-1}{x-3}$$

Factor denominators & determine LCD

$$\frac{x+1}{(x+3)(x-3)}+\frac{x-3}{x-3}\cdot\frac{4}{x+3}-\frac{x+3}{x+3}\cdot\frac{x-1}{x-3}$$

LCD = $(x+3)(x-3)$

$$\frac{x+1}{(x+3)(x-3)}+\frac{4x-12}{(x+3)(x-3)}-\frac{x^2+2x-3}{(x+3)(x-3)}$$

$$\frac{x+1+4x-12-x^2-2x+3}{(x+3)(x-3)}$$

$$\frac{-x^2+3x-8}{(x+3)(x-3)};x\neq-3,3$$

P6D Complex Fractions

- Simplify complex rational expressions

P6 day 3 Objective

Be able to solve complex rational expressions.

1. Get Numerator Fractions over LCD.
2. Get Denominator Fractions over LCD.
3. Factor completely both numerator and denominator.
4. **Keep → Switch → Flip**
5. Cancel and Simplify.
6. State Domain,

   find excluded x values prior to cancelling

(excluded values → set denominator factors ≠ 0).

Complex Rational Expressions

- Complex Rational Expressions- have numerators or denominators containing one or more rational expressions.

  (Fractions over fractions)

Examples:

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

$$\frac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}}$$

Example 1:

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

STEPS:

1. Find LCD for numerator.
2. Find LCD for denominator.
3. Rewrite problem as horizontal division problem.
4. Rewrite as a multiplication and solve.

$$\frac{1}{1}+\frac{1}{x}=\frac{1\bullet x}{1\bullet x}+\frac{1}{x}=\frac{x}{x}+\frac{1}{x}=\frac{x+1}{x}$$

$$\frac{1}{1}-\frac{1}{x}=\frac{1\bullet x}{1\bullet x}-\frac{1}{x}=\frac{x}{x}-\frac{1}{x}=\frac{x-1}{x}$$

$$\frac{\frac{x+1}{x}}{\frac{x-1}{x}}=\frac{x+1}{x}\div\frac{x-1}{x}$$

$$\frac{x+1}{x}\div\frac{x-1}{x}=\frac{x+1}{x}\bullet\frac{x}{x-1}=\frac{x+1}{x-1},\ x\neq 0,\ x\neq 1$$

Example 2:

$$\frac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}}$$

LCD: 2x

LCD: 4x

Find LCD.

Simplify Numerator and Denominator

$$\frac{\frac{2-3x}{2x}}{\frac{4+3x}{4x}} = \frac{2-3x}{2x} \div \frac{4+3x}{4x} = \frac{2-3x}{2x} \bullet \frac{4x}{4+3x}$$

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ANSWER:

$$\frac{2(2-3x)}{4+3x},\ x \neq 0, -\frac{4}{3}$$

You TRY:

State the excluded values

Don't forget to distribute the negative

$$\frac{\dfrac{1}{x+7}-\dfrac{1}{x}}{7}$$

ANSWER:

$$\frac{-1}{x(x+7)},\ x\neq 0,\ x\neq -7$$

Example 3:

$$\frac{\frac{x}{x+2}-\frac{1}{x}}{\frac{3}{x^2-4}+1}$$

Find LCD of Numerator and Combine Terms.

$$\frac{x}{x+2}\bullet\frac{x}{x}-\frac{1}{x}\frac{x+2}{x+2}=\frac{x^2-(x+2)}{x(x+2)}$$

Find LCD of Denominator and Combine Terms.

$$\frac{3}{x^2-4}+1\frac{x^2-4}{x^2-4}\qquad\frac{3+x^2-4}{(x^2-4)}$$

Factor Numerator and Denominator

$$=\frac{x^2-x-2}{x(x+2)}\qquad=\frac{(x-2)(x+1)}{x(x+2)}$$

$$\frac{x^2-1}{(x^2-4)}\qquad\frac{(x+1)(x-1)}{(x+2)(x-2)}$$

Cont'd

$$= \frac{\frac{(x-2)(x+1)}{x(x+2)}}{\frac{(x+1)(x-1)}{(x+2)(x-2)}}$$

Keep → Switch → Flip

$$= \frac{(x-2)\cancel{(x+1)}}{\cancel{x(x+2)}} \times \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+1)}(x-1)}$$

ANSWER:

$$\frac{(x-2)^2}{x(x-1)}, x \neq 0, -2, -1, 1$$

You try

$$\frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1}$$

Answer: $\frac{2(x+2)}{x+1}, x \neq -1, 2, 1, -2$