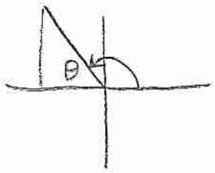
Name Key
Period

Semester 2 Final Exam Review

Precalculus

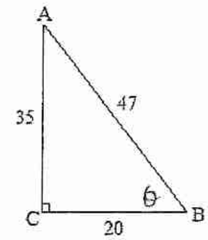
1) Find the reference angle for 135°.



$180 - 135 = 45°$

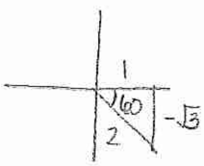
$\theta = 45°$

2) Find $\cos B$:



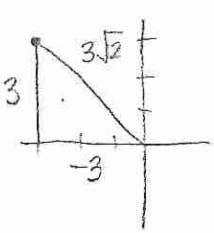
$\cos B = \frac{adj}{hyp} = \frac{20}{47}$

3) Find the value of $\tan 300°$.



$\tan 300 = -\frac{\sqrt{3}}{1} = -\sqrt{3}$

4) Find the value of $\sec\theta$ if the point (-3, 3) lies on the terminal side of the angle.



$(-3)^2 + (3)^2 = c^2$

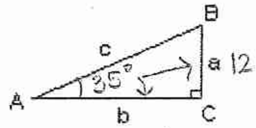
$9 + 9 = c^2$

$18 = c^2$

$c = \sqrt{18} = 3\sqrt{2}$

$\sec\theta = \frac{\cancel{3}\sqrt{2}}{-\cancel{3}} = -\sqrt{2}$

5) Solve for b if $A = 35°, a = 12$.



$(b)\tan 35 = \frac{12}{b}(b)$

$\frac{12 = \tan 35 \cdot b}{\tan 35 \quad \tan 35}$

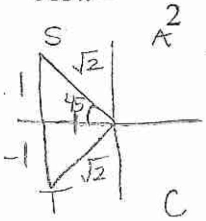
$b = \frac{12}{\tan 35}$

$b = 17.14$

6) Solve for x on the interval $0° \le x \le 360°$.

$\cos x = -\frac{\sqrt{2}}{2}$ – cos neg. in Q2 and Q3

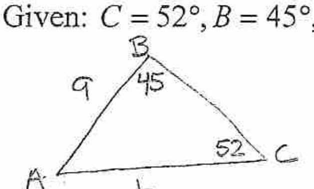
– $\cos\theta = \frac{adj}{hyp}$



So $180 - 45 = \boxed{135°}$

$180 + 45 = \boxed{225°}$

7) Given: $C = 52°, B = 45°, c = 9$, find b.



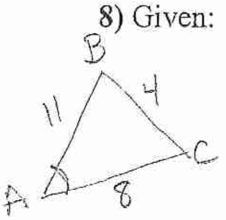
$\boxed{8.08}$

$\frac{\sin 45}{b} = \frac{\sin 52}{9}$

$9\sin 45 = b\sin 52$

$b = \frac{9\sin 45}{\sin 52} = 8.08$

8) Given: $a = 4, b = 8, c = 11$, find A.



Use Law of Cosines

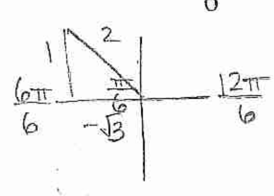
$a^2 = b^2 + c^2 - 2bc\cos A$

$4^2 = 8^2 + 11^2 - 2(8)(11)\cos A$

$16 = 185 - 176\cos A$

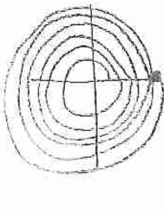
$\cos^{-1}\left(\frac{-169}{-176}\right) = \boxed{16.21°}$

9) Evaluate $\cos\frac{5\pi}{6}$.



$\boxed{\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}}$

10) Find the value of $\sin 12\pi$.



2π (1,0)

cos, sin

$\sin 12\pi = 0$

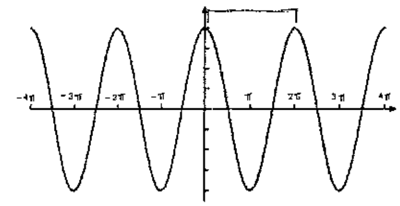
11) Given $f(x) = 2\sin 4x + 1$, identify the amplitude and period

$f(x) = a\sin(bx - c) + k$

amplitude: 2

period: $\frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

12) Find the period and the amplitude of the function:
Write the equation of the function



per: 2π
amp: 4
start @ amp = cos graph

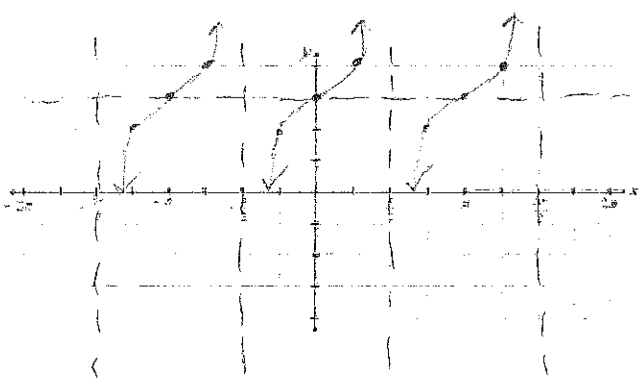
$f(x) = 4\cos x$

13) What direction will the graph, $f(x) = 2\cos(x+\pi)$, be shifted from the parent function $f(x) = \cos x$?

To determine a shift: $\frac{-c}{K} = \frac{-\pi}{1} = -\pi \longrightarrow \boxed{\text{shift Left } \pi}$

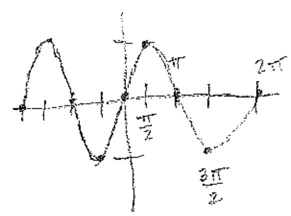
$\hookrightarrow$ moves left

14) Graph $f(x) = \tan x + 3$.
(up 3)



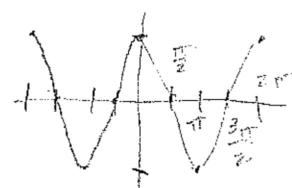
15) For each quadrant, state whether sine and cosine is increasing or decreasing

$f(x) = \sin x$



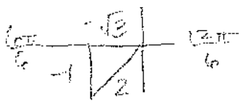
Q1: $0 < \theta < \frac{\pi}{2}$ increase
Q2: $\frac{\pi}{2} < \theta < \pi$ decrease
Q3: $\pi < \theta < \frac{3\pi}{2}$ decrease
Q4: $\frac{3\pi}{2} < \theta < 2\pi$ increase

$g(x) = \cos x$



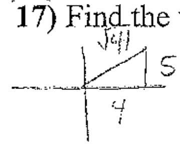
Q1: $0 < \theta < \frac{\pi}{2}$ decrease
Q2: $\frac{\pi}{2} < \theta < \pi$ decrease
Q3: $\pi < \theta < \frac{3\pi}{2}$ increase
Q4: $\frac{3\pi}{2} < \theta < 2\pi$ increase

16) Find the value of $\cos^{-1}\left(\sin\frac{7\pi}{6}\right)$.



$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} = 120°$

17) Find the value of $\sin(\cot^{-1}\frac{4}{5})$.



$\sin\theta = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$

18) Simplify $\frac{\cot x}{\cos x} + \frac{1}{\sin x}$.

$\boxed{\frac{\frac{\cos x}{\sin x}}{\cos x}} + \frac{1}{\sin x} \Rightarrow \frac{1}{\sin x} + \frac{1}{\sin x}$

$= \frac{2}{\sin x} = 2\csc x$

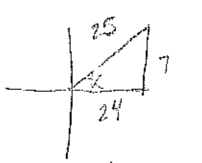
19) Find a numerical value of one trigonometric function if $2\cos x \csc x = 1$.

$\cos x \cdot \frac{1}{\sin x} = \frac{1}{2}$

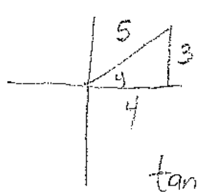
$\frac{\cos x}{\sin x} = \frac{1}{2}$

$\cot x = \frac{1}{2}$

19) Find the exact value of $\tan(x+y)$ if $\sin x = \frac{7}{25}$, $0 < x < \frac{\pi}{2}$, and $\cos y = \frac{4}{5}$, $0 < y < \frac{\pi}{2}$.



$\tan x = \frac{7}{24}$



$\tan y = \frac{3}{4}$

$\tan\left(\frac{7}{24} + \frac{3}{4}\right) = \frac{\frac{7}{24} + \frac{3}{4}}{1 - \left(\frac{7}{24}\right)\left(\frac{3}{4}\right)} = \frac{\frac{25}{24}}{\frac{25}{32}} = \frac{4}{3}$

20) Use the sum and difference identities to find the exact value of $\cos 15°$.

$\cos(45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30$

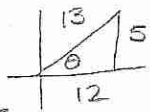
$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

21) Find $\cos 2\theta$ given $\sin\theta = \dfrac{5}{13}$, $0° < \theta < 90°$.

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 =$

$= \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$



22) Solve $\cos^2 x - \sin x \cot x = 0$.

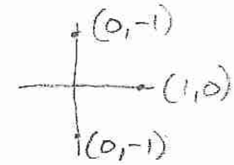
$\cos^2 x - \cancel{\sin x}\,\frac{\cos x}{\cancel{\sin x}} = 0$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0 \qquad \cos x = 1$

$x = \frac{\pi}{2} + 2\pi n,\ \frac{3\pi}{2} + 2\pi n \qquad x = 0 + 2\pi n$

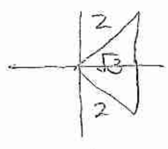


23) Solve $\sin x \cot x = \dfrac{\sqrt{3}}{2}$ for $0° \le x \le 360°$.

$\cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}} = \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30°,\ 330°$



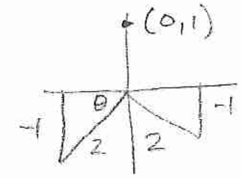
24) Solve $2\sin^2 x - \sin x - 1 = 0$.

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2} \qquad \sin x = 1$

$x = \frac{7\pi}{6},\ \frac{11\pi}{6} \qquad x = 90°$ or $\frac{\pi}{2}$



25) Find the ordered triple that represents $\overrightarrow{AB}$ given $A(4,-2,8)$ and $B(0,-5,12)$.

$\overrightarrow{AB} = \langle 0-4, -5+{+2}, 12-8 \rangle$

$= \langle -4, -3, 4 \rangle$

26) Find the magnitude of $\langle 3,-5 \rangle$.

$\|m\| = \sqrt{3^2 + (-5)^2} = \sqrt{9+25}$

$= \boxed{\sqrt{34}}$

27) Find $3\vec{a} - 2\vec{b}$ if $\vec{a} = \langle -1,4 \rangle$ and $\vec{b} = \langle 6,-2 \rangle$.

$3\langle -1,4 \rangle - 2\langle 6,-2 \rangle$

$\langle -3,12 \rangle + \langle -12,4 \rangle = \boxed{\langle -15, 16 \rangle}$

28) Find the dot product: $\langle 3,-1 \rangle \cdot \langle 6,8 \rangle$.

$(3 \cdot 6) + (-1 \cdot 8)$

$18 + -8 = \boxed{10}$

scalar

29) Find the cross product: $\langle 5,-2,1 \rangle \times \langle -3,0,6 \rangle$.

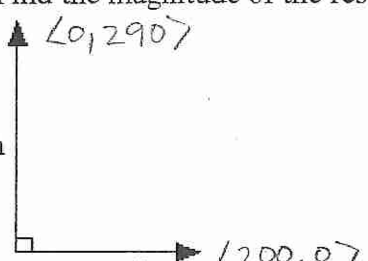
$\begin{vmatrix} i & j & k \\ 5 & -2 & 1 \\ -3 & 0 & 6 \end{vmatrix} = (-12-0)i - (30 - -3)j + (0-6)k$

$-12i - 33j - 6k$

$\langle -12, -33, -6 \rangle$

vector

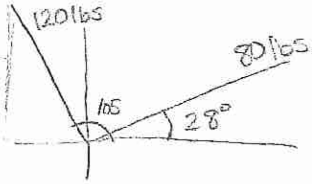
30) Find the magnitude of the resultant vector.



Resultant $= \langle 200+0, 290+0 \rangle = \langle 200, 290 \rangle$

Magnitude $= \sqrt{200^2 + 290^2} = \boxed{352.28}$

31) Two forces with magnitudes of 80 pounds and 120 pounds act on an object at angles of 28° and 105°, respectively, with the positive x-axis. Find the direction of the resultant.



$\langle 80\cos 28, 80\sin 28\rangle$

$\langle 70.64, 37.56\rangle$

$\langle 120\cos 105, 120\sin 105\rangle$

$\langle -31.06, 115.91\rangle$

Resultant: $\langle 70.64 + -31.06, 37.56 + 115.91\rangle$

$\langle 39.58, 153.47\rangle$

extra: $m = \sqrt{39.58^2 + (153.47)^2} = 158.49$

$\tan^{-1}\left(\frac{153.47}{39.58}\right) = \boxed{75.54°}$

$75.54 + 180 = \boxed{225.54°}$

32) Find the distance between the points ~~$P_1\left(4,\frac{\pi}{4}\right), P_2\left(7,\frac{2\pi}{3}\right)$~~ (3,7) and (-2,-5).

$d = \sqrt{(-2-3)^2 + (-5-7)^2}$

$= \sqrt{25+144} = \sqrt{169} = \boxed{13}$

33) Write the polar equation $4r\cos\theta + r^2 = 0$ in rectangular form.

$4\underbrace{r\cos\theta}_{x} + \underbrace{r^2}_{x^2+y^2} = 0$

$4x + x^2 + y^2 = 0$ (solve for y)

$y = \sqrt{-x^2 - 4x}$

34) Find the polar coordinates of the point given in rectangular form. $(-3,3)$. $\rightarrow (r,\theta)$

$r = \sqrt{(-3)^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{3}{-3}\right) = -1 = -\frac{\pi}{4}$

* since (-3) add $\pi \Rightarrow \left(-\frac{\pi}{4} + \frac{4\pi}{4} = \frac{3\pi}{4}\right.$ $\boxed{\left(3\sqrt{2}, \frac{3\pi}{4}\right)}$ or $(3\sqrt{2}, 135°)$

35) Simplify $(3+4i)^2$

$(3+4i)(3+4i)$

$9 + 12i + 12i + 16i^2$

$9 + 24i - 16$

$\boxed{-7+24i}$

36) Express $4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ in rectangular form.

$4\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \frac{\cancel{4}^{2}\sqrt{2}}{\cancel{2}} + \frac{\cancel{4}^{2}\sqrt{2}}{\cancel{2}}i$

$\boxed{2\sqrt{2} + 2i\sqrt{2}}$

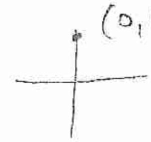
37) Write in rectangular form: $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\cdot 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$.

$2\cdot 5\left(\cos\left(\frac{\pi}{6} + \frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{6} + \frac{\pi}{3}\right)\right)$

$10\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) =$

$10(0 + i(1)) = \boxed{10i}$

cos sin
(0,1)



38) Simplify $\left(1+\sqrt{2}i\right)^7$ and express in rectangular form. Need $r(\cos\theta + i\sin\theta)$

Find r: $\sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$

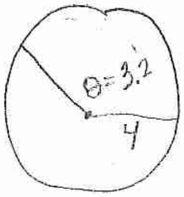
Find θ: $\tan^{-1}\left(\frac{\sqrt{2}}{1}\right) = 54.74°$

$\sqrt{3}(\cos 54.74 + i\sin 54.74)^7$

$\rightarrow (\sqrt{3})^7\left(\cos(7\cdot 54.74) + i\sin(7\cdot 54.74)\right)$

$= \boxed{43 + 18.41i}$

39)The measure of an angle θ is 3.2 radians. If the vertex of the angle is at the center of the circle of radius 4, find the length of the arc formed by angle θ.

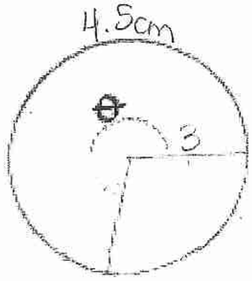


Arc length = $s = \theta r$

$s = 3.2(4)$

$\boxed{\text{Length} = 12.8}$

40) Given a circle with a 3 cm radius, determine θ, the measure of an angle, in radians, whose vertex is at the origin and cuts off an arc length of 4.5 cm



$s = \theta r$

$4.5 = 3\theta$

$\theta = \frac{4.5}{3}$

$\theta = 1.5$ radians

41) Find d for the arithmetic sequence in which $a_1=4$ and $a_6 = 10$.

$\frac{10-4}{6-1} = \frac{6}{5} = d$

$a_n = 4 + \frac{6}{5}(n-1)$

$= 4 + \frac{6}{5}n - \frac{6}{5}$

$\boxed{a_n = \frac{14}{5} + \frac{6}{5}n}$

$n=6$
-3
$3+1=4$

42) Find the sum $\sum_{n=3}^{6}(4n-7)$. Arithmetic

$S_4 = \frac{n}{2}(a_1 + a_n) =$

$S_4 = \frac{4}{2}(5+17)$

$a_3 = 5$ $\quad a_6 = 17$

$\boxed{S_4 = 44}$

43) Find the 15th term in the arithmetic sequence for which $a_1=.75$ and $d = 2.5$

$a_{15} = .75 + 2.5(15-1)$

$\boxed{a_{15} = 35.75}$

$a_1 \quad a_2 \quad a_3$

44) Find the next term for the arithmetic sequence. $4a - 5, 4a, 4a + 5 \ldots$

$d = 4a - (4a-5)$

$d = 4a - 4a + 5$

$d = 5$

$d = 5$

$\boxed{a_4 = 4a + 10}$

$1 \quad 2 \quad 3$

45) Find the common ratio for the geometric sequence. $\mathbf{x^8, x^6, x^4,\ldots}$

$r = \frac{1}{x^2}$

Even terms $=(+)$

46) Find the sixth term of the geometric sequence if $a_4 = -18$ and $a_7 = \frac{2}{3}$ odd terms$=(-)$

$-18 = a_1 r^3$ $\quad \frac{2}{3} = a_1 r^6$ $\quad -\frac{1}{27} = r^3$ $\quad a_1 = \frac{-18}{-1/3} = 54$ $\quad \cancel{\boxed{a_6 = -\frac{2}{9}}}$

$\boxed{a_1 = \frac{-18}{r^3}}$ $\quad \frac{2}{3} = \frac{-18 r^{6 3}}{r^3}$ $\quad r = -\frac{1}{3}$ $\quad a_n = 54\left(-\frac{1}{3}\right)^{n-1}$ $\quad \boxed{a_6 = -2}$

47) A college student needs make their schedule for next semester. The can pick one of two math classes, one of three science courses and one of four history classes. How many schedules are possible?

$\underset{\text{math}}{\underline{2}} \cdot \underset{\text{science}}{\underline{3}} \cdot \underset{\text{history}}{\underline{4}} = \boxed{24 \text{ possiblities}}$

48) Find the probability of tossing a six-sided die twice and getting a sum of 5.

Sum 5 : 1,4 $\quad$ 2,3 $\quad$ 3,2 $\quad$ 4,1 $\quad P(\text{sum } 5) = \frac{4}{36} = \frac{1}{9} = 11\%$

Total : 36

49) Find the probability of tossing a six-sided die twice and getting a sum that is at least 6.

Sum > 6 : 1,6 $\quad$ 2,5 $\quad$ 3,4 $\quad$ 4,3 $\quad$ 5,2 $\quad$ 6,1 $\quad$ 5,6 $\quad$ 6,5

2,6 $\quad$ 3,5 $\quad$ 4,4 $\quad$ 5,3 $\quad$ 6,2 $\quad$ 6,6

3,6 $\quad$ 4,5 $\quad$ 5,4 $\quad$ 6,3 $\quad P(x>6) = \frac{21}{36} \approx 58\%$

4,6 $\quad$ 5,5 $\quad$ 6,4

$P(x \geq 6) = \frac{26}{36} = 72\%$

50) In how many ways can seven textbooks be lined up on the shelf?

$\underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 7! = 5040$