



# ARITHMETIC SEQUENCES AND SERIES

## 9.2

- *Recognize, write, and find the  $n$ th term of an arithmetic sequence*
- *Find the  $n$ th partial sum of an arithmetic series*
- *Use arithmetic sequences to model and solve real-life problems*

# ARITHMETIC SEQUENCE

A sequence is arithmetic when the differences between consecutive terms are the same.

$d$  is the common difference, it can be positive or negative and is added to each term.

To find  $d$  subtract the previous term from a term.

Formula to find the  $n$ th term of an arithmetic sequence:

$$a_n = a_1 + (n - 1)d$$

# EXAMPLE 1

Find the 21<sup>st</sup> term of the arithmetic sequence whose first three terms are 51, 48.1, 45.2

$$a_n = a_1 + (n - 1)d$$

$$n = 21$$

$$a_1 = 51$$

$$d = 48.1 - 51$$

$$d = -2.9$$

$$a_{21} = 51 + (21 - 1)(-2.9)$$

$$a_{21} = 51 + (20)(-2.9)$$

$$a_{21} = 51 - 58$$

$$a_{21} = -7$$

## EXAMPLE 2

If the first term of an arithmetic sequence is 7 and the second term is 20 find the 38th term.

$$a_n = a_1 + (n - 1)d$$

$$n = 38$$

$$a_1 = 7$$

$$d = 20 - 7$$

$$d = 13$$

$$a_{38} = 7 + (38 - 1)(13)$$

$$a_{38} = 7 + (37)(13)$$

$$a_{38} = 7 + 481$$

$$a_{38} = 488$$

## EXAMPLE 3

Find the 11th term if terms one and two are 4 and 9 respectively.

$$a_n = a_1 + (n - 1)d$$

$$n = 11$$

$$a_1 = 4$$

$$d = 9 - 4$$

$$d = 5$$

$$a_{11} = 4 + (11 - 1)(5)$$

$$a_{11} = 4 + (10)(5)$$

$$a_{11} = 4 + 50$$

$$a_{11} = 54$$

## EXAMPLE 4

Find the first 5 terms of the arithmetic sequence whose 4th term is 17 and 13th term is 64.79

$$a_1, a_2, a_3, 17, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, 64.79$$

$$n = (13 - 4) + 1$$

$$n = 10$$

$$a_n = a_1 + (n - 1)d$$

$$64.79 = 17 + (10 - 1)d$$

$$5.31 = d$$

$$1.07, 6.38, 11.69, 17, 22.31, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, 64.79$$

## EXAMPLE 5

Find the  $n$ th term (the general term) of the arithmetic sequence whose common difference is 6 and whose first term is 7.

$$a_n = a_1 + (n - 1)d$$

$$a_n = 7 + (n - 1)(6)$$

$$a_1 = 7$$

$$a_n = 7 + 6n - 6$$

$$d = 6$$

$$a_n = 6n + 1$$

## EXAMPLE 6

Find the  $n$ th term of the arithmetic sequence whose common difference is  $-3$  and whose first term is  $18$ .

$$a_n = a_1 + (n - 1)d$$

$$a_1 = 18$$

$$d = -3$$

$$a_n = 18 + (n - 1)(-3)$$

$$a_n = 18 - 3n + 3$$

$$a_n = -3n + 21$$



# SUM OF ARITHMETIC SERIES

$$S_n = \frac{n}{2} (a_1 + a_n)$$

# EXAMPLE 7

Find the sum:  $3 + 9 + 15 + 21 + 27 + 33 + 39 + 45 + 51 + 57 + 63$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$n = 11$$

$$a_1 = 3$$

$$a_{11} = 63$$

$$S_{11} = \frac{11}{2}(3 + 63)$$

$$S_{11} = \frac{11}{2}(66)$$

$$S_{11} = 11(33)$$

$$S_{11} = 363$$

# EXAMPLE 8

Find the sum of the even integers from 2 to 100.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$n = 50$$

$$a_1 = 2$$

$$a_{50} = 100$$

$$S_{50} = \frac{50}{2}(2 + 100)$$

$$S_{50} = 25(102)$$

$$S_{50} = 2550$$

# EXAMPLE 9

Find the sum of the first 200 terms of the sequence: 4, 7, 10, 13, 16, ...

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$n = 200$$

$$a_1 = 4$$

$$d = 3$$

$$a_{200} = 4 + (200 - 1)(3)$$

$$a_{200} = 4 + (199)(3)$$

$$a_{200} = 601$$

$$S_{200} = \frac{200}{2}(4 + 601)$$

$$S_{200} = 100(605)$$

$$S_{200} = 60,500$$

# EXAMPLE 10

Write the series using sigma (summation) notation:  $4 + 11 + 18 + 25 + 32$

$$a_n = a_1 + (n - 1)d$$

$$a_1 = 4$$

$$d = 11 - 4$$

$$d = 7$$

$$a_n = 4 + (n - 1)(7)$$

$$a_n = 4 + 7n - 7$$

$$a_n = 7n - 3$$

$$\sum_{n=1}^5 (7n - 3)$$