9.3
GEOMETRIC SEQUENCES AND SERIES

- *Recognize, write, and find the nth term of a geometric sequence*
- *Find the sum of a finite geometric sequence*
- *Find the sum of an infinite geometric sequence*
- *Use geometric sequences to model and solve real-life problems*

GEOMETRIC SEQUENCE

A sequence is geometric when consecutive terms have a common ratio.

r is the common ratio, it can be positive or negative and is multiplied to each term.

To find r divide a term by the previous term.

Formula to find the nth term of a geometric sequence:

$$a_n = a_1 \cdot r^{n-1}$$

EXAMPLE 1 –
FIND THE NEXT 3 TERMS OF EACH GEOMETRIC SEQUENCE:

a. 4, 16, 64, ______, ______, ______

$r = \frac{16}{4}$ $r = 4$

$a_n = a_1 \cdot r^{n-1}$

$a_4 = (4) \cdot (4)^{4-1}$

$a_4 = (4)64$

$a_4 = 256$

$a_5 = 256 \cdot 4 \quad = 1024$

$a_6 = 1024 \cdot 4 \quad = 4096$

$4, 16, 64, 256, 1024, 4096$

b. 81, 27, 9, ______, ______, ______

$r = \frac{27}{81}$ $r = \frac{1}{3}$

$a_n = a_1 \cdot r^{n-1}$

$a_4 = (9) \cdot \left(\frac{1}{3}\right) \quad = 3$

$a_5 = (3) \cdot \left(\frac{1}{3}\right) \quad = 1$

$a_6 = (1) \cdot \left(\frac{1}{3}\right) \quad = \frac{1}{3}$

$81, 27, 9, 3, 1, \frac{1}{3}$

EXAMPLE 2 - GIVE THE FIRST 4 TERMS OF EACH GEOMETRIC SEQUENCE DESCRIBED

a. $a_1 = 125 \; and \; r = -\frac{2}{5}$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_1 = (125) \cdot \left(-\frac{2}{5}\right)^{1-1} = 125$$

$$a_2 = (125) \cdot \left(-\frac{2}{5}\right)^{2-1} = -50$$

$$a_3 = (125) \cdot \left(-\frac{2}{5}\right)^{3-1} = 20$$

$$a_4 = (125) \cdot \left(-\frac{2}{5}\right)^{4-1} = -8$$

$125, -50, 20, -8$

b. $a_1 = 4 \; and \; r = 3$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_1 = (4) \cdot (3)^{1-1} = 4$$

$$a_2 = (4) \cdot (3) = 12$$

$$a_3 = (12) \cdot (3) = 36$$

$$a_4 = (36) \cdot (3) = 108$$

$4, 12, 36, 108$

EXAMPLE 3 - FIND THE NTH TERM (THE GENERAL TERM) OF EACH GEOMETRIC SEQUENCE DESCRIBED.

a. $a_4 = 10, r = -\frac{1}{2}, a_{12} = ?$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_1 = 10$$

$$n = (12 - 4) + 1$$

$$n = 9$$

$$a_{12} = 10 \cdot \left(-\frac{1}{2}\right)^{9-1}$$

$$a_{12} = 10 \cdot \left(\frac{1}{2^8}\right)$$

$$a_{12} = 10 \cdot \left(\frac{1}{256}\right)$$

$$a_{12} = \frac{5}{128}$$

b. $a_6 = 5, r = 3, a_{19} = ?$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_1 = 5$$

$$n = (19 - 6) + 1$$

$$n = 14$$

$$a_{19} = 5 \cdot (3)^{14-1}$$

$$a_{19} = 5 \cdot (3)^{13}$$

$$a_{19} = 7{,}971{,}615$$

SUM OF A FINITE GEOMETRIC SERIES

$$S_n = \frac{a_1(1 - r^{n})}{1 - r}; r \neq 1$$

$$S_n = \sum_{i=1}^{n} a_1 \cdot r^{n-1}; r \neq 1$$

EXAMPLE 4

a. Find the sum of the first seven terms of the geometric sequence: 3, 15, 75, 375, …

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$r = \frac{15}{3}$ $\quad r = 5 \quad a_1 = 3$

$$S_7 = \frac{3(1 - 5^7)}{1 - 5}$$

$$S_7 = \frac{3(1 - 478{,}125)}{-4}$$

$$S_7 = 58{,}593$$

EXAMPLE 4

b. Find the sum $\sum_{k=1}^{11} (0.3)^k$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_{11} = \frac{0.3(1 - 0.3^{11})}{1 - 0.3}$$

$r = 0.3$

$a_1 = 0.3$

$n = 11$

$$S_{11} = 0.429$$

SUM OF AN INFINITE GEOMETRIC SERIES

$$S = \frac{a_1}{1-r} \quad or \quad S = \sum_{i=1}^{\infty} a_1 \cdot r^i$$

$$if\ and\ only\ if \quad -1 < r < 1$$

Otherwise there is **NO SUM**

EXAMPLE 5

a. Find the sum of each infinite geometric series, if it exists.

$$\sum_{i=1}^{\infty} 6(0.6)^{i-1}$$

$$S = \frac{a_1}{1-r}$$

$r = 0.6$

$$S = \frac{6}{1-0.6}$$

$-1 < 0.6 < 1$

$$\boxed{S = 15}$$

$a_1 = 6$

EXAMPLE 5

b. Find the sum of each infinite geometric series, if it exists.

$$1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \cdots$$

$$S = \frac{a_1}{1 - r}$$

$$r = -\frac{2}{3}$$

$$-1 < -\frac{2}{3} < 1$$

$$a_1 = 1$$

$$S = \frac{1}{1 - -\frac{2}{3}}$$

$$S = \frac{1}{\frac{3}{3} - -\frac{2}{3}}$$

$$S = \frac{1}{\frac{5}{3}}$$

$$S = \frac{3}{5}$$

EXAMPLE 5

c. Find the sum of each infinite geometric series, if it exists.

$$1 - 2 + 4 - 8 + \cdots$$

$$S = \frac{a_1}{1 - r}$$

$$S = Does\ Not\ Exist$$

$$r = -2$$

$$-1 \nless -2 \nless 1$$

$$a_1 = 1$$

EXAMPLE 5

d. Find the sum of each infinite geometric series, if it exists.

$$\sum_{k=1}^{\infty} 5\left(\frac{9}{7}\right)^{k-1}$$

$$S = \frac{a_1}{1-r}$$

$$S = Does\ Not\ Exist$$

$$r = \frac{9}{7}$$

$$-1 \nless \frac{9}{7} \nless 1$$

$$a_1 = 5$$

EXAMPLE 5

e. Find the sum of each infinite geometric series, if it exists.

$$\sum_{n=1}^{\infty} 9\left(\frac{5}{6}\right)^{n-1}$$

$$r = \frac{5}{6}$$

$$-1 < \frac{5}{6} < 1$$

$$a_1 = 9$$

$$S = \frac{a_1}{1 - r}$$

$$S = \frac{9}{1 - \frac{5}{6}}$$

$$S = 54$$

EXAMPLE 5

e. Find the sum of each infinite geometric series, if it exists.

$$6 + \frac{10}{3} + \frac{50}{27} + \frac{250}{243} + \cdots$$

$$S_n = \frac{a_1}{1 - r}$$

$$r = \frac{\frac{10}{3}}{6} = \frac{5}{9}$$

$$-1 < \frac{5}{9} < 1$$

$$a_1 = 6$$

$$S = \frac{6}{1 - \frac{5}{9}}$$

$$S = \frac{6}{\frac{9}{9} - \frac{5}{9}}$$

$$S = \frac{6}{\frac{4}{9}}$$

$$S = \frac{27}{2}$$