4.3B Properties of Logarithms

- **Condense logarithmic expressions**
- **Use the change-of-base property**

LT: Condense logarithmic expressions

To condense a logarithmic expression, write the sum or difference of two or more logarithmic expressions as a single logarithmic expression. Use the same properties:

$\log_b M + \log_b N = \log_b (MN)$	**Product Rule**
$\log_b M - \log_b N = \log_b \left(\frac{M}{N}\right)$	**Quotient Rule**
$p \log_b M = \log_b M^p$	**Power Rule**

Example 5: Write as a single logarithm:

$\log_4 2 + \log_4 32$

$\log_4 2 \cdot 32$

$\log_4 64$

$\log_4 4^3$

3

b. $\log(4x - 3) - \log x$

$\log \frac{4x - 3}{x}$

Example 6: Write as a single logarithm:

a. $\frac{1}{2}\log x + 4\log(x-1)$

$\log x^{\frac{1}{2}} + \log(x-1)^4$

$\log\left[x^{\frac{1}{2}}(x-1)^4\right]$

b. $3\ln(x+7) - \ln x$

$\ln(x+7)^3 - \ln x$

$\ln\frac{(x+7)^3}{x}$

c. $4\log_b x - 2\log_b 6 - \frac{1}{2}\log_b y$

$\log_b x^4 - \log_b 6^2 - \log_b y^{\frac{1}{2}}$

$\log_b x^4 - \left(\log_b 6^2 + \log_b y^{\frac{1}{2}}\right)$

$\log_b x^4 - \left(\log_b 36y^{\frac{1}{2}}\right)$

$\log_b \frac{x^4}{36y^{\frac{1}{2}}}$ or $\log_b \frac{x^4}{36\sqrt{y}}$

LT: Use the change-of-base property

When the base is **not 10 or *e***, using a calculator can be difficult. So we use the change of base formula:

Change of base using Common Logs: $\log_b M = \frac{\log M}{\log b}$

Change of base using Natural Logs: $\log_b M = \frac{\ln M}{\ln b}$

Example 7: Use common logs to evaluate $\log_5 140$

$$\log_5 140 = \frac{\log 140}{\log 5}$$

$$\log_5 140 \approx 3.07$$

Example 8: Use natural logs to evaluate $\log_5 140$

$$\log_5 140 = \frac{\ln 140}{\ln 5}$$

$$\log_5 140 \approx 3.07$$