4.1 Exponential Functions

- Evaluate exponential functions
- Graph exponential functions
- Evaluate functions with base e

Functions whose equations contain a variable in the exponent are called exponential functions

$$\boldsymbol{f(x) = b^x} \text{ or } \boldsymbol{y = b^x}$$

LT: Evaluate exponential functions

Example 1 The exponential function $f(x) = 13.49(0.967)^x - 1$ describes the number of O-rings expected to fail, $f(x)$, when the temperature is x°F. On the morning the *Challenger* was launched, the temperature was 31°F, colder than any previous experience. Find the number of O-rings expected to fail at this temperature.

$$x = 31$$

$$f(x) = 13.49(0.967)^{31} - 1$$

$$f(x) \approx 3.8 \quad \approx 4$$

Your turn 1 Use the function in Example 1 to find the number of O-rings expected to fail at a temperature of 60°F. Round to the nearest whole number.

$x = 60$

$f(x) = 13.49(0.967)^{60} - 1$

$f(x) \approx 0.8014 \approx 1$

LT: Graph exponential functions

Example 2 Graph: $f(x) = 2^x$

Identify the y-intercept: (0,1)

What is happening at the x-axis?

The curve is above and close to the x-axis on for negative values of x, but then increases rapidly.

Your Turn 2 Graph: $f(x) = 3^x$

Identify the y-intercept: (0,1)

What is happening at the x-axis?

The curve is above and close to the x-axis on for negative values of x, but then increases rapidly.

Example 3 Graph: $g(x) = \left(\frac{1}{2}\right)^x$

Identify the y-intercept:

(0,1)

What is happening at the x-axis?

The curve decreases rapidly, it is above and close to the x-axis on for positive values of x.

Your Turn 3 Graph: $g(x) = \left(\frac{1}{3}\right)^x$

Identify the y-intercept:

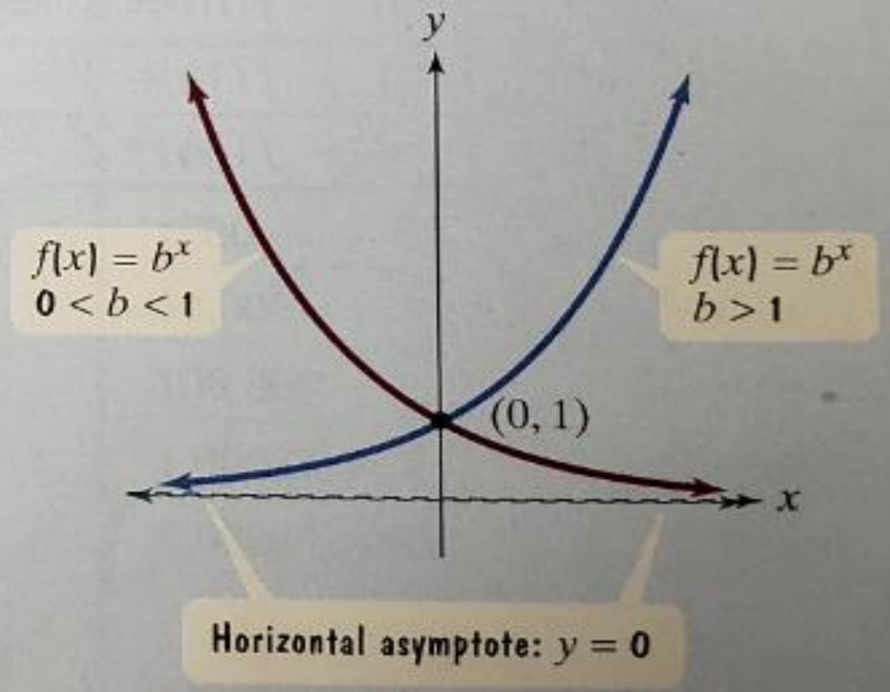
(0,1)

What is happening at the x-axis?

The curve decreases rapidly, it is above and close to the x-axis on for positive values of x.

Characteristics of Exponential Functions of the Form $f(x) = b^x$

1. The domain of $f(x) = b^x$ consists of all real numbers: $(-\infty, \infty)$. The range of $f(x) = b^x$ consists of all positive real numbers: $(0, \infty)$.
2. The graphs of all exponential functions of the form $f(x) = b^x$ pass through the point $(0, 1)$ because $f(0) = b^0 = 1 (b \neq 0)$. The y-intercept is 1.
3. If $b > 1$, $f(x) = b^x$ has a graph that goes up to the right and is an increasing function. The greater the value of b, the steeper the increase.
4. If $0 < b < 1$, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function. The smaller the value of b, the steeper the decrease.
5. $f(x) = b^x$ is one-to-one and has an inverse that is a function.
6. The graph of $f(x) = b^x$ approaches, but does not touch, the x-axis. The x-axis, or $y = 0$, is a horizontal asymptote.



LT: Evaluate functions with base e

Natural Base *e* is an irrational number. $\boldsymbol{e = 2.718281827}$

Natural Exponential Function $\boldsymbol{f(x) = e^x}$

Example 6 In a report entitled Resources and Man, the US National Academy of Sciences concluded that a world population of 10 billion "is close to (if not above) the maximum that an intensely managed world might hope to support with some degree of comfort and individual choice. "At the time the report was issued in 1969, world population was approximately 3.6 billion, with a growth rate of 2% per year.

The function $f(x) = 3.6e^{0.02x}$ describes the world population, in billions, x years after 1969. Use the function to find world population in the year 2020. Is there a cause for alarm?

$x = 51$

$f(x) = 3.6e^{0.02(51)}$

$f(x) \approx 9.98$

Close to 10 billion, yes there is cause for concern.

Your Turn 4 The function $f(x) = 6.4e^{0.0123x}$ describes the world population in billions, x years after 2004 subject to a growth rate of 1.23% annually. Use the function to predict the world population in 2050.

$x = 46$

$f(x) = 6.4e^{0.0123(46)}$

$f(x) \approx 11.27$

Close to 11 billion, **yes** there is cause for concern.